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DUCTILE FRACTURE PHENOMENON DURING EXTRUSION OF BIMETAL RODS[1]

ZJAWISKO PĘKANIA PODCZAS WYCISKANIA PRĘTÓW BIMETALOWYCH

The analysis of plastic flow in the extrusion of bi-metalic rods from the point of view of identification of parameters influencing fracture phenomena of core in core – sleeve system. The concept of FEM Marc Mode has been presented with results which may indicate the possibility of assessment of existing fracture criteria for composite material. Programme of experimental work dealing with extrusion of mono material end co – extrusion of various kind of composed material have been curried out with use of flat dies leading to different extrusion ratios. It has been shown, that existing analyzed criteria for mono materials (uniform materials) are not satisfactory for heterogeneous composite materials.

Keywords: fracture, extrusion, composites

Przedmiotem pracy jest analiza plastycznego płynięcia podczas wyciskania prętów bimetalowych z punktu widzenia identyfikacji czynników wpływających na zjawisko pękania. Przedstawiono koncepcję modelu wyciskania wlewków o układzie rdzeń – powłoka oraz wyniki obliczeń wskazujących na możliwość oceny istniejących kryteriów pękania w zastosowaniu do materiałów kompozytowych. Przedstawiono analizę znanych kryteriów pękania z punktu widzenia poszukiwania kryterium odpowiedniego dla przewidywania zjawiska pękania w procesie wyciskania bimetalicznych warstwowych materiałów złożonych. Zaprezentowano wyniki obliczeń numerycznych z zastosowaniem komercyjnego programu MSC MARC opartego na metodzie elementów skończonych oraz wyniki przeprowadzonych badań eksperymentalnych wyciskania materiałów monometalicznych i materiałów złożonych z zastosowaniem matryc płaskich prowadzących do dwóch różnych współczynników wydłużenia λ. Zastosowano różne aranżacje geometryczne bimetalicznych wlewków do wyciskania. Wykazano, że kryteria stosowane do materiałów jednorodnych nie dają zadowalających wyników w zastosowaniu do materiałów niejednorodnych, do których należą kompozyty warstwowe.

1. Introduction

The mechanical behaviour of material in metal forming process depends on mode of plastic flow, structural and mechanical effects and factors dealing with conditions of the process. The means of controlling the material structure and determining the influence of material properties on metal flow are basic information needed to describe physical nature of plastic deformation of metallic materials. It is essential to be able to predict mechanics of plastic flow and in consequence controlling the metal forming operation. Among the simplest forms of composite materials are bi-metal rods or wires, which can be obtain in the course of co-extrusion process. Even in this simple case metal flow is very complicated especially because of non-homogeneity of the initial material [1, 2, 3].

The deformation degree results from the size of the deformation zone. The global degree λ_g (the extrusion ratio of composite) is kind of averaging of the deformation degree of the core λ_c (the extrusion ratio of the core) and of the deformation degree of the core λ_s. Their values depend on the features of the materials mainly; for hard materials λ_c or λ_s (the extrusion ratio of the sleeve) is smaller then λ_g whilst for soft material they are greater then λ_g. The volume ratio of the components influences the moment of the beginning of the second stage of extrusion – steady flow. Non – homogeneity of component deformation in comparison with the composite extrusion ratio arises from the plastic non-homogeneity

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of the composite billet (distinct core and sleeve) and the characteristic non- uniform velocity distribution in the initial material. The some material deformed as monomaterial behaves in a different manner, unlike metal simultaneously deformed together with another material. For a sound composite product it is important not to exceed the limit of deformation degree. The following are conditions for sound co-extrusion of non-uniform flow:

$$\lambda_g \neq \lambda_s \neq \lambda_c; \qquad \lambda_s < \lambda_{slimit}; \qquad \lambda_c < \lambda_{climit}.$$

Total equalization of these values ($\lambda_g = \lambda_s = \lambda_c$ for assumed homogeneous flow) is impossible because of technological restrictions and technological requirements. These values are important for evaluation of the actual parameters of simultaneous deformation without defective flow. Considering mechanical behaviour of two different materials deformed together it is necessary to take into account two possibilities in the interface core – sleeve: sticking or sliding phenomena. It depends on the conditions and parameters of the co-extrusion process. Relative thickness of layers of the composite (components volume ratio),flow stress ratio ($\sigma_{p\ \mathrm{hard\ component}}/\sigma_{p\ \mathrm{soft\ component}}$), the combination of flow types of the components and the conditions of contact and interface (core – sleeve) friction and die shape are the main factors influencing plastic flow in extrusion. Analysis of theoretical mechanical behaviour allows to verify the features of material under conditions of compression tests of composite initial material. It allows to define (for considered model of composite material) the most important material constant and evaluate other parameters e.g. factor of friction on contact surface metal – tool, or on the interface between core and sleeve. One of specially criterion for prediction of fracture of core or sleeve in the extrusion process of composite is Avitzur criterion [4], which has been based on assumption that product emerges without defect in the case of proportional flow only. It means that degrees of deformation of core and sleeve are equal. The results presented in [3] show that it is possible to get product without any defects in cases differentiated flow of components of composite.

The forms of the plastic zones in the extrusion of layered composite of sleeve – core system are dependent on many factors and finally may lead to non-proportional flow or defective flow with sleeve or core fracture (Fig. 1).

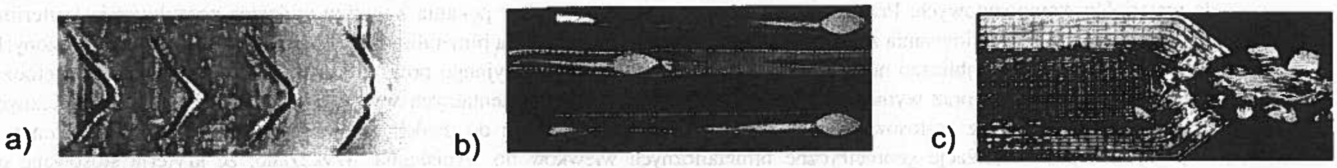


Fig. 1. Fracture phenomena in extrusion process, a) internal fracture of monomaterial, b) fracture of the core of composite, c) fracture of the sleeve

To avoid such negative phenomenon it is important to use the theoretical method for prediction the conditions when fracture may occur. FEM may be applied to model and control metal forming process or predict possible negative phenomena. There are numerous papers concerning such problem [5, 7, 8, 16] but up to now there is no enough good solution taking into account the real behavior of various materials or choice of proper criterion of fracture. There are many criteria of various formulas (e.g. Table 1) and their application in the modeling leads to different results. Functioning of these criteria may be checked in the bulk forming processes for example in the co-extrusion of various metals deformed together. Well known fracture criteria [e.g. 6, 9-18] like: formula according to Brozzo, Cockroft-Latham, Ayada, Freudenthal or Clift include different factors, so indication of the moment of fracture depends of chosen formula. Up to now there is no enough good solution, which could be applied for composite material. Existing formulas for prediction of fracture of bimaterial (e.g. acc. to Avitzur) [4] are rather complicated and based on over simplifying assumptions.

The aim of this work is to select the criteria, which could be applied in prediction of ductile fracture of layered composite materials in the extrusion process.

Some of the criteria commonly used to predict ductile fracture are presented in Table 1. These criteria are based on macro mechanical considerations, in which fracture occurs when a stress function over the effective strain field reaches a critical value.

TABLE 1

Comparison of some ductile fracture criteria

criterion	formula of criterion	σ_H	$\bar{\sigma}$	$\sigma_{\max}$	$\bar{\varepsilon}$	ε_f
Brozzo et.all (1)	$\int_0^{\bar{\varepsilon}_f} \frac{2\sigma_{\max}}{3(\sigma_{\max}-\bar{\sigma})} d\varepsilon = C_1$	-	$\bar{\sigma}$	$\sigma_{\max}$	-	ε_f
Cockroft-Latham (2)	$\int_0^{\bar{\varepsilon}_f} \frac{\sigma_{\max}}{\bar{\sigma}} d\bar{\varepsilon} = C_2$	-	$\bar{\sigma}$	$\sigma_{\max}$	$\bar{\varepsilon}$	ε_f
Ayada (3)	$\int_0^{\bar{\varepsilon}_f} \left\{ \frac{\bar{\sigma}}{\sigma_H} \right\} d\bar{\varepsilon} = C_3$	σ_H	$\bar{\sigma}$	-	$\bar{\varepsilon}$	ε_f
Freudenthal (4)	$\int_0^{\bar{\varepsilon}_f} \sigma_H d\bar{\varepsilon} = C_4$	σ_H	-	-	$\bar{\varepsilon}$	ε_f
Clift et.all (5)	$\int_0^{\bar{\varepsilon}_f} \bar{\sigma} d\bar{\varepsilon} = C_5$	-	$\bar{\sigma}$	-	$\bar{\varepsilon}$	ε_f

σ_H – hydrostatic stress, $\sigma_{\max}$ – maximum stress, $\bar{\sigma}$ – equivalent stress, $\bar{\varepsilon}$ – equivalent strain $\bar{\varepsilon}_f$ – equivalent strain of fracture, C – constant

The object of this study is to examine some known criteria in order to predict ductile fracture during co-extrusion process and to profit such results to determine the forming conditions that enable plastic deformation of a bimetal rod composed of two dissimilar materials to take place without core fracture.

2. Finite element analysis – model description

FEM modeling of extrusion process of composite material and monomaterial for different geometrical arrangement has been done. The axisymmetric finite element analysis was performed using MSC MARC commercial programme for direct extrusion. In general, hot working and cold working are used for Coulomb friction equation and shear friction equation, respectively. In this study Coulomb friction equation is used for finite element analysis. The finite element analysis was performed with friction factor between die and workpiece (m_1 = 0,1) and between core and sleeve (m_2 = 0,25). The calculated extrusion forces are compared with experimental data. The die and ram were assumed as rigid body in finite element simulation. FEM model was verified by comparison with experimental results. The test of uniaxial tension for materials that was used in experiments was made in order to obtain flow characteristics of materials for modeling (lead, aluminum), (Fig. 2). Some of the mechanical properties of materials used in experiment are presented in Table 2.

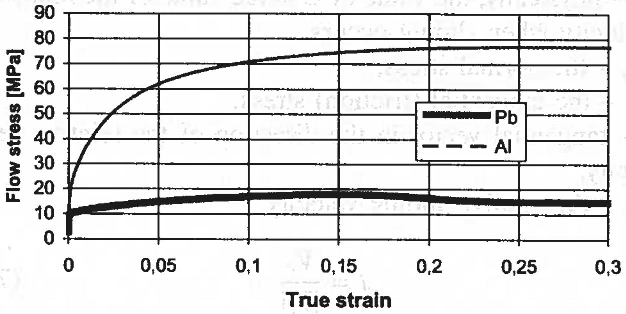


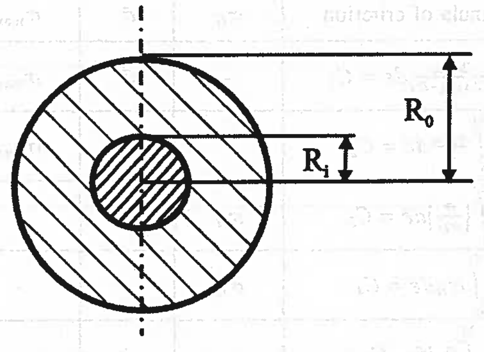
Fig. 2. Flow stress-thrue strain curves for aluminium and lead

TABLE 2

Some mechanical properties of materials used in the investigations

material	Chemical composition [%]	Yield stress [MPa]	Young modulus [MPa]	Poisson's ratio	Brinell Hardness HB
Lead	99,98 Pb; 0,05 Ag; 0,05As; 0,01Sb; 0,05Sn; 0,05Cu; 0,05Fe; 0,05Zn; 1,0Bi	5	14000	0,43	4,4
Aluminum 1050	99,5 Al; 0,4Fe; 0,3Si; 0,05Cu; 0,07Zn; 0,05Ti	26	70000	0,33	22,4

The analysis results of FEM modeling were compared with experimental data. Different arrangement of components in the billet was used in the experimental work (Fig. 3).



R_i/R_0	0,1	0,28	0,3	0,4	0,55	0,6

Fig. 3. Geometrical arrangement of metal composite billets

Boundary conditions

Friction models in metal forming analysis:

- *Coulomb friction equation*

$$\sigma_t \leqslant -\mu\sigma_n \frac{2}{\pi} \arctan\left(\frac{V_r}{C}\right) \cdot t, \qquad (6)$$

where:

C – physically, the value of C is the value of the relative velocity when sliding occurs,
σ_n – the normal stress,
σ_t – the tangential (friction) stress,
t – tangential vector in the direction of the relative velocity,
V_r – the relative sliding velocity.

$$t = \frac{V_r}{|V_r|}. \qquad (7)$$

- *Shear friction model*

The shear based model states that the frictional stress is a fraction of the equivalent stress in the material:

$$\sigma_t \leqslant -m\frac{\bar{\sigma}}{\sqrt{3}} \cdot \frac{2}{\pi} \arctan\left(\frac{V_r}{C}\right) \cdot t. \qquad (8)$$

This model is available for all elements using the distributed load approach [20].

Constitutive equations

Flow – stress functions may be divided into a number of groups differing in the way of taking into account the parameters describing the conditions and the course of deformation and the initial state of material. Functions $\sigma_p = f(\varphi)$ take into consideration the value of the current strain (φ), and in some forms the stress (σ_0) or strain (φ_0) of the initial state of the material. These functions are used in simulation programmes for cold – working processes. For processes characterized with a deformation course that is close to proportional and monotonic over the whole volume of the formed material, these functions fulfill their task in a satisfactory way [19]. Some examples of constitutive equations are as follow:

- *Hollomon power – law equation,*

$$\sigma_p = K\varphi^n, \qquad (9)$$

From the point of view of simple form and correct description of phenomenon of strengthening of many metals it is very often used in the modeling.

- *Krupkowski – Swift equation,*

$$\sigma_p = K(\varphi_0 + \varphi)^n, \qquad (10)$$

It is applicable for materials, which exert preliminary strengthening.

- *Grosman equation,*

$$\sigma_p = K\varphi^n \exp(n_1\varphi)\,\dot{\varphi}^m. \qquad (11)$$

The following symbols are used in the above equations: σ_p – the flow stress, n, n_1 – coefficient of strain hardening, m – strain rate sensitivity, φ_0 – pre-strain, $\dot{\varphi}$ – strain rate, K – experimentally determined parameter.

Idealization

Applied FEM analysis includes main assumption:

- Mechanical characteristic of the Al, Pb materials:

$$\sigma_p = K\varphi^n,$$

- contact conditions – Coulomb friction model
- Huber-Misses yield criterion
- Prandtl-Reuss flow rule

$$d\varphi^p = d\bar{\varphi}^p \frac{\partial \bar{\sigma}}{\partial \sigma}, \qquad (12)$$

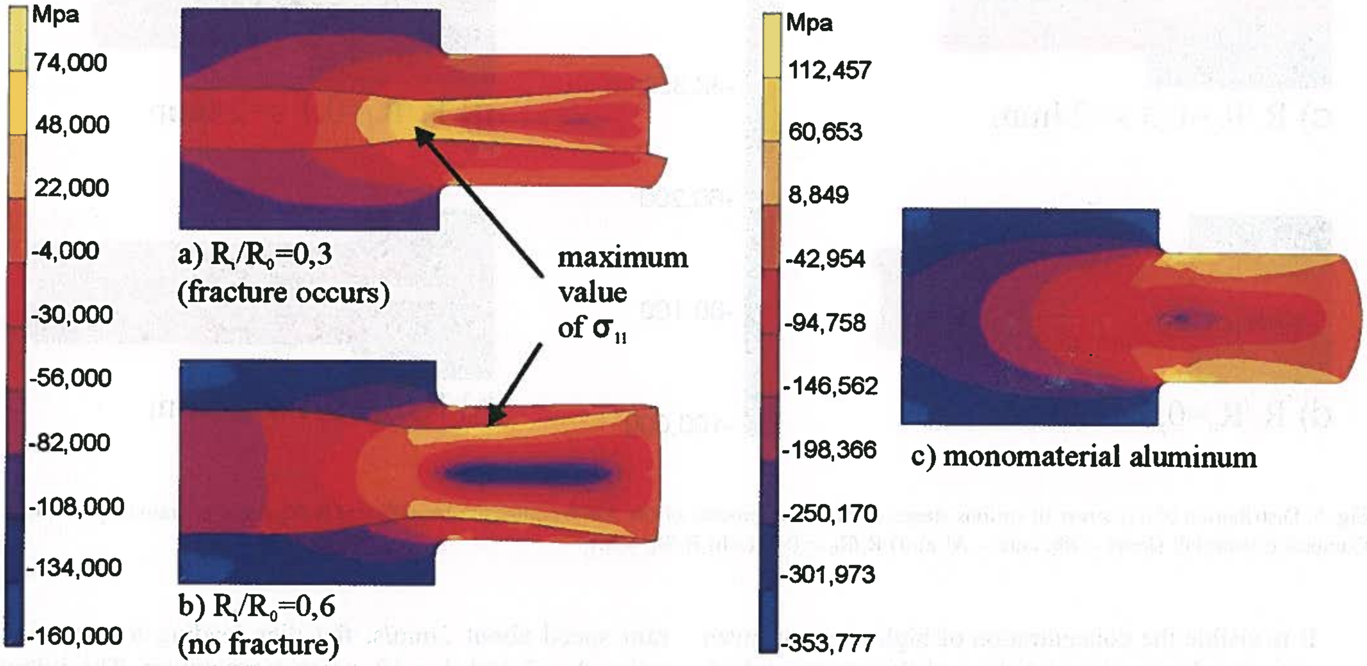
where $d\varphi^p$ and $\bar{\sigma}$ are equivalent plastic strain increment and equivalent stress, respectively.

- number of elements-5000
- automatic re-meshing
- axisymmetric model.

Selection of constitutive equations was made basing on the result of publication and conditions of experimental work taking into consideration parameters of the extrusion process. The equation that excludes influence of temperature can be used because of cold deformation. Influence of strain rate was excluded and it can have a certain value in case of lead deforming like was presented in [21]. Because of the axisymmetric scheme of deformation, this process can be idealized to an axisymmetric model.

3. FEM modeling -result examples

Stress and strain conditions have conclusive sense in the analysis of ductile fracture phenomena. Stress distribution indicates important differences in localization of maximal tension stresses in the cases of different value of R_i/R_0 ratio in comparison with monometallic one (Fig. 4).



a) R_i/R_0=0,3 (fracture occurs)

b) R_i/R_0=0,6 (no fracture)

c) monomaterial aluminum

Fig. 4. Stress σ_{11} distribution on the longitudinal section of the billet and of extrudate of sleeve (Pb) – core (Al) system

Distribution of σ_{11} stress (stress in the axis of extrusion) for constant ram displacement and for two cases of components volume ratio in the billet has been done. It is shown that an examples of deformation in extrusion when different components volume ratios (indicated by R_i/R_0 on the longitudinal sections) cause such stress σ_{11} distribution which may produce core fracture (e.g. in Fig. 4a) or avoid fracture (e.g. in Fig. 4b). Different mechanical behaviour of Al as monomaterial in comparison with its behaviour as a component (core) in the Al/Pb composite is presented in Fig. 4c. Fig. 5 shows evolution of σ_{11} stress in particular stages of extrusion process.

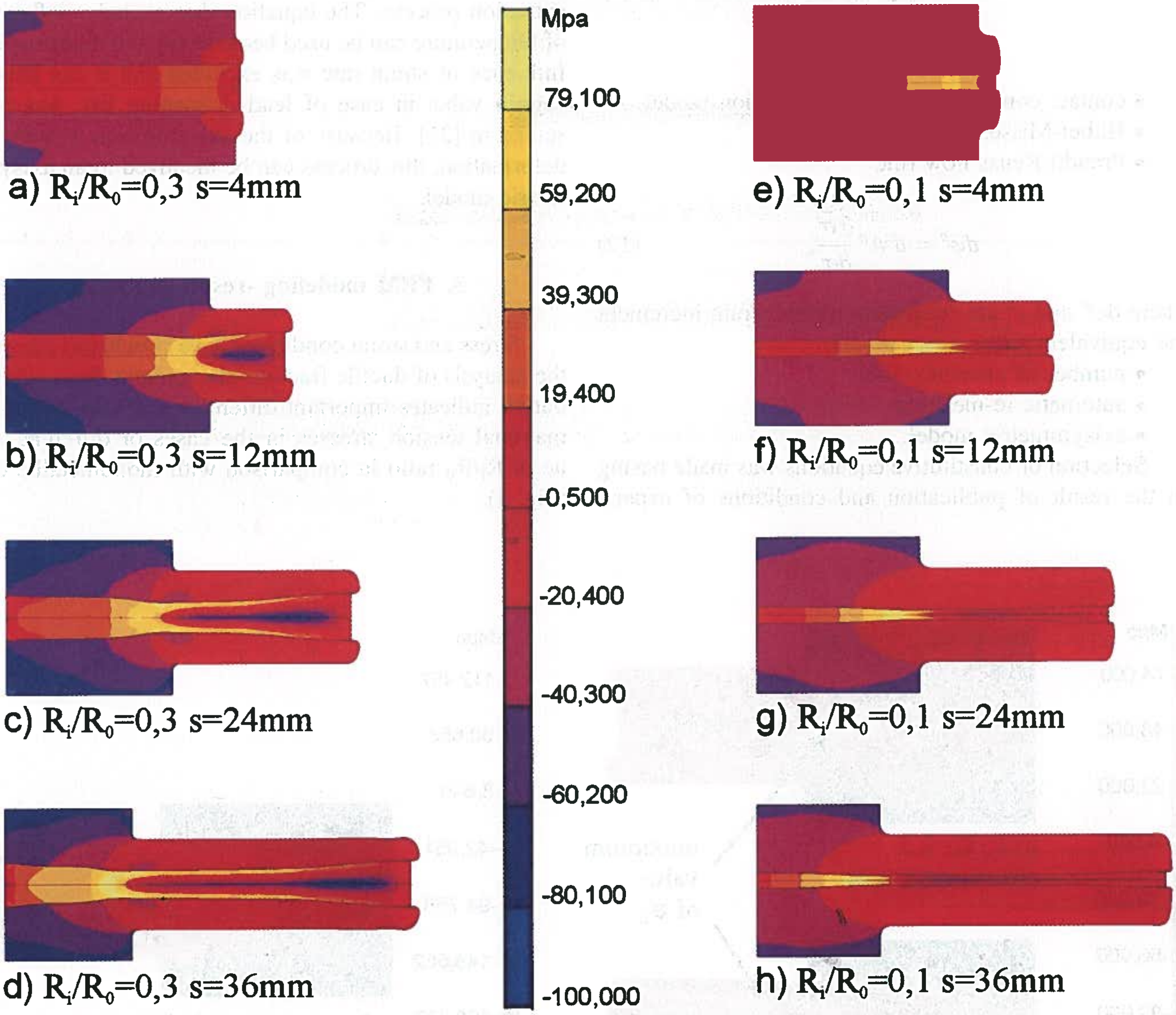


Fig. 5. Distribution of σ_{11} stress in various stages of extrusion process of the Al/Pb composite for different R_i/R_0 ratio, s – ram displacement. Composite material: sleeve – Pb; core – Al a)-d) R_i/R_0 = 0,3; e)-h) R_i/R_0 = 0,1

It is visible the concentration of high stress at given component volume ratio and stage of the process which may cause future fracture in this region.

4. Experimental results and discussion

Experimental investigations have been made to identify a ductile fracture phenomenon in composite material extrusion process and to define a moment of core fracture for analyzing such case in order to compare with theoretical results (using chosen fracture criterion).

Forward extrusion of layered composite rods were carried out using set of tools in a hydraulic press. Conditions of the process were as follow: forward extrusion, ram speed about 2mm/s, flat dies leading to extrusion ratios λ = 3 and λ = 12, room temperature. The initial material (billet) was composed of aluminum (the core layer) and lead (the sleeve layer).

The stress-strain relationships of the materials have been obtained by a tension test. Figure 2 shows the true stress-true strain curves for these materials. The flow stresses for lead and aluminum can be expressed approximately by the Hollomon power law and are given by: $\sigma = 118*\varepsilon^{0,254}$ for aluminum, $\sigma = 24*\varepsilon^{0,218}$ for lead.

Other process parameters are presented in Table 3. The values of parameters for modeling were chosen by matching procedure of comparison FEM and real results in experimental work.

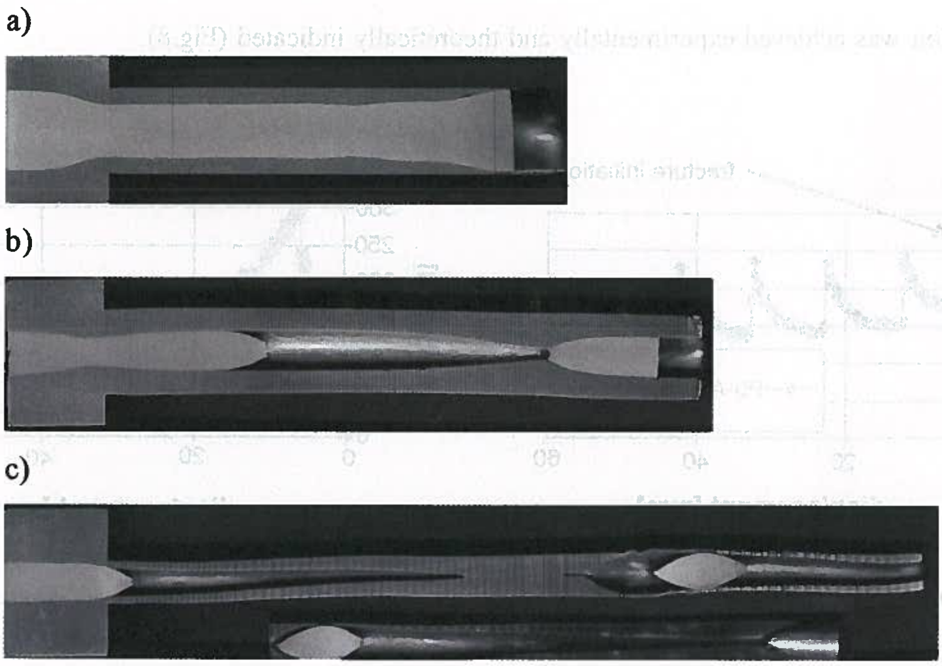


Fig. 6. Longitudinal section of the composed billet and extrudate; a) $R_i/R_0 = 0{,}55$, $\lambda = 3$; b) $R_i/R_0 = 0{,}28$, $\lambda = 3$; c) $R_i/R_0 = 0{,}28$, $\lambda = 12$. b), c) Fracture of the inner material (core) with periodic cracking

TABLE 3
Process parameters used in experimental work and modeling procedure

Billet length	72 mm
Billet diameter	36 mm
Temperature	20°C
Extrusion ratio	3 and 12
Length of the bearing area	2 mm
Die semi- angle	90°
Friction coefficient between core and sleeve	0,25
Friction coefficient between core and tool (punch)	0,1
Friction coefficient between sleeve and tool	0,1

Products of experimental extrusion process have been cut along the symmetry axis in purpose to observe the obtaining effect (Fig. 6a-c).

In order to use the fracture criterion it is necessary to know the constant value (C) and fracture strain (ε_f). Fracture strain was obtained in tension test (for aluminum $\varepsilon_f = 1{,}2$) (Fig. 7). Values of stresses and strains were received from computer simulation of the process. A critical value was calculated for each criterion.

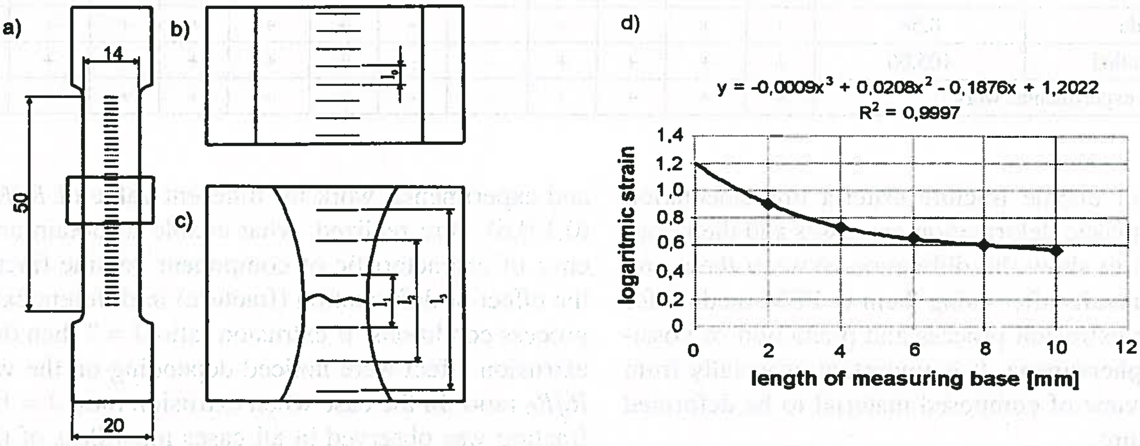


Fig. 7. The method of determination of strain of fracture, a) view of sample, b) scale before tension test, c) scale after tension test, d) assignment of value of strain of fracture for aluminium

Fracture initiation was achieved experimentally and theoretically indicated (Fig.8).

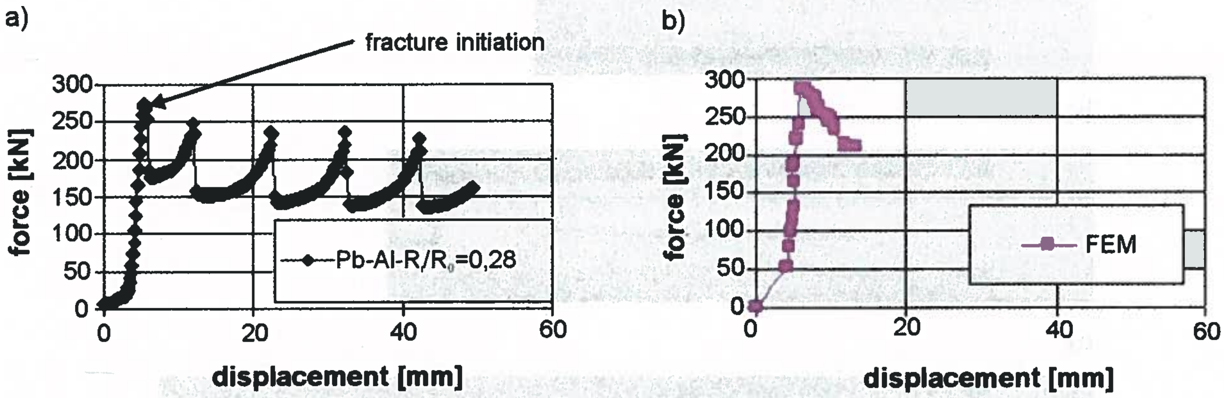


Fig. 8. a) Initiation of fracture in experimental work (Pb/Al; R_i/R_0 = 0,28), b) Relationship between force and displacement acc. to FEM (Pb/Al; R_i/R_0 = 0,28)

Numerical simulation for other cases of geometrical arrangement of the billets (various R_i/R_0 ratios) for two different values of extrusion ratio λ (3, 12) was realized. Experimental and theoretical results have been compared in Table 4.

Constant C value defined acc. to given criterion was obtained from calculations using criterion formula at fracture (obtained experimentally for every geometrical case of the composed material) and compared with critical value C. Matching of functioning of a criterion was succeeded with the procedure of calculation of critical value for one case of geometrical arrangement of initial material (R_i/R_0 = 0,28). Other cases were analyzed to verify each criterion.

TABLE 4

Prediction of fracture using criteria equations and experimental data; "+" fracture occurs, "-" no fracture

Criterion	critical value C	$\lambda = 3$						$\lambda = 12$					
		R_i/R_0						R_i/R_0					
		0,1	0,28	0,3	0,4	0,55	0,6	0,1	0,28	0,3	0,4	0,55	0,6
Cockroft and Latham	0,29	+	+	+	+	-	-	+	+	+	+	+	+
Clift et al.	109,62	+	+	+	+	-	-	+	+	+	+	+	+
Brozzo et al.	5552	+	+	-	-	-	-	-	-	-	+	+	+
Ayada	0,58	+	+	+	+	-	-	+	+	+	+	+	+
Freudenthal	105,06	+	+	+	+	-	-	+	+	+	+	+	+
experimental work		+	+	+	-	-	-	+	+	+	+	+	+

Review of ductile fracture criteria implementation into specific plastic deformation processes and their conditions let us to show the difference between them and between the results after using them in FEM method for modeling the extrusion process and prediction of possible fracture phenomena. It is important especially from the point of view of composed material to be deformed without fracture.

FEM data was used to calculate C value for particular ductile fracture criterion. Table 4 shows comparison of experimental and theoretical data. Calculation and experimental work for different value of R_i/R_0 ratio (0,1-0,6) were realized, what enable to obtain an influence of characteristic of component volume fraction on the effect of deformation (fracture) in different extrusion process conditions. If extrusion ratio $\lambda = 3$ then different extrusion effect were noticed depending on the value of R_i/R_0 ratio. In the case when extrusion ratio $\lambda = 12$, core fracture was observed in all cases regardless of the size of R_i/R_0 ratio. For extrusion ratio $\lambda = 3$ and R_i/R_0 ratio smaller then 0,4 ductile fracture occurs in the experiment. It was confirmed by calculation results when four of examined criteria were used. For R_i/R_0 greater than 0,4 calculation results are the same as results of experiment work for all cases. For extrusion ratio $\lambda = 12$ and $R_i/R_0 < 0,4$ there is not good convergence between calculation and experimental work for one criteria. The others indicate properly that core fracture occurs. Comparison between experimental and theoretical results using particular criterion gives some differences but there is no criterion that indicates core fracture in all experimental cases properly (Table 4).

Stress distribution indicates important differences in localization of maximum tensile stresses in the cases of different value of R_i/R_0 in comparison with monometallic one (Fig. 4). Figure 5 enables to observe evolution of stress σ_{11} distribution in the extrusion process stages for constant ram displacement for some cases of geometrical arrangement of billet with the great concentration of maximum value σ_{11} in symmetry axis area. For lower value of R_i/R_0 ratio (Fig.5.e-h) σ_{11} stress reaches higher value and appears earlier in comparison with the case of higher value of R_i/R_0 ratio (Fig. 5.a-d).

5. Conclusions

Basing on the analysis of the modeling of composite flow the following conclusions can be drawn:

- The presented FEM model reflects the flow and character of deformation of the composite of layered structure (core – sleeve system) in comparison with real phenomenon in co – extrusion of the composed material and extrusion of monomaterial.
- Modeling the distribution of stress for extrusion of layered composite (of various components volume ratios) gives the possibility to indicate the danger regions in the composite from the point of view of concentration of high stress which may cause possible fracture. It is very important factor to predict occurrence of ductile fracture of core or sleeve. Where the R_i/R_0 ratio is lower than the probability of fracture occurrence is higher.
- Experimentally and theoretically described relationships between force of extrusion and ram displacement may suggest the fracture initiation.
- The prediction of fracture of the core of layered composite acc. to various criteria is not enough good from the point of view of coincidence with results of real process. They indicate the core fracture but in different stages of the process and different geometrical parameters of composed billet. Presented results are the first stage of research work on method of prediction of fracture during extrusion of composite material of core – sleeve system with assumed non-uniformity of such material.

REFERENCES

[1] H. Tokuno, K. Ikeda, J. Mat. Proc. Techn. **26**, 323-335 (1991).

[2] R. Śliwa, J. Zasadziński, Non-homogeneous plastic deformation of metal composites, Proc. EURO MAT'91, Cambrige U.K. p. 158 June 1991.

[3] R. Śliwa, Odkształcenie plastyczne kompozytów metalowych podczas wyciskania, Zeszyty Naukowe PRz, 32, (1992).

[4] B. Avitzur, R. Wu, S. Talbert, Y. T. Chou, Criterion of the prevention of core fracture during extrusion of bimetal roods. (1982).

[5] B. Avitzur, Handbook of Metal-Forming Processes, Wiley, New York, 1983.

[6] A. Atkins, Fracture in forming: Proc. Int. Conf. On Adv. in Mat. and Proc. Tech., Dublin, AMPT (1993).

[7] G. Ryzińska, I. Nowotyńska, R. Śliwa, Niejednorodność plastycznego odkształcenia warstwowych materiałów złożonych w próbie ściskania, Rudy i Metale Nieżelazne, **1**, (2004).

[8] R. Sowerby, N. Chandrasekaran, The upsetting and free surface ductility of some commercial steels, J. Appl. Metalworking, **3**, 257-263 (1984).

[9] V. Vujovic, A. Shabaik, A new workability criterion for ductile metals, J. Eng. Mat. Tech. **108**, 245-249 (1986).

[10] A. Shabaik, V. Vujovic, On the stress formability index, International Conference on Mechanical Design and Production, Cairo, (1988).

[11] F. McClintock, A criterion for ductile fracture by the growth of holes, J. Appl. Mech. ASME, **90**, 363-371 (1968).

[12] M. Oyanne, T. Sato, K. Okimoto, S. Shima, Criteria for ductile fracture and their applications, J. Mech. Work. Tech. **4**, 65-81 (1980).

[13] M. Cockroft, D. Latham, Ductility and workability of metals, J. Int. Met. **96**, 33-39 (1968).

[14] P. Brozzo, B. DeLuca, R. Rendina, A new method for the prediction of formability limits in metal sheets, Sheet metal Forming and Formability, Proc. of the 7-th Conference of the International Deep Drawing Research Group, (1972).

[15] D. Norris, J. Reaugh, B. Moran, D. Quinnonesa, A plastic strain mean- stress criterion for ductile fracture, J. Eng Met. Tech, **100**, 279-286 (1978).

[16] A. Atkins, Possible explanation for the unexpected departure in hydrostatic tension – fracture strain relations, Met. Sci, Feb. 81 (1981).

[17] H. Moritoki, The criterion for central bursting and its occurrence in drawing and extrusion under plane strain, Int. J. Plast. **7**, 713 (1991).

[18] H. Moritoki, Free surface ductility in upsetting, Int. J. Plast. **9**, 507-523 (1993).

[19] F. Grosman, Flow-stress functions for the computer simulation of metal forming, J. Mat. Proc. Tech. **106**, 45-48 (2000).

[20] MSC MARC User's Guide, MSC Software Corporation 2004.

[21] Z. Pater, Ołów jako materiał modelowy do symulacji procesów obróbki plastycznej na gorąco, Obróbka Plastyczna 4, 41-48 (2003).

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