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ON THE EVALUATION OF MECHANICAL PROPERTIES FROM THE AXISYMMETRIC TENSILE TEST

WYZNACZANIE WŁAŚCIWOŚCI MECHANICZNYCH Z PRÓBY ROZCIĄGANIA

This paper deals with analytical modelling of the classical tensile test which is still considered as one of the main experimental procedures to determine the flow curve of elasto-plastic materials. Accurate numerical simulation of the process allowed us to recognise the errors introduced by the well-known classical formulae which are used to correct the experimental data under the stage of neck formation. Modifications to the analytical models make it possible to eliminate some of the questionable classical assumptions and to construct a new formula. Comparison of the new results with the well-known classical ones indicates significant improvement.

Keywords: tensile test, material properties, flow curve, yield stress, analytical analysis, FEM simulation, Bridgman formula, Davidenkov-Spiridonova formula

Praca dotyczy analitycznego modelowania klasycznej próby rozciągania, która jest wciąż uważana za jedną z podstawowych procedur eksperymentalnych do wyznaczania krzywej umocnienia materiałów sprężysto-plastycznych. Dokładna symulacja numeryczna procesu pozwoliła rozpoznać błędy wynikające z zastosowania powszechnie znanych wzorów klasycznych podczas korygowania danych eksperymentalnych od momentu pojawienia się szyjki. Wprowadzenie modyfikacji do modelu analitycznego umożliwiło wyeliminowanie pewnych wątpliwych uproszczeń klasycznych i skonstruowanie nowego wzoru na podstawie deformacyjnej teorii plastyczności. Porównanie rezultatów otrzymanych w wyniku zastosowania nowego wzoru z powszechnie znanymi wzorami klasycznymi, wskazuje na osiągnięcie znaczącego usprawnienia.

1. Introduction

Tensile testing with axisymmetric specimens is a simple and an important standard engineering procedure which is effective to determine elastic and plastic properties of materials. Up to the stage of neck formation, it is characterised by a homogeneous 1-D stress state that provides a unique opportunity to evaluate the properties of the specimen material. However, when the neck appears in the sample, the stress state becomes essentially 3-D and the neck shape has to be taken into account in the yield stress evaluation. An analysis of possible stress distributions in the neck of an axisymmetric sample under tensile test and respective formulae for the yield stress from the moment of neck creation can be considered as classic results obtained by Bridgman, Siebel, Davidenkov-Spiridonova, and slightly later by Szczepinski. Their formulae are presented in numerous textbooks devoted to mechanical engineering [11, 14, 23] and theory of plasticity [9, 12].

Bridgman and Davidenkov-Spiridonova derived their formulae in frames of the deformation theory of plasticity in Euler's coordinates under Huber-Mises or Tresca yield criteria. Accurate mathematical derivation of the Bridgman formula in the frame of the plastic flow theory can be found in Hill's monograph [9]. All the authors additionally employed a set of simplifying assumptions which assumed to be valid in a neighbourhood of the minimum cross-section. Namely, they a) neglected elastic properties at the stage of the neck creation (and respectively postulated material incompressibility within the plastic region), b) proposed to accept the hypothesis that the circumferential stress is equal to the radial stress, c) assumed that the yield stress, k, is a constant value at every time increment (but changing in time) and finally d) used different formulae describing the radius of curvature of the longitudinal stress trajectory, $\rho = \rho(r)$, see Fig. 1.

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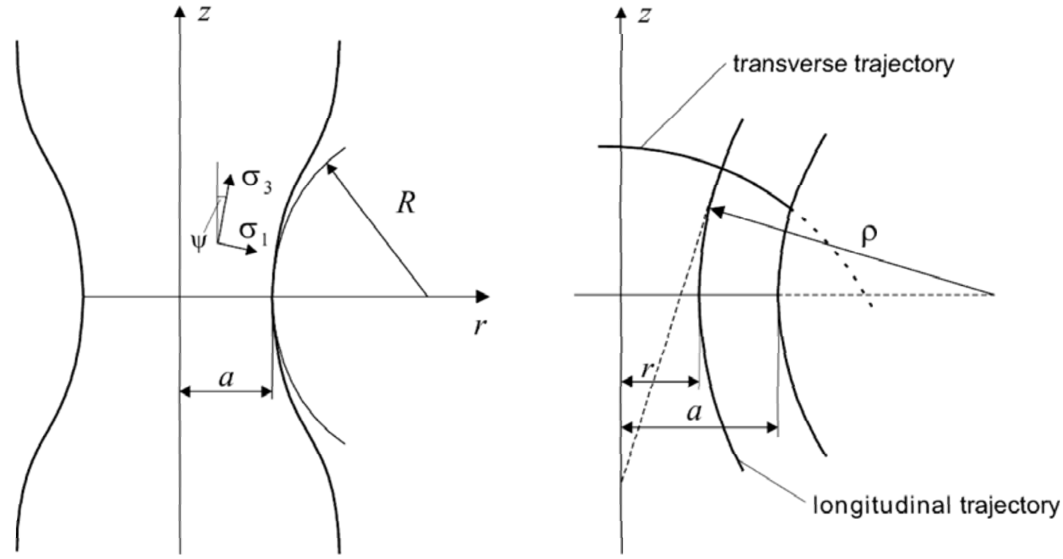

Fig. 1. Neck geometry of a tensile specimen (a); main stress trajectories in the meridian plane (b)

These assumptions allowed them to derive the following correction formulae for the normalised axial stress in the minimum cross section:

$$\begin{aligned} &\text{Bridgman: } \frac{\bar{\sigma}_z}{k} = \left(1 + \frac{2R}{a}\right)\ln\left(1 + \frac{a}{2R}\right), \\ &\text{Davidenkov – Spiridonova}: \frac{\bar{\sigma}_z}{k} = 1 + \frac{a}{4R}, \\ &\text{Szczepinski: } \frac{\bar{\sigma}_z}{k} = \frac{2R}{a}\left[\exp\left(\frac{a}{2R}\right) - 1\right] \end{aligned} \qquad (1)$$

where R is the external neck radius at point $r = a$ and $z = 0$ (see Fig. 1a). Let us note that the same result, even in a more general form in comparison with that obtained by Davidenkov-Spiridonova, has been earlier derived by Siebel [22]:

$$\frac{\bar{\sigma}_z}{k} = 1 + \frac{a}{(n+3)R}. \qquad (2)$$

It is clear that the Davidenkov-Spiridonova solution coincides with Siebel's one if $n = 1$. However, Siebel published his paper in German in an unfortunate time and as a result it is not well-known. In addition, Siebel did not provide enough information to choose the value of parameter n and finally came to the same assumption as Davidenkov-Spiridonova, i.e. $n = 1$ (see [22]).

Let us remind that also Szczepinski formula (13) was derived under the classical simplifying assumptions a) – d) together with a simple engineering balance law (see [23]) instead of exchanging accurate equilibrium equations. This result can be hardly found in engineering literature. Finally, it is important to underline that all the formulae from (1) lead to the same main asymptotic approximation for small ratio a/R with accuracy of the order (a/R^2).

Short time after publishing the classical formulae, many attempts have been made to verify the formulae utilising experimental data (see for example [15]). Since it is impossible to directly determine the yield stress from experiments, most of the authors compared the real stress instead identifying them with (1). It looked like that the Bridgman formula revealed a constant accuracy during the whole tensile test, while outcomes received from Davidenkov-Spiridonova formula were not so accurate for values of a/R greater than 0.6. On this basis, the conclusion was drawn that application of Bridgman formula provides better results, which obviously must have not always been true and required further investigation.

Just to support this conclusion, let us remind here that Bridgman, Siebel, Davidenkov-Spiridonova and Szczepinski utilised some simplifying assumptions during the derivation of the formulae for the yield stress whose accuracies were not well enough estimated. Since Finite Element Method (FEM) started to be the main engineering instrument, numerous papers (for example [2, 16, 17, 21, 26]) were published with attempt to verify the assumptions numerically. Two of them, equality of the radial and circumferential stresses and constancy of the yield stress along the minimum cross-section were questioned. Simultaneously, the assumptions were analysed also theoretically and recognised to be not enough accurate in many textbooks [9, 12, 23]. Outcomes of our own semi-analytical and numerical results concerning accuracy of the assumption are presented in the second section.

Besides verification of any particular simplifying assumption, it is equally important to check the correctness of the classical formulae (1) as every simplifying assumption can lead to mutually complementary results in such a sense that the total error may be less than (maximal/minimal) error of each individual assumption. Such estimations were reported in [1-3, 20] where the authors utilised numerical simulation. They claim that a considerable discrepancy exists between results obtained from the numerical simulation and those received from classical formulae.

Apart of the earliest work mentioned above, the first statement that Davidenkov-Spiridonova formula provides better result in comparison with the Bridgman's one has been given by Jasienski [10], who quoted results from C. Rossarda, P. Blama and F. A. Hodierne'a. Our own results (see section 2) also support this hypothesis. Despite this fact, Bridgman's formula is the most often applied in practice until now to evaluate the flow curve from the moment of the neck formation [13, 18, 19, 25, 26]. Moreover, some authors made use of the classical formulae in cases when they have not been sufficiently justified (for instance to create damage accumulation models of plastic materials [18, 19] or for samples with cross-sections different from the circle).

Difficulties in the profile measurement of the deformed sample and imperfectness in determination of the curvature radius of the neck contour R were noticed even by Bridgman as well as many others [4, 10, 13, 23, 25, 26]. This error influences the value of the yield stress determined from any of the classic formula. Attempts have been made to link other easy measurable values during the experiment [4, 10] with the formulae (1). However, outcomes did not bring an expected effect. Fortunately, recent measurements given by modern sensitive extensometers and laser scanners overcome this problem.

Short time after the publication of the classical formulae, several unsuccessful attempts have been made to improve them [2, 10, 16, 17, 21, 26]. During the last years, determination of the yield stress from axisymmetric sample under tensile test from the moment of neck creation becomes again of great interest. Thus, Ling applied the method of weight average to evaluate the flow curve in paper [13]. However, this method requires the knowledge on the highest and lowest limitations of the flow curve and thus, causes difficulties during its application in practice. It is worth mentioning different approaches to determine material properties. First of all it is the inverse analysis method [5, 24]. This method allows to reconstruct the properties even from more complicated processes, but a disadvantage of this method is time consuming computations and limited accuracy of coefficients reconstruction (an untypical flow curve cannot be, in fact, recognisable at all). An additional weakness of the method is unavailability for industrial engineers possessing equipment for materials testing.

All the aforementioned arguments prove the necessity to revise the classic formulae for the yield stress evaluation. In fact, with modern computers and software allowing computation accuracy at level of percent fraction and measurements realised with sensitive extensometers and lasers on the same level, nobody should accept information on material properties evaluated by the classic formulae without knowing the accuracy.

In the next sections, our recent results concerning evaluation of the mechanical properties from the tensile test are discussed. In particular, we estimate the accuracy of the most questionable assumptions and derive a more accurate formula for the yield stress than the classical ones. Comparison of classical and the more adequate formula obtained has been done with accurate numerical simulations.

2. Evaluation of a new formula

In paper [7], three of the four questionable assumptions b), c) and d) were verified by means of numerical simulation. The first assumption, a), was deliberately omitted as elastic strains play any visible role only at an initial stage of the neck formation and that is why we also believe that its influence on the final results should be marginal. An accurate numerical model has been prepared to simulate errors resulted from the application of a particular simplification were estimated on the basis of theoretical analysis and numerical simulations. As expected simplifying assumptions used to derive the classical formulae cannot be fully acceptable. It turned out that the highest error exhibits the assumption of equality of the radial and circumferential stresses. The smallest error is generated by the utilised formulae for the curvature radius of the longitudinal stress trajectory. Surprisingly, the simplification of constant yield stress along the minimum cross-section occurred to be quite accurate.

In [6], we additionally checked the formula for the logarithmic plastic strain widely used instead of strain intensity (for example [1]). It provides even better accuracy then one can expect (0.6%).

Our first attempt to improve the classical formulae has been made in [7]. Together with estimation of the aforementioned error caused by application of specific formulae for the curvature of the longitudinal stress trajectory, we derived a new formula:

$$\frac{\bar{\sigma}_z}{k} = 1 + \frac{a}{4R} + \frac{a(1-\beta)\alpha}{4R(4-\alpha)}, \tag{3}$$

where conditions for parameters α and β were determined assuming a monotonic behaviour of the curvature radius distribution in the minimum cross-section. Unfortunately, it turned out that it is not possible to find a unique set of suitable parameters for all possible loads. This effectively means that the parameters should depend on a, R and probably some other measured values. However, in spite of this fact, we selected a parameter set as $\alpha = 0.5$ and $\beta = 0.5$, which gave a better approximation than any classical formula for all material laws tested in [7].

On the other hand, formula (3) is identical to Siebel one (2) and allows to find up to now unknown value of the parameter n as a function of these new parameters α, β : $n = [4(4 - \alpha)]/[4 - \alpha + \alpha(1 - \beta)] - 3$. In case $\alpha = \beta = 0.5$ one gets $n = 11/15$ and thus, $n = 1$ looks not to be the best assumption.

In the same paper [7], we tried to evaluate a new formula using the deformation theory of plasticity under Lagrange's coordinate approach. We assumed incompressibility of the material in the minimal cross section, but none of the simplifying assumptions were additionally applied. We have used intensively asymptotic methods with some assumption on coefficients appearing in computations. However, the new formula for small deformation was better than classical ones but behaved worst than the empirical suggestions (3).

In [8], we have made the similar analysis under Euler's approach. The two new non-empirical formulae derived in [7, 8] give better results in comparison with Szczepinski formula (the best from the classical ones, as it follows from our own analysis, compare Figs. 2). The formula derived under Euler's approach has a wider range of applicability than that derived for the Lagrange approach and reveals for small deformation better accuracy in comparison with all other suggestions, (1)-(3). Unfortunately, both of these new non-empirical formulae are only valid for small plastic deformations.

Our present approach is aimed to expand the applicability of the latter formula obtained by Euler approach in [8]. For this reason, coefficients appeared in the formula are determined in a slightly different way. Namely, analysing semi-analytically the stress state in the neighbourhood of the symmetrical part of the sample (where the highest plastic deformation occurs), we derived a differential equation describing the trajectory of the main stress. The neck contour was approximated then with a polynomial of fourth degree. Coefficients of the polynomial were selected in such a manner to satisfy asymptotically all boundary conditions on the neck surface and symmetry axis. Neglecting technical details, the new formula finally takes the form:

$$\frac{\bar{\sigma}_z}{\bar{k}} = 1 - \frac{5\Lambda}{7(1+5\Lambda)} - \frac{2(1-6\Lambda)}{7(1+5\Lambda)} + \frac{2}{7} + \frac{30\Lambda(8\Lambda-\delta-5\delta\Lambda)}{49\delta(1+5\Lambda)^2} + \frac{3(8\Lambda-\delta-5\delta\Lambda)}{7\delta(1+5\Lambda)}\left(\frac{2(1-6\Lambda)}{7(1+5\Lambda)} - \frac{2}{7} - \frac{30\Lambda(8\Lambda-\delta-5\delta\Lambda)}{49\delta(1+5\Lambda)^2}\right)\ln\left|1 + \frac{7\delta(1+5\Lambda)}{3(8\Lambda-\delta-5\delta\Lambda)}\right|, \quad (4)$$

where $\delta = a/R, \Lambda = 1 - a_0/a$.

It is worth noticing here that in two specific cases when $\Lambda \to 0$ with a fixed parameter δ and $\delta \to 0$ with a fixed parameter $\Lambda < 0$ formula (4) leads to the same results $\overline{k} = \overline{\sigma}_z$, which is in agreement with expectations.

3. Comparison of the new formula with numerical simulations and discussions

To verify (4), the numerical simulations were based on the commercial finite element code MSC.Marc under the option of the large deformation and strains. The utilised model for the numerical simulation is the same as that described in details in [7]. The material of the considered specimen was assumed to be elasto-plastic with Young's modulus of 210 GPa, Poisson's ratio of 0.3 and the initial yield stress of 200 MPa. We carried out the simulations for three different flow curves modelling a variety of possible material properties. Namely, linear hardening with a plastic modulus of 150 MPa, nonlinear hardening with yield stress $k(\overline{\varepsilon}^p) = 100 + 100(1 + 14.24775\overline{\varepsilon}^p)^{0.5}$, and finally the ideal plasticity model. Computations were verified with respect of both the stability of the obtained solution and the accuracy of the results which was controlled to be less than 0.1%.

By means of appropriate indicated markers, the relations between values of $\overline{\sigma}_z/\overline{k}$ and a/R obtained from numerical simulations are shown in Figs. 2 together with respective results obtained from the new formula (4). (Fig. 2a, 2b and 2c corresponds to the three materials under consideration). For illustrative reasons, the curves corresponding to the classical Bridgman, Siebel-Davidenkov-Spiridonova and Szczepinski formulae are also drawn.

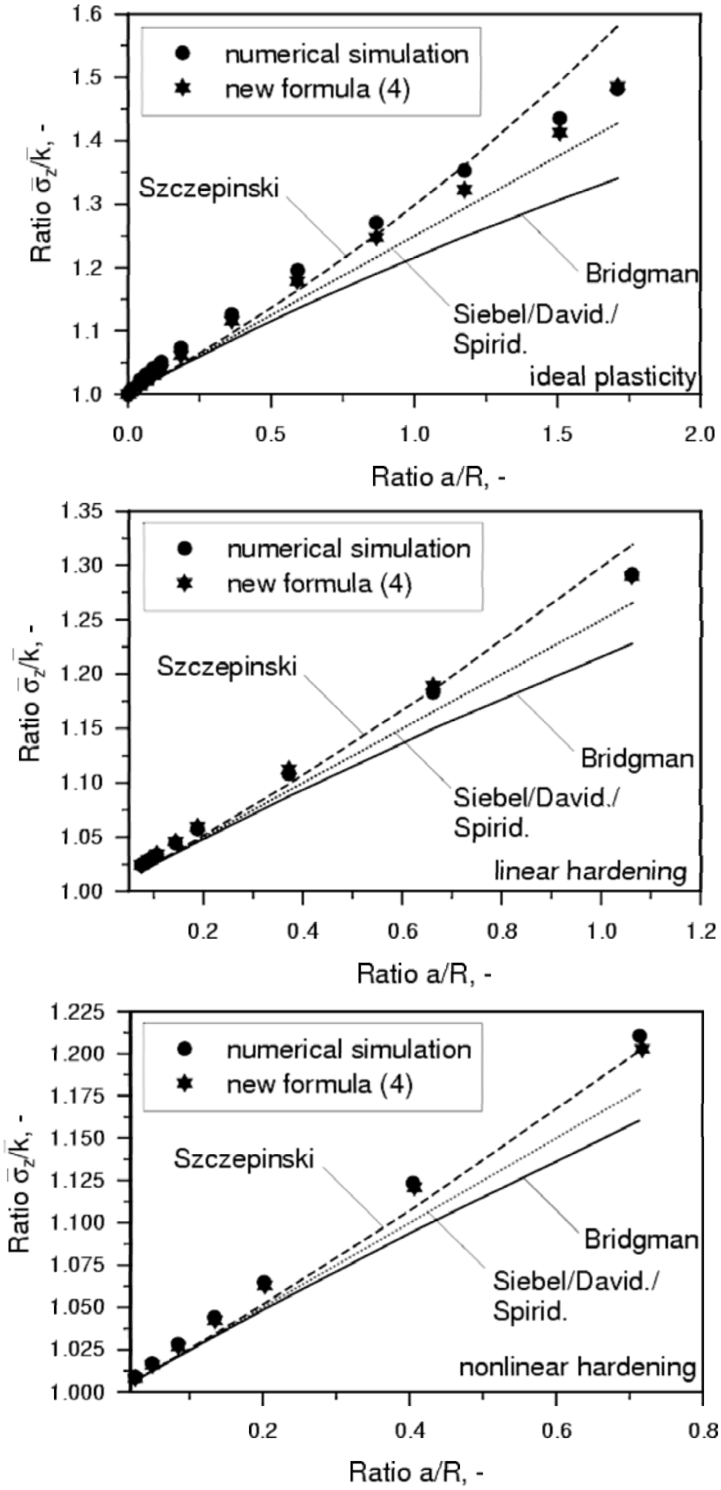

Fig. 2. Ratio $\overline{\sigma}_z/\overline{k}$ as a function of a/R obtained from FEM simulation and its approximations by the classical formulae by Bridgman, Siebel-Davidenkov-Spiridonova and Szczepinski and by the new formula (4). Fig 2a), 2b and 2c correspond to the ideal plastic, linear hardening and nonlinear hardening material, respectively

As it follows from the numerical simulations, a significant improvement has been achieved by using the new formula (4). It is important to note that the new formula additionally incorporates into analysis the initial radius of the sample a_0. As a result, the curves corresponding to different materials do not coincide to each other in contrast with the classical formulae.

Similarly to all other considerations, we have neglected in the analysis an influence of elastic strains at the stage of the neck creation. Further improvement of the derived solutions may be possible by taking into account this factor. However, one should not have a high expectation from this as it may rather have some influence only at an initial stage of the neck formation when the elastic strains can still play a certain role.

REFERENCES

[1] D. J. Celentano, E. E. Cabezas, C. M. Garcia, Journal of Applied Mechanics **72**, 149-152 (2005).

[2] W. H. Chen, Int. J. Solids Structures **7**, 685-717 (1971).

[3] C. Garcia-Garino, F. Gabaldon, J. M. Goicolea, Finite Elem. Anal. **42**, 1187-1197 (2006).

[4] W. Gaudig, K. Bothe, A. K. Bhaduri, K. Maile, Testing Evaluation **24**, 161-167 (1996).

[5] J. Gawąd, R. Kuziak, Ł. Madej, D. Szeliga, M. Pietrzyk, Steel Research International **76**, 131-137 (2005).

[6] M. Gromada, G. Mishuris, A. Öchsner, Advanced Computational Engineering Mechanics, Proceedings of the 2. Workshop Erlangen, Germany, June 30-July 2, 53-62 (2005).

[7] M. Gromada, G. Mishuris, A. Öchsner, Arch Metall Mater **52**, 231-238 (2007).

[8] M. Gromada, G. Mishuris, A. Öchsner, Israel Journal of Chemistry **47**, 329-335 (2007).

[9] R. Hill, The mathematical theory of plasticity, Clarendon Press, Oxford (1950).

[10] Z. Jasieński, Archiwum Hutnictwa **10**, 189-239 (1965).

[11] S. Katarzyński, S. Kocańda, M. Zakrzewski, Badanie własności mechanicznych metali, WNT, Warszawa (1967).

[12] Л. Каханов, Основы теории иластичности, Гостехиздат, Москва (1956).

[13] Y. Ling, AMP Journal of Technology **5**, 37-48 (1996).

[14] N. N. Malinin, J. Rżysko, Mechanika materiałów, PWN, Warszawa (1981).

[15] E. R. Marshall, M. C. Shaw, Transactions of American Society for Metals **44**, 705-725 (1952).

[16] A. Needleman, J. Mech. Phys. Solids **20**, 111-127 (1972).

[17] D. M. Norris, B. Moran, Jr, J. K. Scudder, D. F. Quinones, J. Mech. Phys. Solids **26**, 1-19 (1978).

[18] A. Öchsner, J. Gegner, W. Winter, G. Kuhn, Material Science and Engineering **A 318**, 328-333 (2001).

[19] L. Ramser, W. Winter, G. Kuhn, Workshop on Advanced Computational Engineering Mechanics, Maribor, 9-11 October (2003).

[20] G. L A Rosa, G. Mirone, A. Risitano, Metallurgical and Materials Transactions **34**, 615-624 (2003).

[21] M. Saje, Int. J. Solids Structures **15**, 731-742 (1979).

[22] E. Siebel, Werkstoffaus, 71, (in German) (1925).

[23] W. Szczepiński (praca zbiorowa): Metody doświadczalne mechaniki ciała stałego, PWN, Warszawa (1984).

[24] D. Szeliga, J. Gawąd, M. Pietrzyk, R. Kuziak, Steel Research International **76**, 807-814 (2005).

[25] P. F. Thomason, Int. J. Mech. Sci. **11**, 481-490 (1969).

[26] K. S. Zhang, Z. H. Li, Engng Fract Mech. **49**, 235-241 (1994).

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