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INFLUENCE OF THE ASYMMETRY OF VIBRATORS RESISTANCE TO MOTION ON THE CORRECTNESS OF THE VIBRATION DISTRIBUTION ON WORKING SURFACES OF VIBRATORY MACHINES

WPŁYW ASYMETRII OPORÓW ŁOŻYSKOWANIA NA PRAWIDŁOWOŚĆ ROZKŁADU DRGAŃ NA POWIERZCHNI ROBOCZEJ MASZYN WIBRACYJNYCH

The present paper elucidates the irregularity of vibration distribution on the working surface of vibratory machines (vibratory conveyers and vibrating screens, self-feeding vibrating grates) being the cause of an anomalous transport of a material feed along the machine body.

Out of various possible reasons of this phenomenon, angular oscillations of machine bodies occurring due to non-cophasal running of driving vibrators were chosen for analysis. Such non-cophasal running occurs in two-drive systems in which either the resistance of bearings or the driving moments of vibrators are not maintained equal.

The dependence of vibrators disphasing on the diversification of their anti-torques was determined by means of averaging the equation of motion of vibrators in the non-inertial system related to the vibrating machine body. The dependencies enabling to determine the amplitude of angular oscillations of the machine body – as a function of machine parameters and differences in the resistance to motion of vibrators – are also given in the present paper.

Praca dotyczy przyczyn nierównomierności rozkładu drgań na powierzchni roboczej maszyn wibracyjnych (przenośników i przesiewaczy wibracyjnych, samopodających krat wstrząsowych itp.) będącej przyczyną nieprawidłowego transportu nadawy wzdłuż korpusu. Spośród rozmaitych przyczyn tego zjawiska poddano analizie wahania kątowe korpusów powstające na skutek niewspółfazowego biegu wibratorów napędowych, występujące w układach dwunapędowych, w których nie zachowano równości oporów łożyskowań lub. momentów napędowych wibratorów. Metodą uśredniania równań ruchu wibratorów w układzie nieinercjalnym związanym z drgającym korpusem maszyny wyznaczono zależność rozfazowania wibratorów od zróżnicowania ich momentów oporów oraz podano zależności pozwalające na wyznaczenie amplitudy wahań kątowych korpusu w funkcji parametrów maszyny i różnic oporów ruchu wibratorów.

1. Introduction

Very often a feed transportation variability – along the machine body – is observed in vibratory machines of the self-feeding vibration grids type, sifters or vibratory conveyers supported in a way allowing a movement of some degrees of freedom.

This phenomenon can lead to a local elevation of a loose material feed and in extreme cases to a stoppage of a feed flow from the machine and even to a local reversing of its transportation direction.

There might be several reasons for such disturbances:

- Constructional or assembling errors causing that the direction of the resulting force of a dual mass inertial drive does not pass exactly through the machine mass centre;
- Coupling between a translatory motion along a horizontal and vertical axis and the angular motion of the machine body caused by improper elastic supporting system of the machine body;
- Asymmetry of vibrators' movements due to a different orientation of their force of gravity versus unbalanced masses;
- Superposition of bending and working vibrations – in the case of a body whippiness;
- Influence of collisions with a feed on body rotational vibrations or vibrator running;
- Asymmetry of driving moments and vibrators resistance to motion caused by assembling or exploitation factors.

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Apart from the banding body vibrations, an irregular vibration distribution along the working surface of a machine is related to the occurrence of a body angular motion (oscillations). This motion creates additional components of velocity and displacements resulting from rotations, which disturb and differentiate vibrations in particular regions of a working surface.

Phenomena related to an improper orientation of a vibrator set versus a mass centre or to coupling of various vibration forms via a supporting elastic system are not difficult to analyse. However, a problem of body oscillations due to a partial dissynchronisation of vibrators caused by differentiations of driving moments and resistances to motion has not been solved, up to the present.

The development of the mathematical model of body oscillations due to a dissynchronisation of vibrators – resulting from differentiations of electromagnetic and resistive moments of both driving units – is the aim of the hereby research.

The analysed phenomenon is relatively complex. Differentiations of driving and resisting moments of both vibrators cause their deviation from symmetric running, forced by the operation of vibratory synchronizing moments. Those synchronizing moments are formed when certain conditions are fulfilled [1] in a system, in which phase angles of vibrators are not the same. An asymmetry of vibrator positions is the reason that the resulting excitation forces are deviated from the direction passing through the mass centre and excite body rotational vibrations. In turn, those vibrations are responsible for uneven distribution of vibrations on the body surface resulting the differentiation of transporting velocity. Additionally, a disturbed feed motion influences inversely the system causing a certain correction of rotational vibrations of the body.

2. Theoretical analysis of the phenomenon

Let us consider the scheme of a vibratory machine, either of a vibratory conveyer type, or a vibrating screen of a linear vibration trajectory or self-feeding vibratory grids, in which the drive constitutes two independent inertial vibrators set in motion by means of induction motors – Fig.1.

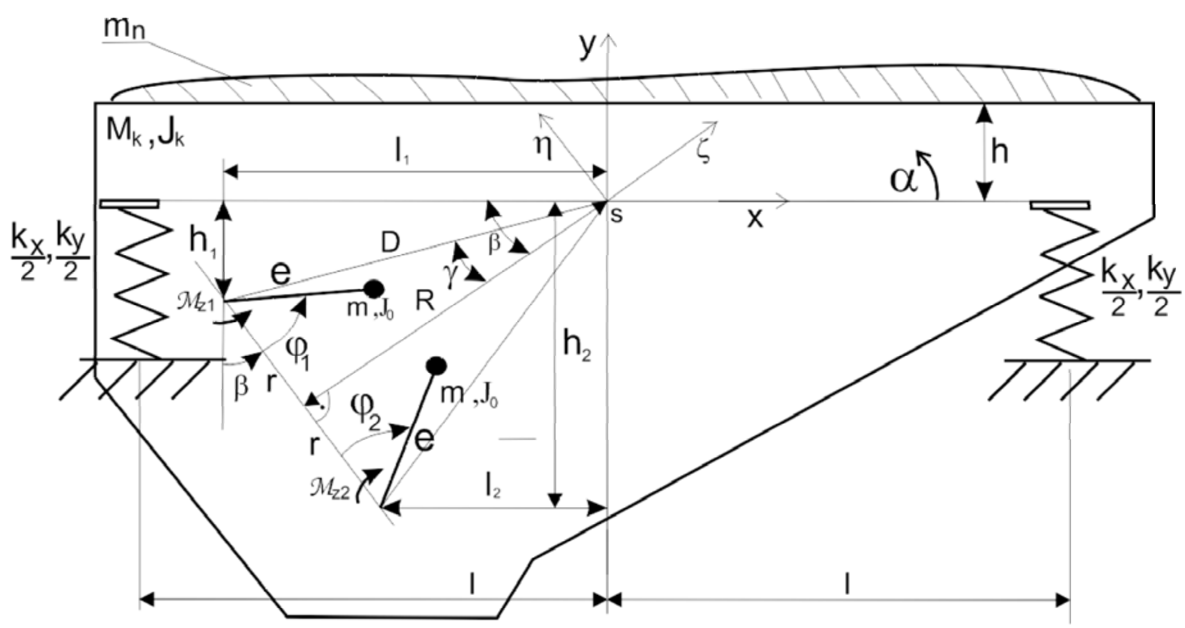

Fig. 1. Analysed model of a conveyer

where:
M_k, J_k – mass and a central moment of a body inertia,
m, e – mass and eccentricity of an individual vibrator,
J_0 – vibrator moment of inertia versus its axis of rotation,
m_n – mass of a material feed,
k_x, k_y – coefficient of elasticity of the body suspension along x and y axis,
$\mathcal{M}_{z1}, \mathcal{M}_{z2}$ – external moments applied to rotating masses, originated from the drive and from resistance to motion of vibrators.

The condition for a proper operation of such device is that both vibrators are counter and cophasal running generating the resulting force in the working directions (ζ). The direction of this force should pass through the mass centre s of the machine, which assures a lack of excitations for angular oscillations of the body (when an elastic supporting system is symmetrical versus the machine mass centre).

Conditions of occurrence a tendency for the needed synchronous, cophasal running of vibrators can be determined on the basis of the integral criterion developed by I.I.Blechman [1]:

$$B(\phi_1-\phi_2,\phi_1-\phi_3,...,\phi_1-\phi_n)=\frac{1}{T}\left[\int_0^T(E-V)dt-\int_0^T(E_w-V_w)dt\right]=\min \quad (1)$$

In accordance with this criterion the system of phase angles is stable around values $\Delta\phi_{12},\Delta\phi_{13},...,\Delta\phi_{1n}$, if for these values function B – determined by Eq. (1) – assumes a local minimum where:

$\phi_1,\phi_2,...,\phi_n$ – angles of rotation of individual vibrators versus their initial mutually parallel positions,

$T=\frac{2\pi}{\omega}$ – forced vibrations period,

E – kinetic energy of a machine body with rotor masses concentrated in the pivoting points,

V – potential energy of the supporting system of the machine body,

E_w, V_w – kinetic and potential energy – respectively – of the constrains between vibrators (if they exist).

For machines operating in the far over-resonance mode, for which influence of elastic forces in suspension can be neglected and which correspond to the scheme presented in Fig. 1 the above given condition leads [6] to equation:

$$B>0. \quad (2)$$

However, this criterion does not decide on an occurrence and synchronisation accuracy in the case of the counter acting factors [2]. The range of allowable disphasing angles of vibrators - for various types of vibratory machines - given in paper [6] indicates the importance of a vibrator dissynchronising for the time-history of the working process.

$\Delta\varphi \leqslant 3-5°$ for vibrating screens,

$\Delta\varphi \leqslant 5-12°$ for feeders,

$\Delta\varphi \leqslant 12-16°$ for vibratory conveyers and self-feeding vibratory grids.

Let us write equations of motion of the body in the absolute system ζ, η, denoting an angle of rotation as α and assuming the equality of rotational speeds of both vibrators $\dot{\phi}_1=\dot{\phi}_2=\omega=const$ and disregarding an influence of a feed:

$$M\ddot{\xi}+k_\xi\xi=me\omega^2[\sin\phi_1+\sin\phi_2] \quad (3a)$$

$$M\ddot{\eta}+k_\eta\eta=me\omega^2[\cos\phi_2-\cos\phi_1] \quad (3b)$$

$$J\ddot{\alpha}+k_yl^2\alpha=me\omega^2r[\sin\phi_2-\sin\phi_1]+me\omega^2R[\cos\phi_1-\cos\phi_2] \quad (3c)$$

where:

$M=M_k+2m$ – mass of a vibrating part of the machine,

J – main central moment of inertia of a machine with unbalanced masses brought to the axis of rotation of vibrators,

$k_\zeta=k_x\cos^2\beta+k_y\sin^2\beta$ – coefficient of elasticity in ζ direction

$k_\eta=k_y\cos^2\beta+k_x\sin^2\beta$ – coefficient of elasticity in η direction

– remaining markings are the same as in Fig 1.

Denoting $\varphi_1-\varphi_2=\Delta\varphi$= constant and assuming $\Delta\varphi<<1$ and $\varphi_2=\omega t$, ω=constant, equations (3a, 3b, 3c) can be presented in an approximate form:

$$M\ddot{\xi}+k_\xi\xi=2me\omega^2\sin\left(\omega t+\frac{\Delta\phi}{2}\right) \quad (4a)$$

$$M\ddot{\eta}+k_\eta\eta=me\omega^2\Delta\phi\sin(\omega t) \quad (4b)$$

$$J\ddot{\alpha}+k_yl^2\alpha=-me\omega^2D\cdot\Delta\phi\sin(\omega t+\gamma) \quad (4c)$$

where: tg γ = r/R - Fig.1.

The particular integrals of those equations, determining the steady state are as follows:

$$\xi(t)=\frac{2me\omega^2}{k_\xi-M\omega^2}\sin\left(\omega t+\frac{\Delta\phi}{2}\right) \quad (5a)$$

$$\eta(t)=\frac{-me\omega^2\Delta\phi}{M\omega^2-k_\eta}\sin(\omega t) \quad (5b)$$

$$\alpha(t)=\frac{me\omega^2D\cdot\Delta\phi}{J\omega^2-k_yl^2}\sin(\omega t+\gamma) \quad (5c)$$

Let us also determine the second time derivatives in a steady motion for these coordinates:

$$\ddot{\xi}(t) = \frac{2me\omega^4}{M\omega^2 - k_\xi}\sin\left(\omega t + \frac{\Delta\phi}{2}\right) \quad (6a)$$

$$\ddot{\eta}(t) = \frac{me\omega^4\Delta\phi}{M\omega^2 - k_\eta}\sin(\omega t) \quad (6b)$$

$$\ddot{\alpha}(t) = \frac{-me\omega^4 D\cdot\Delta\phi}{J\omega^2 - k_y l^2}\sin(\omega t + \gamma) \quad (6c)$$

Dynamic equations, analysed so far, were describing body vibrations at the assumption that the vibrators' angular motion – at the steady state - can be considered a uniform one. Such assumption is equivalent to disregarding the influence of body vibrations on vibrators running. Presently we will write equations of motion of vibrators while taking into account those couplings, i.e. in a non-inertial system related to the machine body performing the vibrations described above.

Applying moments from a transportation force of inertia to vibrators we obtain equations of their angular motion in the following form:

$$J_0\ddot{\phi}_1 = M_{z1} - me\ddot{\xi}_1\cos\phi_1 - me\ddot{\eta}_1\sin\phi_1 \quad (7a)$$

$$J_0\ddot{\phi}_2 = M_{z2} - me\ddot{\xi}_2\cos\phi_2 + me\ddot{\eta}_2\sin\phi_2, \quad (7b)$$

where :

$\mathcal{M}_{z1,2}$ – external moments (difference of a driving moment and anti-torque),

J_o – moment of inertia of a vibrator versus its axis of rotation.

Let us mark as **vibratory moments** $\mathcal{M}_{wi}$, i-1,2 , expressions:

$$\mathcal{M}_{w1} = -me(\ddot{\xi}_1\cos\phi_1 + \ddot{\eta}_1\sin\phi_1) \quad (8a)$$

$$\mathcal{M}_{w2} = -me(\ddot{\xi}_2\cos\phi_2 - \ddot{\eta}_2\sin\phi_2) \quad (8b)$$

On the basis of the previously determined solutions of body motions (not taking into account the influence of vibratory moments on vibrators running) we will determine components of axes acceleration of both vibrators:

$$\ddot{\xi}_1 \cong \ddot{\xi} - \ddot{\alpha}D\sin\gamma \quad (9a)$$

$$\ddot{\eta}_1 \cong \ddot{\eta} - \ddot{\alpha}D\cos\gamma \quad (9b)$$

$$\ddot{\xi}_2 \cong \ddot{\xi} - \ddot{\alpha}D\sin\gamma \quad (9c)$$

$$\ddot{\eta}_2 \cong \ddot{\eta} - \ddot{\alpha}D\cos\gamma \quad (9d)$$

Taking into consideration equations (6) and substituting them into (8) we will obtain the following equations for vibratory moments:

$$\begin{aligned}\mathcal{M}_{w1} = -m^2e^2\omega^4[&\tfrac{2}{M\omega^2-k_\eta}\sin\left(\omega t + \tfrac{\Delta\phi}{2}\right)\cos(\omega t + \Delta\phi) + \tfrac{D^2\Delta\phi\sin\gamma}{J\omega^2-k_y l^2}\sin(\omega t+\gamma)\cos(\omega t + \Delta\phi) + \\ &+\tfrac{\Delta\phi}{M\omega^2-k_\eta}\sin(\omega t)\sin(\omega t+\Delta\phi) + \tfrac{D^2\Delta\phi\cos\gamma}{J\omega^2-k_y l^2}\sin(\omega t+\gamma)\sin(\omega t+\Delta\phi)]\end{aligned} \quad (10a)$$

$$\begin{aligned}\mathcal{M}_{w2} = -m^2e^2\omega^4[&\tfrac{2}{M\omega^2-k_\eta}\sin\left(\omega t + \tfrac{\Delta\phi}{2}\right)\cos(\omega t) - \tfrac{D^2\Delta\phi\sin\gamma}{J\omega^2-k_y l^2}\sin(\omega t+\gamma)\cos(\omega t) - \\ &-\tfrac{\Delta\phi}{M\omega^2-k_\eta}\sin^2(\omega t) - \tfrac{D^2\Delta\phi\cos\gamma}{J\omega^2-k_y l^2}\sin(\omega t+\gamma)\sin(\omega t)].\end{aligned} \quad (10b)$$

Presently we will calculate the averaged – for a period $T=2\pi/\omega$ – value of the vibratory moment acting on the vibrator No 1 and use the assumption that $\Delta\varphi <<1$:

$$\begin{aligned}\mathcal{M}_{w1r} &= \tfrac{1}{T}\int_0^T M_{w1}(t)dt = -m^2e^2\omega^4\tfrac{\omega}{2\pi}[\tfrac{2}{M\omega^2-k_\xi}\int_0^{2\pi/\omega}\sin\left(\omega t+\tfrac{\Delta\phi}{2}\right)\cos(\omega t+\Delta\phi)dt + \tfrac{D^2\Delta\phi\sin\gamma}{J\omega^2-k_y l^2}\int_0^{2\pi/\omega}\sin(\omega t+ \\ &+\gamma)\cos(\omega t+\Delta\phi)dt + \tfrac{\Delta\phi}{M\omega^2-k_\eta}\int_0^{2\pi/\omega}\sin(\omega t)\sin(\omega t+\Delta\phi)dt + \tfrac{D^2\Delta\phi\cos\gamma}{J\omega^2-k_y l^2}\int_0^{2\pi/\omega}\sin(\omega t+\gamma)\sin(\omega t+\Delta\phi)dt] = \\ &= \tfrac{-m^2e^2\omega^4}{2}[\tfrac{2}{M\omega^2-k_\xi}\sin\left(-\tfrac{\Delta\phi}{2}\right) + \tfrac{D^2\Delta\phi\sin\gamma}{J\omega^2-k_y l^2}\sin(\gamma-\Delta\phi) + \tfrac{\Delta\phi}{M\omega^2-k_\eta}\cos(\Delta\phi) + \tfrac{D^2\Delta\phi\cos\gamma}{J\omega^2-k_y l^2}\cos(\gamma-\Delta\phi)] = \\ &= \tfrac{-m^2e^2\omega^4}{2}[\tfrac{-\Delta\phi}{M\omega^2-k_\xi} + \tfrac{D^2\Delta\phi\sin\gamma}{J\omega^2-k_y l^2}(\sin\gamma - \Delta\phi\cos\gamma) + \tfrac{\Delta\phi}{M\omega^2-k_\eta} + \tfrac{D^2\Delta\phi\cos\gamma}{J\omega^2-k_y l^2}(\cos\gamma+\Delta\phi\sin\gamma)]\end{aligned} \quad (11)$$

Disregarding terms containing $(\Delta\varphi)^2$ we finally obtain:

$$\mathcal{M}_{w1r} = \frac{-m^2e^2\omega^4}{2}\left[\frac{D^2}{J\omega^2 - k_y l^2} + \frac{1}{M\omega^2 - k_\eta} - \frac{1}{M\omega^2 - k_\xi}\right]\Delta\phi \tag{12}$$

Likewise, calculating the averaged - for a period $T=2\pi/\omega$ – value of the vibratory moment for the vibrator No 2 we obtain:

$$\mathcal{M}_{w2r} = \frac{1}{T}\int_0^T M_{w2}(t)dt = -m^2e^2\omega^4\frac{\omega}{2\pi}[\frac{2}{M\omega^2-k_\xi}\int_0^{2\pi/\omega}\sin\left(\omega t+\frac{\Delta\phi}{2}\right)\cos(\omega t)dt - \frac{D^2\Delta\phi\sin\gamma}{J\omega^2-k_y l^2}\int_0^{2\pi/\omega}\sin(\omega t+\gamma)\cos(\omega t)dt -$$
$$-\frac{\Delta\phi}{M\omega^2-k_\eta}\int_0^{2\pi/\omega}\sin^2(\omega t)dt - \frac{D^2\Delta\phi\cos\gamma}{J\omega^2-k_y l^2}\int_0^{2\pi/\omega}\sin(\omega t+\gamma)\sin(\omega t)dt] =$$
$$= \frac{-m^2e^2\omega^4}{2}[\frac{2}{M\omega^2-k_\xi}\sin\left(\frac{\Delta\phi}{2}\right) - \frac{D^2\Delta\phi\sin\gamma}{J\omega^2-k_y l^2}\sin(\gamma) - \frac{\Delta\phi}{M\omega^2-k_\eta} - \frac{D^2\Delta\phi\cos\gamma}{J\omega^2-k_y l^2}\cos(\gamma)] =$$
$$= \frac{m^2e^2\omega^4}{2}\left[\frac{D^2}{J\omega^2-k_y l^2} + \frac{1}{M\omega^2-k_\eta} - \frac{1}{M\omega^2-k_\xi}\right]\Delta\phi. \tag{13}$$

As can be seen, the values of both moments are equal and their directions reverse. Thus, the difference of these moments is:

$$\Delta\mathcal{M}_w = \mathcal{M}_{w2r} - \mathcal{M}_{w1r} = m^2e^2\omega^4\left[\frac{D^2}{J\omega^2 - k_y l^2} + \frac{1}{M\omega^2 - k_\eta} - \frac{1}{M\omega^2 - k_\xi}\right]\Delta\phi. \tag{14}$$

The above expression constitutes a measure of the system ability to generate the synchronising moment, when the system of a natural tendency for a synchronised cophasal running - due to a certain reason – operates dissynchronised by angle $\Delta\varphi$.

Let us assume, that e.g. anti-torque in the vibrator bearing No 2 is larger (or – what is equivalent – that the electromagnetic moment of this motor is smaller) by $\Delta\mathcal{M}_z$ than the analogous moment for the vibrator No 1, which causes difference in angles circled by both vibrators: $\Delta\varphi = \varphi_1 - \varphi_2$.

Equating $\Delta\mathcal{M}_z$ to $\Delta\mathcal{M}_w$ given by Eq. (14) we obtain formula (15) determining the disphasing angle of vibrators due to the differences of the driving and anti-torque moments of both vibrators:

$$\Delta\phi = \frac{\Delta\mathcal{M}_z}{m^2e^2\omega^4\left[\frac{D^2}{J\omega^2-k_y l^2} + \frac{1}{M\omega^2-k_\eta} - \frac{1}{M\omega^2-k_\xi}\right]}. \tag{15}$$

This allows to determine amplitudes of the body angular oscillations – on the basis of Eq. (5c):

$$A_\alpha = \frac{me\omega^2 D}{J\omega^2 - k_y l^2}\Delta\phi \tag{16}$$

It is usually possible to assume – without committing essential error – that the difference:

$$\frac{1}{M\omega^2 - k_\eta} - \frac{1}{M\omega^2 - k_\xi}$$

from Eq. (15) is equal zero. Then substituting the simplified Eq. (15) into (16) we finally obtain Eq. (17) determining the amplitude of angular oscillations of the body caused by the diversification of the driving and anti-torque moments of both vibrators:

$$A_\alpha = \frac{\Delta\mathcal{M}_z}{me\omega^2 D}. \tag{17}$$

Equation (17) alone can be considered a measure of the operational correctness of the machine. In the cases, when it is recommended, a time-history of the body oscillations can be estimated on the basis of Eq. (5c) and then the resulting displacements of individual points of the working surface calculated. Those values can be added geometrically to displacements caused by a translatory motion of the machine and then the distribution of an average transport velocity along the body can be estimated.

Thus, e.g. for the machine presented schematically in Fig.1, vibrations x(t), y(t) of the point positioned at a height “h” above the centre of the machine mass and being at a distance “w” to the right of y-axis can be presented as a combination of motions along coordinates ζ and η given by equations (5a) and (5b) as well as components in directions x and y resulting from a swinging motion α of the body (5c):

$$x(t) = \xi(t)\cos(\beta) - \eta(t)\sin(\beta) - \alpha(t)h \tag{18a}$$

$$y(t) = \xi(t)\sin(\beta) + \eta(t)\cos(\beta) + \alpha(t)w. \tag{18b}$$

The knowledge of these components can be used for the determination of the average transport velocity in the given place of the body [3].

3. Simulation investigations

Simplifying assumptions, from the presented above analysis, indicate the usefulness of the verification of the obtained dependencies by means of the computer simulation of the system motion. The model of the system shown in Fig.2 was assumed for the purpose of computer simulations.

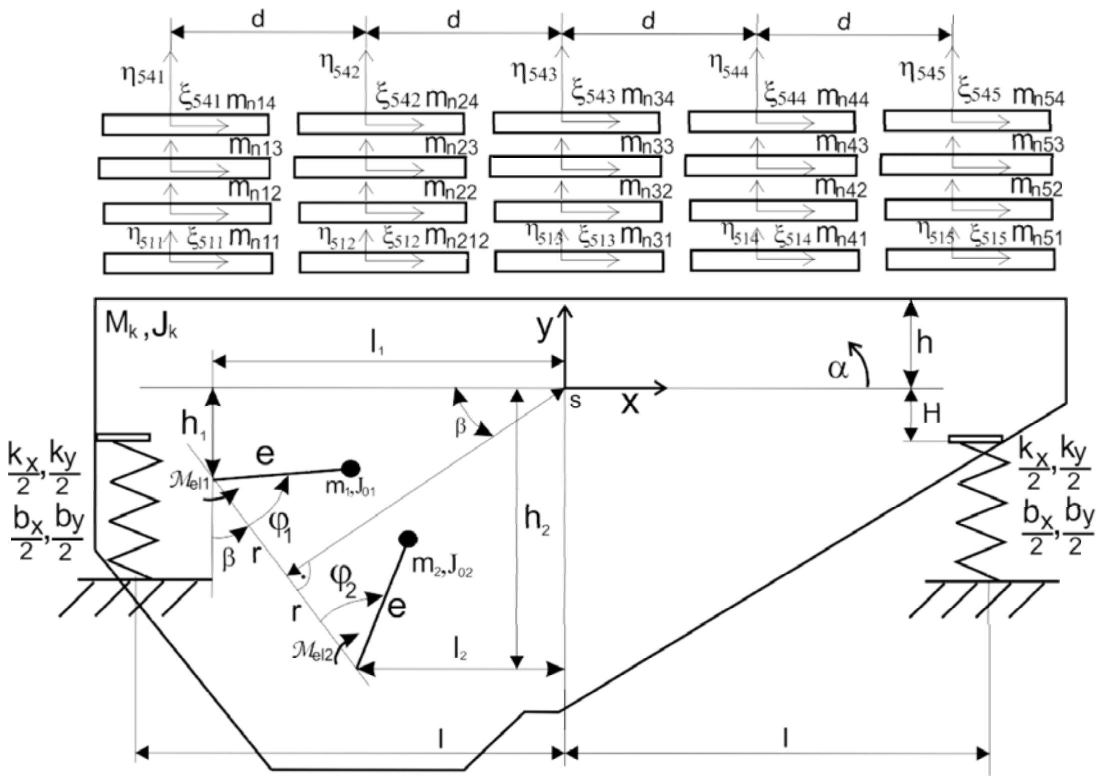

Fig. 2. Model of the conveyer together with the material feed

The system consists of two inertial vibrators of independent induction drives – described by means of a static characteristics – of the machine body – performing a plane motion – supported on a system of vertical coil-springs and of five four-layer models of a loose material feed distributed in various points of the working surface [5]. Influence of the gravity force on the vibrators angular motion was taken into account in the model.

The mathematical model of such system consists of several elements: matrix equation (19) circumscribing the machine motion, equations (26) describing electromagnetic moments of driving motors, equations (25) determining a motion of the successive feed layers as well as equations (23) and (24) concerning normal and tangent reactions between feed layers themselves and between a material feed and the machine body.

$$M \cdot \ddot{q} = Q, \tag{19}$$

where:

$$M = \begin{bmatrix} M_k + m_1 + m_2 & 0 & m_1 h_1 + m_2 h_2 & m_1 e_1 \cos(\beta + \phi_1) & m_2 e_2 \cos(\phi_2 - \beta) \\ 0 & M_k + m_1 + m_2 & -m_1 a_1 - m_2 a_2 & m_1 e_1 \sin(\beta + \phi_1) & -m_2 e_2 \sin(\phi_2 - \beta) \\ m_1 h_1 + m_2 h_2 & -m_1 l_1 - m_2 l_2 & \begin{matrix} m_2 h_2^2 + m_2 l_2^2 + m_1 h_1^2 + \\ + m_1 l_1^2 + J_k \end{matrix} & \begin{matrix} m_1 h_1 e_1 \cos(\beta + \phi_1) - \\ m_1 l_1 e_1 \sin(\beta + \phi_1) \end{matrix} & \begin{matrix} m_2 h_2 e_2 \cos(\phi_2 - \beta) + \\ m_2 l_2 e_2 \sin(\phi_2 - \beta) \end{matrix} \\ m_1 e_1 \cos(\beta + \phi_1) & m_1 e_1 \sin(\beta + \phi_1) & \begin{matrix} m_1 h_1 e_1 \cos(\beta + \phi_1) - \\ m_1 a_1 e_1 \sin(\beta + \phi_1) \end{matrix} & J_{01} & 0 \\ m_2 e_2 \cos(\phi_2 - \beta) & -m_2 e_2 \sin(\phi_2 - \beta) & \begin{matrix} m_2 h_2 e_2 \cos(\phi_2 - \beta) \\ m_2 l_2 e_2 \sin(\phi_2 - \beta) \end{matrix} & 0 & J_{02} \end{bmatrix} \quad (20)$$

$$\ddot{q} = [\ddot{x}\ \ddot{y}\ \ddot{\alpha}\ \ddot{\phi}_1\ \ddot{\phi}_2]^T \quad (21)$$

$$Q = \begin{bmatrix} -m_2 e_2 \dot{\phi}_2^2 \sin(\phi_2 - \beta) - m_1 e_1 \dot{\phi}_1^2 \sin(\beta + \phi_1) - k_x(x + H\alpha) - b_x(\dot{x} + H\dot{\alpha}) - T_{101} - T_{102} - T_{103} - T_{104} - T_{105} \\ m_2 e_2 \dot{\phi}_2^2 \sin(\phi_2 - \beta) + m_1 e_1 \dot{\phi}_1^2 \cos(\beta + \phi_1) - \frac{1}{2} k_y(y + l_1\alpha) - \frac{1}{2} k_y(y - l_2\alpha) - \frac{1}{2} b_y(\dot{y} + l_1\dot{\alpha}) - \frac{1}{2} b_y(\dot{y} - l_2\dot{\alpha}) - F_{101} - F_{102} - F_{103} - F_{104} - F_{105} \\ -m_1 h_1 e_1 \dot{\phi}_1^2 \sin(\beta + \phi_1) - m_1 l_1 e_1 \dot{\phi}_1^2 \cos(\beta + \phi_1) - m_2 h_2 e_2 \dot{\phi}_2^2 \sin(\phi_2 - \beta) + m_2 l_2 e_2 \dot{\phi}_2^2 \cos(\phi_2 - \beta)) - k_x H^2 \alpha - k_x H x - b_x H \dot{x} - b_x H^2 \dot{\alpha} - \\ \frac{1}{2} k_y(y + l\alpha)l + \frac{1}{2} k_y(y - l\alpha)l - \frac{1}{2} b_y(\dot{y} + l\dot{\alpha})l + \frac{1}{2} b_y(\dot{y} - l\dot{\alpha})l + (T_{101} + T_{102} + T_{103} + T_{104} + T_{105})h_n + F_{101} 2d + F_{102} d - F_{104} d - F_{105} 2d \\ M_{el1} - b_{s1} \dot{\phi}_1^2 sign(\dot{\phi}_1) - m_1 g e_1 \sin(\beta + \phi_1) \\ M_{el2} - b_{s2} \dot{\phi}_2^2 sign(\dot{\phi}_2) + m_2 g e_2 \cos(\phi_2 - \beta) \end{bmatrix} . \quad (22)$$

where:
$F_{j,j-1,k}$ – normal component of the j^{th} layer pressure on $j-1$ layer – in the k^{th} column,
$T_{j,j-j,k}$ – tangent component of the j^{th} layer pressure on $j-1$ layer – in the k^{th} column,
j – material feed index, $j = 0$ concerns the machine body,
k – material feed column index.
$m_i e_i^2 + J_{0ic} = J_{0i}$, $i = 1, 2$
J_{0ic} – central moment of inertia of m_i, i = 1,2
it was further assumed that:
$J_{01} = J_{02} = J_0$.

If successive layers of a material feed j and $j-1$ (in the given column) are not in contact, the contact force in the normal direction $F_{j,j-1,k}$ and in the tangent direction $T_{j,j-1,k}$ between these layers equals zero:

$$F_{j,j-1,k} = 0, \quad T_{j,j-1,k} = 0 \quad \text{for} \quad \eta_{j,k} \geqslant \eta_{j-1,k}.$$

Otherwise, the contact force in the normal direction between layers j,k and $j-1,k$ of a material feed occurs (or in the case of the first layer: between the layer and the body), which model is of the form:

$$F_{j,j-1.k} = (\eta_{j-1,k} - \eta_{j,k})^p \cdot k_H \cdot \left\{1 - \frac{1-R^2}{2}\left[1 - \mathrm{sgn}(\eta_{j-1,k} - \eta_{j,k}) \cdot \mathrm{sgn}(\dot{\eta}_{j-1,k} - \dot{\eta}_{j,k})\right]\right\} \quad (23)$$

and the force originated from friction in the tangent direction:

$$T_{j,j-1,k} = -\mu F_{j,j-1,k} \mathrm{sgn}(\dot{\xi}_{j,k} - \dot{\xi}_{j-1,k}), \quad (24)$$

where:
R – restitution coefficient of normal impulses at collision,
k_H, p – Hertz-Stajerman constants.

The form of dependence (23) was developed in [4] on the basis of the Hertz-Stajerman contact forces model modified by taking into account a material damping. Parameters of the hysteresis loop were assumed in such a way as to have the ratio – of the bodies relative velocity after the collision to their velocity before the collision – equal R. It means formula (23) ensures that this ratio is equal to the assumed restitution coefficient value.

Equations of motion of individual layers in directions ξ and η, with taking into consideration the influence of the conveyer on the lower layers of a material feed are in the following form:

$$\begin{aligned} m_{nj,k}\ddot{\xi} &= T_{j,j-1,k} - T_{j+1,j,k}, \\ m_{nj,k}\ddot{\eta} &= -m_{nj,k} g + F_{j,j-1,k} - F_{j+1,j,k}. \end{aligned} \quad (25)$$

$\mathcal{M}_{eli}$ – electromagnetic moment generated by the i^{th} motor, assumed in the form corresponding to the static characteristic of the motor:

$$\mathcal{M}_{eli} = \frac{2M_{ut}(\omega_{ss} - \dot{\phi}_{i1}) \cdot (\omega_{ss} - \omega_{ut})}{(\omega_{ss} - \omega_{ut})^2 + (\omega_{ss} - \dot{\phi}_i)^2} i = 1.2, \quad (26)$$

where
$\mathcal{M}_{ut}$ – stalling torque of driving motors
ω_{ss} – synchronous frequency of driving motors
ω_{ut} – stalling frequency of driving motors

The simulation was performed for the following parameter values:

$l = 0.5[m]$
$l_1 = 1[m]$
$l_2 = 0.5[m]$
$H = 0.0\ [m]$
$h = 0,0[m]$
$h_1 = 0.5[m]$
$h_2 = 1[m]$
$b_x = by = 400[Ns/m]$
$k_x = ky = 150000[N/m]$
$m_1 = m_2=5[kg]$
$M_k = 120[kg]$
$J_{01} = J_{02} = 0.0021\ [kg\ m^2$
$J_k = 25[kgm^2]$
$e_1 = e_2 = 0.02[m]$
$D=1.118[m]$
$\mathcal{M}_{ut} = 50[Nm]$
$\omega_{ss} = 50\ [rad/s]$
$\omega_{ut} = 15.9{*}2\ [rad/s]$
b_{si} - *coefficient of resistance to motion of vibrators:*
$b_{s1} = 0.00009\ [Ns^2/m]$
$b_{s2} = variable.$

The simulation model developed for the verifications of analytical solutions takes into consideration factors contained in analytical solutions as well as other phenomena having a significant influence on body oscillations such as e.g. gravity or collisions with the feed material. Furthermore, there are no limitations for disphasing angle values, and vibratory moments are treated as variables – not being restricted to their averaged values.

4. Simulation results

Time-histories of the examined parameters obtained during simulations are presented in Figures 3 to 12. The time-history of the angle of body oscillations $\alpha(t)$ is shown in Fig. 3 for the case of the diversified resistance to motion of vibrators (in a ratio 1:2).

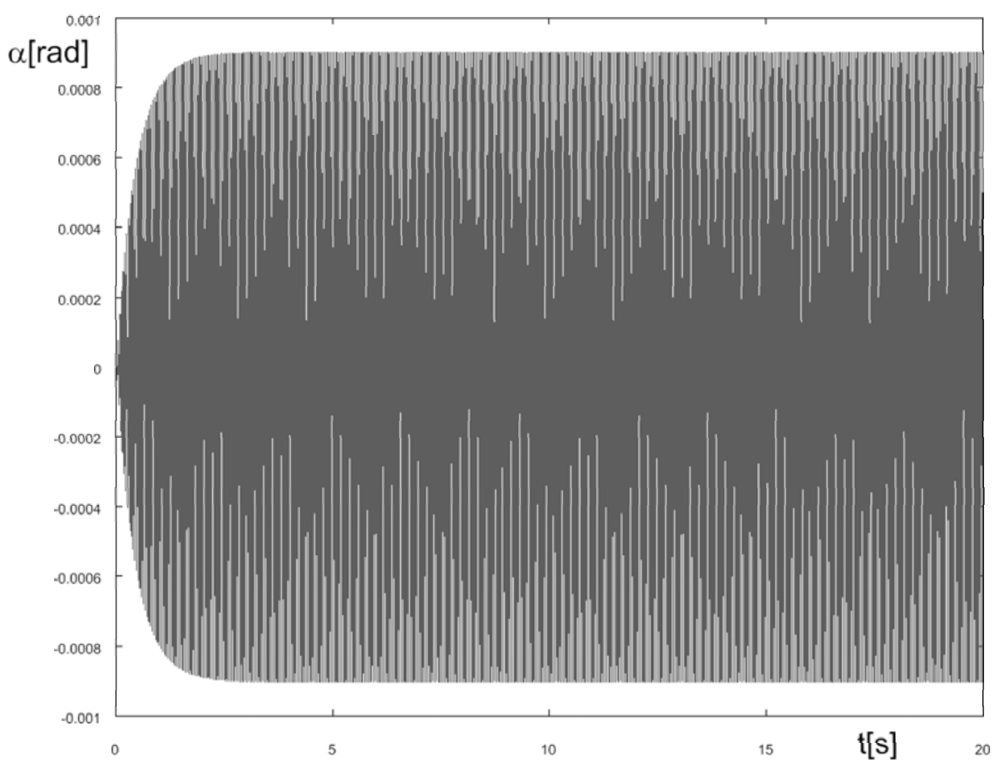

Fig. 3. Angle α versus time, when the diversification of vibrators resistance to motion $b_{s1} = 2b_{s2}$, was taken into consideration

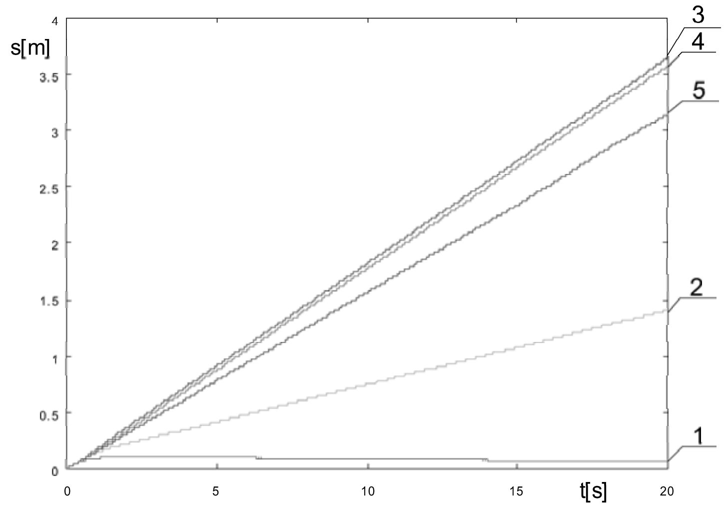

Fig. 4. Transport in individual columns, when the influence of the diversification of vibrators resistance to motion $b_{s1} = 2b_{s2}$, on dissynchronisation of vibrators, was taken into account

The direction of rotation of vibrators – in the theoretical and simulation investigations described above - was assumed in accordance with a positive direction of calculation of their angles of rotation, shown in Fig.1. In order to verify an influence of rotation direction on the amplitude of angular vibrations and the transport along the machine body, the series of simulations for the reverse direction of vibrations was performed. The results are shown in Fig. 5 and 6.

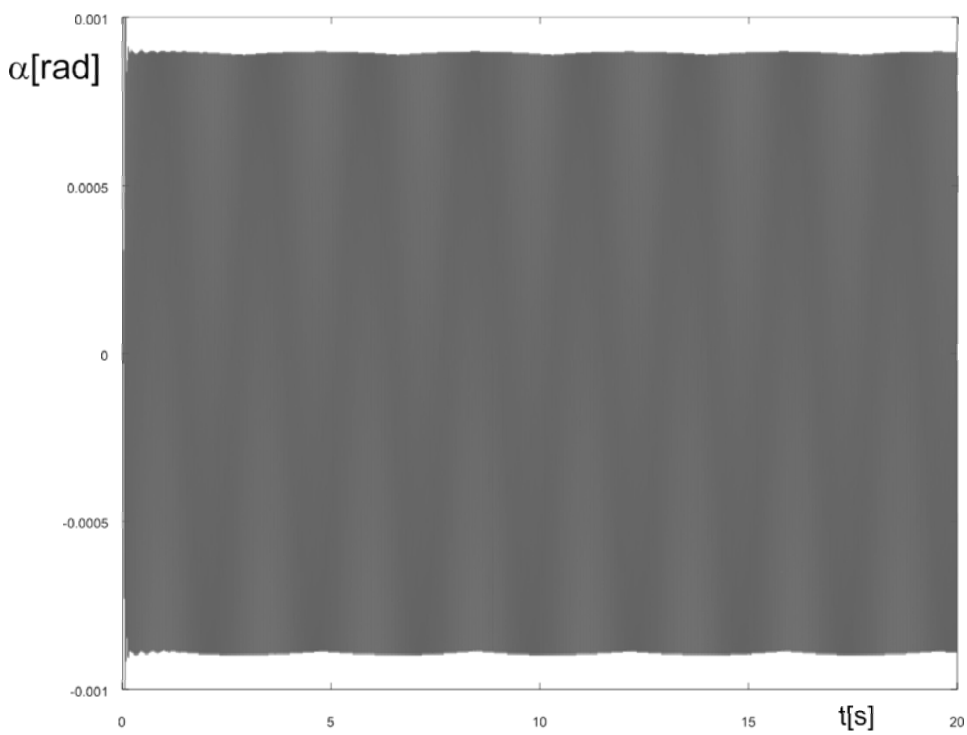

Fig. 5. Angle α versus time, when the influence of the diversification of vibrators resistance to motion $b_{s1} = 2b_{s2}$ was taken into account and reverse direction of rotation was forced

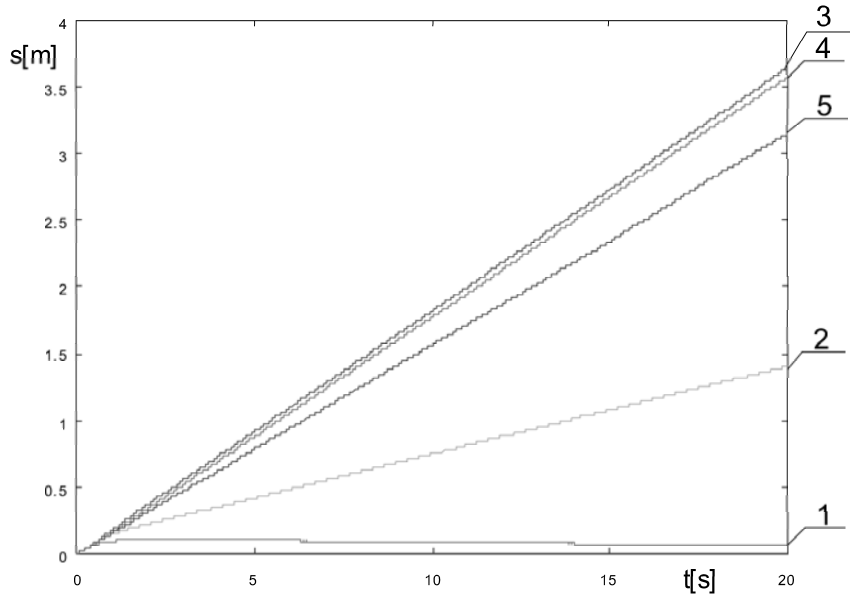

Fig. 6. Transport in individual columns, when the influence of the diversification of vibrators resistance to motion $b_{s1} = 2b_{s2}$ was taken into account and reverse direction of rotation was forced

As can be seen, the direction of vibrators rotation does not influence significantly the amplitude of angular oscillations of the machine body in the steady state, which allows for the application of equation (17) for an arbitrary direction of vibrations. However, significant differences occur in the transportation velocity in body parts being far from the mass centre of the machine. They result from different summing of translational and rotational vibrations of the body and are more pronounced when the asymmetry of driving or anti-torque moments of vibrators is larger. This effect can be utilised in practical applications. The proper selection of the direction of vibrator rotation can favourably influence the amplitude distribution along the body, related to diversification of driving or anti-torque moments of vibrators.

Comparison of the results obtained analytically with the simulation ones

Analytical calculations for parameter values assumed in the simulation, performed according to Eq. (17), give $A_\alpha = 8{,}06 \cdot 10^{-4}$, which is in accordance with the simulation results - Fig.3 and Fig.5.

Assessment of a participation of the asymmetry of driving moments in generating rotational vibrations of the machine body

In order to prove that the diversification of vibrators bearing resistance is the main reason of disphasing of vibrators and angular oscillations of the body – the special simulation experiment was performed. The start of the system was at the ratio of resistances to bearing being 1:1.5 (in the time range: $0 \leqslant t \leqslant 10$ *[s]*) and then those resistances were equalized for $t > 10$ *[s]*. The obtained in such a way time-histories of $\varphi_2 - \varphi_1$ and α are shown in Fig. 7 and 8. They indicate the decisive influence of differences in the driving and resisting moments on the analysed phenomena.

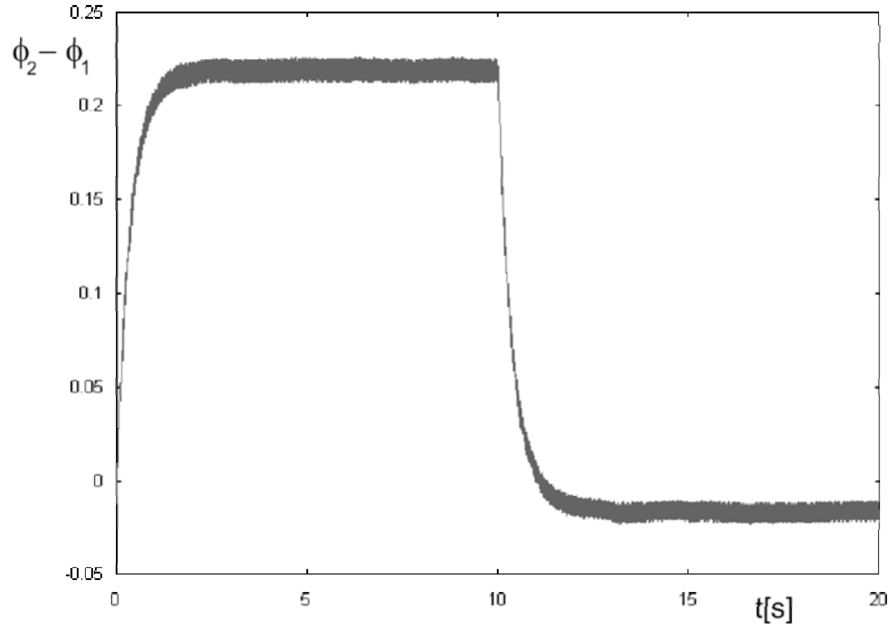

Fig. 7. Disphasing angle of vibrators at a variable diversification of their resistance to motion. The influence of collisions with the material feed as well as the influence of gravity – taken into account

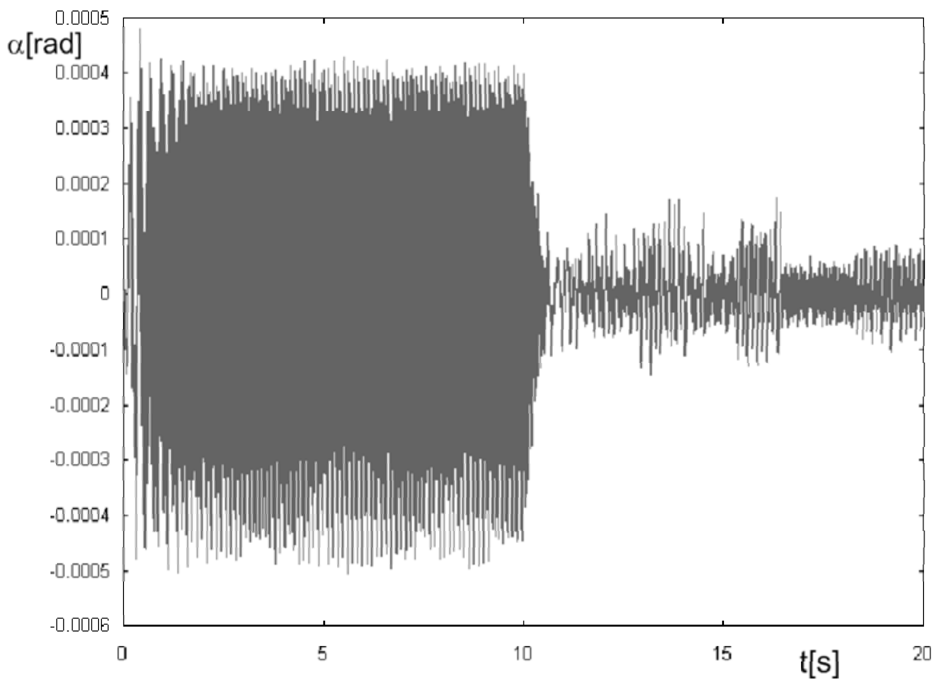

Fig. 8. Angle of rotation α of the machine body as the time function at a variable diversification of vibrators resistance to motion. The influence of collisions with the material feed as well as the influence of gravity – taken into account

5. Conclusions

1. The fact that the diversification of the driving and resistance moments of synchronised vibrators – constituting the drive of the vibratory machine – causes angular oscillations of the machine body and the variability of the material feed transportation velocity along the machine working surface – was proved in the hereby paper.
2. Analytical formulas derived in the paper allow to determine amplitudes of rotational vibrations of vibratory machine bodies caused by the diversification of the driving and anti-torque moments of both vibrators (Eq. (17)). They can constitute the basis for

an assessment the transport velocity distribution of the material feed (Eq. (5) and (18)).

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