THE METHODOLOGY OF STRAIN – STRESS CURVES DETERMINATION FOR STEEL IN SEMI-SOLID STATE

M. HOJNY*, M. GŁOWACKI**

The main target of the presented paper is the presentation of possibilities of computer aided strain-stress curve construction for semi-solid steels on the basis of compression tests. The experimental work was done using the Gleeble 3800 simulator in the Institute for Ferrous Metallurgy in Gliwice, Poland. The testing machine allows the physical deformation of samples while solidification of their central part is still in progress. The essential aim of the simulation was the reconstruction of both temperature changes and strain evolution for specimen subjected to simultaneous deformation and solidification. The paper includes a discussion concerning the methodology of strain-stress curve definition together with accompanying problems. Some example results are presented in the paper as well. The strain-stress relationship is crucial for future development of 3D computer simulation systems of newest, very high temperature rolling technologies like being part of integrated casting and rolling processes.

Keywords: finite element method, inverse analysis, strain-stress curves

1. Introduction

The mathematical and experimental modeling of steel deformation in semi-solid state is an innovative topic regarding the very high temperature range deformation processes. Tracing the related papers published in the past 10 years, one can find many papers dealing with experimental results for non-ferrous metals tests [1-3]. The first results regarding steel deformation at extra high temperature were presented during last few years [4-6]. Most of the problems concerning semi-solid steel testing are caused by the very high level of steel liquidus and solidus temperatures in comparison with non-ferrous metals. The deformation tests for non-ferrous metals are much easier. The rising abilities of thermo-mechanical Gleeble simulators enable investigation of steel samples and as a result both computer simulation and development of new, very high temperature rolling technologies like Arvedi ISP and AST processes. The main goal of the mentioned new steel plate rolling technologies is the significant lowering of the rolling forces and reaching very favourable temperature field inside the plate in comparison with traditional processes. However, certain problems specific for such metal treatment arise. The central part of the material is usually still mushy. This results in changes in material density and occurrence of several characteristic temperature ranges, in which the plastic behaviour of steels varies significantly. A vital problem is also the lack of material’s thermal and mechanical properties (affecting the rheology and heat transfer) and significant influence of density on simulation results [7-10]. The material behaviour above the solidus line is
strongly temperature-dependent. There are a few characteristic temperature values between solidus and liquidus. The nil strength temperature (NST) is the temperature in which material strength drops to zero while the steel is being heated above the solidus temperature. Another temperature associated with NST is the strength recovery temperature (SRT), which is specified during cooling. At this temperature the material regains strength greater than 0.5 N/mm². Nil ductility temperature (NDT) represents the temperature at which the heated material loses its ductility. The ductility recovery temperature (DRT) is the temperature at which the ductility of the material (characterized by reduction of area) reaches 5% while it is being cooled. Over this temperature the plastic deformation is not allowed at any stress tensor configuration [7]. The temperature levels mentioned above are usually contractual. During the measurement procedures leading to determination of individual characteristic temperatures the thermocouples are located on the sample surface. The results are close to real values in case of metals such as aluminium or copper which have very high heat conductivity. The standards of the investigations lead in case of steel to lower levels of characteristic temperatures due to its low heat conductivity. The measured temperature values are usually lower than equilibrium value of liquidus temperature, although leftovers of liquid phase are observed in the sample centre. Figure 6 shows that an example temperature difference between the sample centre and surface for steel samples reaches 50°C. Taking all that into consideration one can explain the seemingly low level of characteristic temperatures in case of steel samples.

As mentioned before, very important for plastic behaviour is the material’s density. It varies with temperature and depends on the cooling rate. The solidification process causes non-uniform density distribution in the controlled volume. There are three main factors causing density changes: solid phase formation, thermal shrinkage and liquid phase flow inside the mushy zone. The density plays an important role in both mechanical and thermal solutions.

Strain-stress relationship is extremely important and has crucial influence on the metal flow path. Keeping temperature constant during the whole experiment procedure is difficult. There are also some difficulties with interpretation of measurement results. Lack of good methods of metal flow simulation and significant inhomogeneity in strain distribution in the deformation zone leads to weak accuracy of resulting stress field. On the other hand, the strain-stress curves, necessary for the mechanical model, can be constructed only on basis of a series of experiments conducted on physical simulator [7]. It points to the inverse analysis as a method of solving the problem of plastic deformation of semi-solid materials.

The inverse analysis which was used for calculation of strain-stress relationship allows the measurement of real shape of the curve due to application of numerical modelling of the deformation process and due to the fact that calculated temperature changes were controlled at the thermocouple locations on the sample surface. The computation of strain field was done using a dedicated model with an analytical form of mass conservation condition forced directly on the velocity field.

2. Dedicated FEM system

Developed by authors dedicated thermal-mechanical FEM system with variable density is related to the proposed methodology of strain-stress curves construction. The solver of the system allows simulation of compression tests of samples being in semi-solid state. Inverse solution model was developed in order to make a study of mechanical properties of selected steels. The thermal part of the applied model is also able to bring closer both resistance heating and cooling of a sample in the Gleeble equipment for different heating-cooling variants. The system includes supporting modules: input/output data interface, module dedicated to visualization of numerical results and optimization and approximation module. Most of the rigid-plastic FEM systems apply the rigid-plastic variational approach, which allows the calculation of strain field and deviatoric part of stress tensor distribution according to optimisation of functional consisting of three main parts:

\[ J' [v(r,z)] = W_\sigma + W_\lambda + W_t, \tag{1} \]

In (1) \( W_\sigma \) is the plastic deformation power, \( W_\lambda \) – the penalty for the departure from the incompressibility or mass conservation conditions and \( W_t \) – the friction power. The main idea of the presented solution is the lack of the second part of functional (1). The incompressibility condition for solid regions and the mass conservation conditions, which affect the semi-solid area are given in an analytical form, constraining the velocity field components. The second part of the right-hand side of equation (1) is unnecessary and the functional takes the following shape:

\[ J' [v(r,z)] = W_\sigma + W_t. \tag{2} \]

In (1) and (2) \( v \) describes the velocity field distribution function in the deformation zone. For solid regions of the sample the incompressibility condition can be described in cylindrical coordinate system with an equation:
by following equation:
\[
\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0,
\]
where \(v_r\) and \(v_z\) are the velocity field components in cylindrical coordinate system \(r, \theta, z\). For the mushy zone equation (3) must be replaced by the mass conservation condition, which takes a form:
\[
\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} - \frac{1}{\rho} \frac{\partial \rho}{\partial r} = 0,
\]
where \(\rho\) is the temporary material density and \(\tau\) the time variable. Both the strain and stress models are based on Levy-Misses flow criterion. Condition (4), which is more general than relationship (3), was used for the purpose of the model. The analytical form of the mass conservation condition involves the radial and longitudinal components of the velocity field making both of them density dependent [11]. Moreover, the optimization of functional (2) is much more effective than the optimization of functional (1) because numerical form of incompressibility condition generates a lot of local minima and leads to wide flat neighbourhood of the global optimum. The accuracy of the proposed solution is much better because of negligible volume loss. This is important for materials with changing density. In classical solutions the numerical errors which are caused by volume loss can be comparable to those coming from real density changes. All that leads to solution with low accuracy. The model with analytical incompressibility condition is free from the shortcomings mentioned above.

The mechanical model is completed with numerical solution of Navier stress equilibrium equations in aim to calculate the whole stress tensor components. The temperature field is a solution of Fourier-Kirchhoff equation with convection and variable density. The combined Hankel’s boundary conditions have been adopted for the presented model. The details of the presented solution were published in [11].

During the tests the samples were melted down as a result of resistance heating and then cooled to the testing temperature. The heat generated due to direct current flow was calculated according to the Joule-Lenz law given by following equation:
\[
Q = I^2 R \tau,
\]
where \(I\) is the current intensity, \(R\) is the electrical resistance and \(\tau\) is the time. The resistance was calculated according the well known formula:
\[
R = \xi_w \frac{l}{A}.
\]

In equation (6) \(l\), \(A\) and \(\xi_w\) are the sample: length, area of the cross-section and specific resistance, respectively. The temperature changes have influence on specific resistance. In the presented solutions the empirical equation was used to predict the specific resistance at a desired temperature:
\[
\xi_w = \xi_0 [1 + \alpha (T - T_0)].
\]

In equation (7) \(\xi_0\) is the specific resistance at temperature \(T\), whereas \(\alpha\) is a coefficient.

One of the most important parameters of the solution is the density. It has influence on both mechanical and thermal parts of the presented model and strongly depends on temperature. The knowledge of effective density distribution is very important for modelling the deformation of porous materials. In the presented solution an empirical model was used. Density changes were calculated according to a function being the approximation of the experimental data. It leads to slightly less accurate results, in comparison to numerical solution of Darcy differential equation, but is much faster and simplifies the total solution.

The inverse solution was applied in order to make a study of yield stress curve of the selected steel grade. In the past the method was successfully applied for the calculation of yield stress functions of steels subjected to traditional rolling [12]. In the presented solution the objective function of the inverse analysis was defined as a norm of discrepancies between calculated \((F_c)\) and measured \((F_m)\) loads in a number of subsequent stages of the compression according to following equation:
\[
\phi (x) = \sum_{i=1}^{n} [F_{ci} - F_{mi}]^2.
\]

In the present work the heart of the inverse analysis was the developed FEM solver allowing accurate computation of strain, stress and temperature fields for materials with variable density and hence the total force \(F_c\) playing role in objective function (8). In the presented approach a number of the non-gradient optimization methods were used (Hooke-Jeeves, Rosenbrock and Nelder-Mead methods) without limitation. More details concerning the presented mathematical and inverse model was published in [7,8].

3. The methodology

The proposed methodology of strain-stress curves investigation for a steel sample deformed in semi-solid state consists of following five steps:

- **Preparing sample and mounting thermocouple** using Gleeble thermomechanical equipment. The length of the sample is 120 mm and its diameter is equal to 10 mm. Figure 1 shows the shape of the
sample and thermocouple locations. In the following pages the temperature of the TC1 thermocouple is considered as the deformation temperature in aim to keep conformity with NDT temperature.

• **Prediction of NDT temperature** [13]

The prediction of NDT temperature is realized by using standard Gleeble equipment.

![Fig. 1. Scheme of samples used for the experiments. TC1 – thermocouple](image)

**Experiment course:**

- heating of the sample to solidus temperature with velocity 20±50°C/s, and above solidus temperature with velocity 1°C/s to temperature from range T±NST,
- holding for 5 seconds in order to level the temperature in the cross section of the sample,
- deformation until the sample becomes broken and necking measurement.

The predicted NDT temperature is the temperature, at which the sample does not show any necking.

- **Tension tests** in temperature range 1200°C to 1400°C (or maximum to NDT),
- **Smoothing data using Fast Fourier Transformation method**, 
- **Classical calculation of yield function** on the basis of results coming from the previous step and prediction curves coefficients by using dedicated CAE system, 
- **Inverse analysis** at temperature 1420°C and over based on compression tests results.

4. Example results

The experimental work was done at the Institute for Ferrous Metallurgy in Gliwice, Poland. The low carbon steel (18G2A) was investigated using Gleeble 3800 simulator. The chemical composition of the steel is presented in Table.

![Fig. 2. The standard Gleeble equipment allowing NDT temperature prediction](image)

<table>
<thead>
<tr>
<th>Element content (in mass%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>0.16</td>
</tr>
</tbody>
</table>

The equilibrium liquidus and solidus temperatures for carbon steel grades having similar chemical composition of the investigated steel are 1513°C and 1465°C, respectively [7]. The Nil Ductility Temperature (NDT) temperature value was measured during dynamic tests taking into consideration the conditions of real rolling process. The measured value of NDT temperature of the investigated steel is 1420°C-1425°C. Although its level is much lower than equilibrium solidus temperature we must remember that it is the temperature of the sample surface. In the central region the temperature is 50-60°C higher. Hence the remainder amount of liquid phase is observed in the sample central part [7]. Both the values – liquidus and solidus temperatures – are not fully adequate for dynamic processes and the problem requires further examination.

For the purposes of stress-strain curves computation the experiment was improved and the temperature differ-
ence in the sample cross-section was much lower than in case of experiments leading to determination of NDT and its measured value did not exceed 4°C. Strain-stress curve is one of the most important relationships having crucial influence on the metal flow path. The strain-stress curves, which are necessary for the mechanical model, were constructed on basis of a series of experiments conducted on Gleeble simulator, as well [7,8]. For the temperature range under 1400°C traditional testing methods give good results. The usual testing procedure was applied for the discussed temperature range. The curves fitting in the strain stress relationship were described by following Voce’a equation:

\[
\sigma_p = w_4 + w_5 \left[ 1 - \exp \left( -w_2 \epsilon \right) \right]^n - w_1 \left[ 1 - \exp \left( -w_3 \frac{\epsilon}{\epsilon_y} \right) \right]^m \\
\epsilon_c = w_6 \epsilon^n Z \epsilon_s \\
\epsilon_p = w_7 \epsilon_c, 
\]  

(9)

where \( w_i \) (\( i = 1, \ldots, 9 \)), \( n \) and \( m \) are the coefficients calculated by approximation of experimental data, is the logarithmic strain and \( \epsilon \) is the logarithmic strain and \( Z \) is the Zener-Holomon parameter defined as:

\[
Z = \dot{\epsilon} \exp \left( \frac{Q}{RT} \right). 
\]  

(10)

In equation (10) \( R \) is the gas constant, \( T \) - temperature, \( Q \) - activation energy, \( \dot{\epsilon} \) - strain rate. The presented flow curve is applicable in the temperature range below 1400°C. It is not easy to construct isothermal experiments for temperatures 1400°C and higher. Several serious experimental problems arise. First of all, keeping such a high temperature constant during the whole experimental procedure is extremely difficult. There are also severe difficulties concerning interpretation of the measurement results. The significant inhomogeneity in the strain distribution in the deformation zone and barrelling of the central part of the sample lead to poor accuracy of the stress field calculated using traditional methods, which are good for lower temperatures [7,8]. The inverse analysis is the only method of calculation of good coefficients of formula (9) for temperatures 1400°C and higher. Figure 3 summarises the results of an example inverse procedure applied for the identification of stress-strain curve parameters. The comparison between the calculated and measured loads is presented in Figure 4, showing good agreement between both loads.

![Graph](image-url)

Fig. 3. Flow stress vs strain at subsequent temperature values from the range above 1400°C and tool velocity 1 and 20 mm/s
Using previously presented curves, example simulations of compression of cylindrical samples with mushy zone have been performed. The potential length of the deformation zone was 67 mm (the rest of the sample was mounted in the testing equipment jaws), but only a part of it is subjected to the deformation due to temperature inhomogeneity. Example samples were melted at 1470°C and then deformed at temperatures of 1425°C and higher. During the tests each sample was subjected to 10 mm reduction of height.

In aim to demonstrate the agreement between the measured and calculated temperature fields the comparison between experimental and theoretical temperature versus time curve is presented in Figure 5 for the steering thermocouple (TC1 in Figure 1).
In Figure 6 the temperature distribution in the cross-section of the sample just before deformation at 1425°C is presented. Taking into account the value of NDT temperature as well as the temperature difference between the surface and central part of the sample one can state the existence of mushy zone in the sample volume. The non-uniform temperature distribution along the sample has a great influence on the strain field in the deformation zone. The inhomogeneity of the strain field leads to inhomogeneous stress distribution.

![Fig. 6. Initial temperature distribution (right before the deformation) in the 1/4 cross-section of the sample deformed at 1425°C](image)

In Figure 7 the calculated initial density distribution (just before the compression test) in the 1/4 cross-section of a sample deformed at 1425°C is presented.

![Fig. 7. Initial density distribution in the 1/4 cross-section of a sample deformed at 1425°C](image)

The analysis of the strain shows maximal values of strain in the central region of the sample (Figure 8).

![Fig. 8. Strain distribution in the 1/4 cross-section of the sample deformed at 1425°C](image)

### 5. Verification

Two comparative criteria were used for the verification:

- comparison between the measured and calculated length of zone, which is not subjected to the deformation,
comparison the measured and calculated maximal diameter of sample.

Figures 9 and 10 show example application of the 1st and 2nd criterion, respectively. The figures confirm good agreement between theoretical and experimental results.

Fig. 9. The comparison of the measured and calculated length of zone which was not subjected to the deformation – experiments at temperature 1425°C and tool velocity 1, 20 and 100 mm/s

Fig. 10. The comparison of the maximum measured and calculated diameters of the sample – experiments at temperature 1425°C and tool velocity 1, 20 and 100 mm/s

6. Conclusions

In the paper the possibilities of computer aided methodology of strain-stress curve construction for semi-solid steels on the basis of compression and tension tests were presented. The applied methodology which is closely related to dedicated CAE system is very helpful and may enable the right interpretation of mechanical results of very high temperature tests. Good predictive ability regarding both maximum measured diameters and size of the deformation zone has been observed. Hence, the computed yield stress functions of the tested steel being in semi-solid state seem to have right shape due to implementation of inverse analysis which allowed the calculation of strain-stress relationships for strongly inhomogeneous strain distribution. Simulation of compression process has given right results due to application of right, sophisticated mathematical model of the compression process.

Concluding one can state that the inverse analysis, which was used together with developed dedicated model of steel deformation in semi-solid state has delivered real yield stress functions for the temperature range above 1400°C. The calculation of strain-stress curve shape at very high temperature is possible only as a result of collaboration of good experimental technique and computer simulation. The temperature difference between the sample centre and surface can be significant and the measurement technique should be subjected to further improvement in aim to get more adequate experimental data leading to better accuracy of the yield stress function for semi-solid steel. Nevertheless the application of the inverse analysis and suitable shape of the fitting curve as well as appropriate model of the compression test leads to realistic results of the simulation.

Acknowledgements

The work has been supported by the Ministry of Science and Higher Education Grant No N R07 0018 04.

REFERENCES


Received: 10 December 2008.