A R C H I V E S O F M E T A L L U R G Y A N D M A T E R I A L S
Volume 56 2011 Issue 2
DOI: 10.2478/v10172-011-0060-6

A. SKRZAT*

FUZZY LOGIC APPLICATION TO STRAIN-STRESS ANALYSIS IN SELECTED ELASTIC-PLASTIC MATERIAL MODELS

ZASTOSOWANIE LOGIKI ROZMYTEJ DO ANALIZY ODKSZTAŁCEŃ I NAPRĘŻEŃ DLA WYBRANYCH MODELI MATERIAŁÓW SPRĘŻYSTO-PLASTYCZNYCH

"As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality."
Albert Einstein

The common engineering practice is the selection of certain physical factors such as geometry, loading, boundary conditions, and material properties. In reality these data are more or less uncertain. In many engineering problems this uncertainty should be taken into consideration. Fuzzy set theory allows to determine randomness model response to the external loading. Two methods - extension principle and α-level optimization are presented and compared in this paper. Advantages and disadvantages of these approaches are discussed on example of mapping function. Next the extension principle and α-level optimization are applied to the analysis of stresses in the bar made of elastic-plastic material subjected to uniaxial tension. The yield stress and hardening modulus are fuzzy variables. The influence of fuzziness of numerical procedure (large integration step) to the result space is also discussed.

The main part of this paper is the investigation of influence of selected fuzzy parameters of Bodner-Partom material model to stress analysis in the stretched bar. BP material model allows to take into consideration elastic and plastic material properties, isotropic and kinematic hardening, visco-plastic effects, as well as creep and relaxation. The proper response of the material to external loading requires the correct selection of fourteen material constants. The experimental determination of these data is problematic, and gives imprecise results. The increase of accuracy of BP material parameters may be achieved by application of optimization procedures or genetic algorithms. In this research for the first time the fuzzy sets theory is applied to accomplish this goal.

Keywords:fuzzy logic, unified plasticity, Bodner-Partom material

Powszechną praktyką w obliczeniach inżynierskich jest przyjmowanie za dokładne pewnych wielkości typu geometria, obciążenia, warunki brzegowe czy właściwości materiału. W rzeczywistości wielkości te charakteryzują się pewnym rozrzutem, co w przypadku wielu problemów powinno być uwzględniane. Teoria zbiorów rozmytych pozwala na określanie rozmytej odpowiedzi układu na działanie obciążeń przy założeniu niepewności danych wejściowych i modelu matematycznego bądź jego numerycznej implementacji. W artykule przedstawiono i porównano ze sobą dwa podejścia logiki rozmytej: zasadę rozszerzeń i optymalizację α -przekrojami. Zalety i wady obydwu podejść omówiono na przykładzie funkcji mapującej. Obie metody zastosowano następnie w analizie naprężeń w rozciąganym pręcie wykonanym z materiału sprężysto-plastycznego. Granica plastyczności i moduł umocnienia stanowiły zmienne rozmyte. Przedyskutowano wpływ rozmycia procedury numerycznej (zbyt duży krok całkowania) na rozmycie rozwiązania.

Zasadniczą część artykułu stanowi badanie wpływu wybranych rozmytych parametrów modelu materiału Bodnera-Partoma na poziom naprężeń w pręcie rozciąganym w zakresie sprężysto-plastycznym. Model materiału BP pozwala na równoczesne uwzględnienie w analizie właściwości sprężystych i plastycznych, umocnienia izotropowego i kinematycznego, efektów lepko plastycznych, pełzania oraz relaksacji. Właściwa odpowiedź materiału na działanie obciążeń zewnętrznych zależy od prawidłowego przyjęcia wartości czternastu stałych materiałowych. Określenie wartości tych stałych na podstawie badań eksperymentalnych jest problematyczne i mało precyzyjne. Zwiększenie dokładności wyznaczania stałych materiału BP może być osiągnięte poprzez zastosowanie procedur optymalizacji oraz algorytmów genetycznych. W niniejszej pracy po raz pierwszy w tym celu zastosowano zasady logiki rozmytej.

* RZESZÓW UNIVERSITY OF TECHNOLOGY, DEPARTMENT OF MATERIALS FORMING AND PROCESSING, 35-959 RZESZÓW, 8 POWSTAŃCÓW WARSZAWY STR., POLAND

1. Introduction

There are many situations where imprecise or incomplete information about a problem is available until an approximate solution is obtained. In numerical simulations of metal forming processes many parameters e.g. estimated and experimental loads, friction coefficient and material properties are imprecise and often uncertain. Soft computing-based methods accept the presence of imprecision and uncertainty, while attempting to find reasonable solution. The most popular soft computing approaches are fuzzy sets, neural networks and genetic algorithms. Fuzzy set theory allows to determine randomness model response to the external loading. Two fuzzy logic approaches - extension principle and α-level optimization are used as the numerical tool in this paper. Both methods are introduced in benchmark test of the analytically defined mapping function. Advantages and disadvantages of these approaches are discussed.

Next the extension principle and α-level optimization are applied to the elastic-plastic analysis of stress in the bar subjected to uniaxial tension. The purpose of this fuzzy analysis is the prediction of the bar response to the external load while the uncertainty of material parameters is considered. Obtained results provide the information not only about stress changeability but also about the level of acceptance (reliability) of bar response. Conclusions gained in this one-dimensional test may be extended to sophisticated three-dimensional problems.

Three different elastic-plastic material models namely: Prager-Ziegler, Armstrong-Federick and Bodner-Partom are considered. For the first two material models (which represent the classical theory of plasticity) the yield stress and hardening modulus are assumed to be fuzzy variables. The Bodner-Partom material model which is an example of materials considered in unified theory of plasticity allows to predict elastic-plastic material response while simultaneously isotropic and kinematic hardening, visco-plastic effects, as well as creep and relaxation are respected. Bodner-Partom material model may be applied as universal tool in the analysis of metal forming processes, therefore. The proper response of the BP material to external load unfortunately requires the correct selection of fourteen material constants (some of them are assumed in this paper to be fuzzy variables). The experimental determination of these data is problematic, and gives imprecise results. The increase of accuracy of BP material parameters may be achieved by application of sophisticated optimization procedures or genetic algorithms. In this research for the first time the fuzzy sets theory is applied to accomplish this goal.

2. Uncertain structural analysis

The realistic analysis of structures requires reliable input data as well as consistent computational models. As a rule both the data and mathematical model contain uncertainties. Unlike in deterministic structural analysis fuzzy structural analysis takes both data and model randomness into consideration. A typical example of fuzziness is presented in Fig. 1.

The fuzzy input variables x_1, x_2 are described by membership functions $\mu(x_1)$, $\mu(x_2)$. No general algorithm exist in order to indicate the membership functions. Membership functions for structural parameters may be specified on the basis of samples [1]. The measured values, possible measurement errors, experience gained from comparable problems and additional information are helpful in the derivation of the membership functions. The membership function may be interpreted as gradual assessment of the truth content of a set of measured values. They provide the information about the scattering of input parameters and about the level of acceptance of this disperse. The triangular, trapezoidal, Gauss (infinite support) or modified Gauss (finite support) functions are usually applied to describe the membership functions.

The aim of fuzzy structural analysis is the mapping of fuzzy input parameters into the result space with the aid of an analysis algorithm resulting from the mathematical model. The obtained results are also fuzzy quantities described by result membership functions. In the fuzzy sets theory the extension principle is usually applied as a general approach to derive the membership function associated with the fuzzy result variable. Unfortunately, in the case of sophisticated mapping models the extension principle is unusable. The application of α-level optimization approach provides much better results with the same computational effort.

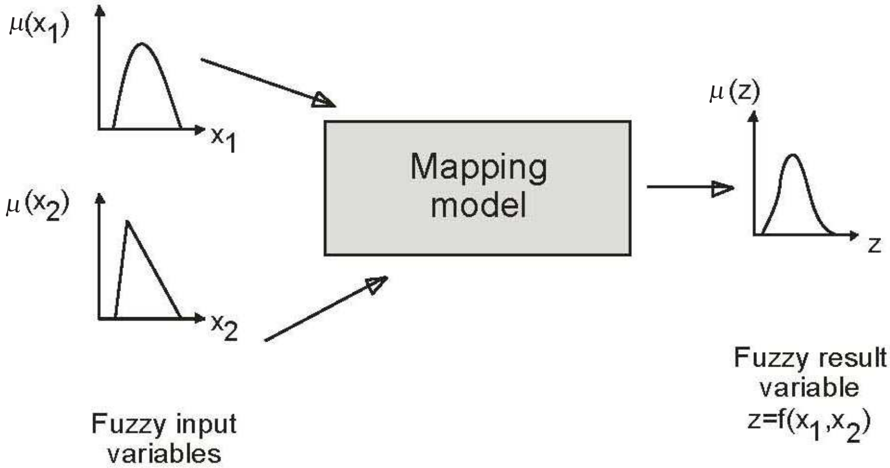

Fig. 1. Mapping of fuzzy input variables into result space

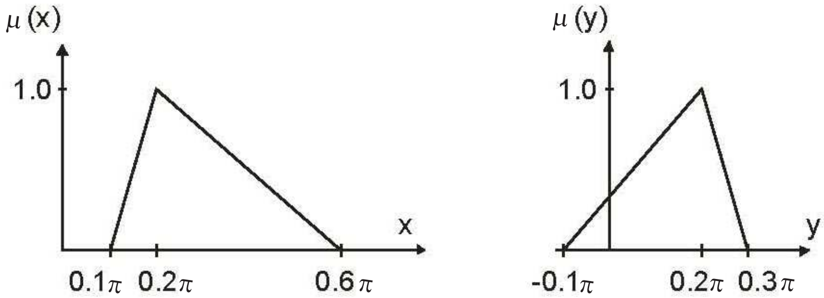

Fig. 2. Benchmark test membership functions

3. The extension principle versus α-level optimization.

The extension principle and α-level optimization approaches are presented in this chapter. These two methods are compared in a benchmark test in which the mapping model (Fig. 1) receives the form of an analytical function (1):

$$z = f(x, y) = \sin x \cos y \tag{1}$$

Arguments x and y are assumed to be fuzzy variables. Two triangle membership functions associated with x and y parameters are shown in Fig. 2.

It is assumed that the range of x argument is $[0.1\pi, 0.6\pi]$. The most reliable x value is 0.2π for which the membership function is unit. Fig. 2 shows that $x < 0.1\pi$ and $x > 0.6\pi$ are not accepted (membership function equals zero).

Optimal membership function can be estimated by means of machine learning (neural networks) or optimization methods (genetic algorithms). Although normal (Gaussian) distribution is adequate to the most of real problems, usually linear distribution is assumed. For linear membership functions which easily satisfy partition of unity condition [1, 2] the fuzzy results are reliable, and the numerical procedure is effective.

In this benchmark test the magnitude of z is searched for $x \approx y \approx 0.2\pi$($x$ and y are about 0.2π). The uncertainty of input variables causes the fuzziness of resulting z variable.

On the basis of the extension principle the mapping function $f(x, y)$ leads to the fuzzy set with the membership function

$$\mu(z) = \sup \min [\mu(x), \mu(y)] \ , \ \exists z = f(x, y) \tag{2}$$

For ranges of fuzzy variables x and y divided into n=50 evenly subdomains the following solution is obtained – Fig. 3.

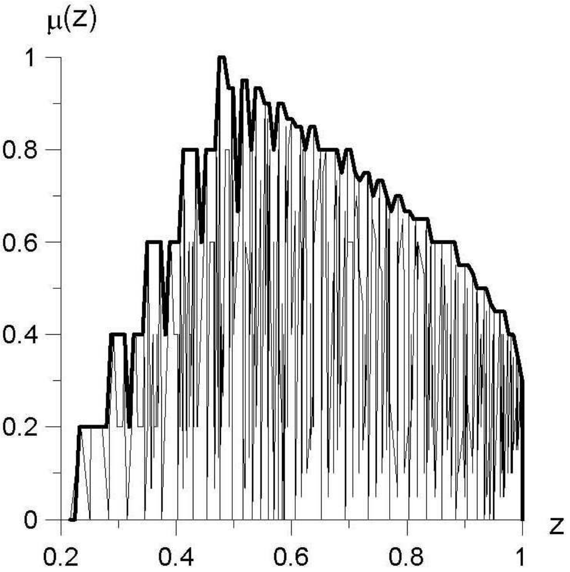

Fig. 3. Fuzzy result and smoothed solution according to the extension principle *n*=50

The application of the *sup* operator in (2) is represented in Fig. 3 by thick line envelope. For floating-point numbers the maximum of $\min\left[\mu(x), \mu(y)\right]$ may be only derived for the range $z \pm \varepsilon$, where ε is the assumed precision. For the small number of subdomains such smoothing causes a jagged solution.

Better smoothing may be achieved by assuming $n > 100$. For n=400 the smoothed solution is comparable with the theoretical one.

The extension principle is hardly applicable in complex mapping models. The principle is very sensitive to:

- the number of combinations of elements from the fuzzy input variables
- the assumed precision ε when two close elements z_1 and z_2 are considered equal or not
- the max-min operator applied to discrete combinations of elements from the fuzzy input sets. This reduces the accuracy of the membership values and favors lower bounds for the graph of the actual membership function.

In order to develop a suitable method for processing fuzzy input variables the concept of α-discretization is adopted [2]. An alternative representation of fuzzy sets based on sufficiently high number of α-levels is developed. The subspace assigned to α_K level $\alpha_K \in [0, 1]$ is determined by extreme $x_{\alpha_k l}$, $x_{\alpha_k r}$ values as shown in Fig. 4.

For the α-level representation of fuzzy input variables with the aid of the mapping operator, the minimum $z_{\alpha_k l}$ and maximum $z_{\alpha_k r}$ elements can be found. The search for smallest and largest elements is formulated as an optimization problem (3)

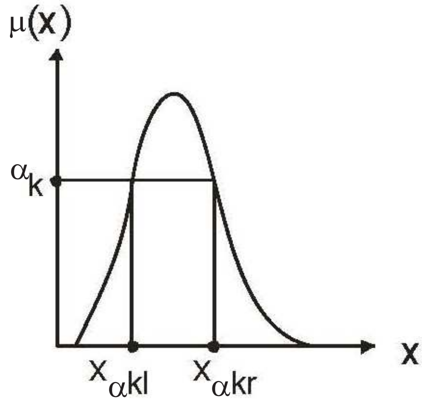

Fig. 4. α-discretization of the fuzzy set

$$z = f(x, y) \Rightarrow Max \quad (x, y) \in X_{\alpha k} \tag{3}$$
$$z = f(x, y) \Rightarrow Min \quad (x, y) \in X_{\alpha k}$$

The requirements $(x, y) \in X_{\alpha k}$ represent constraints of the optimization problem. The optimization problem (3) performed for all α-levels and all fuzzy result values is referred as α-level optimization. The optimization is possible unless the mapping operator is continuous and unique, and fuzzy result space is convex [3]. The α-level representation is thus an alternative way (when compared to the extension principle) to find fuzzy result membership function. Obtained result membership functions are much smoother when compared to ones provided by application of extension principle.

The fuzzy result space obtained for function (1) by application of α-level optimization procedure is shown in Fig. 5.

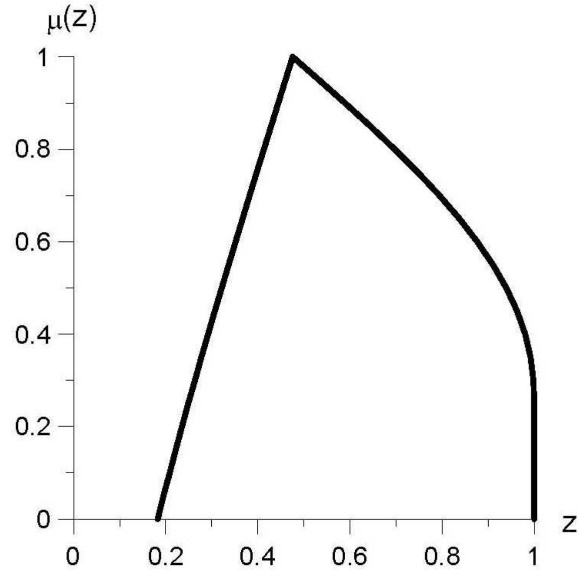

Fig. 5. Fuzzy result according to the α-level optimization

The analysis shows that acceptable range of fuzzy result z is [0.18602, 1.0]. The most reliable (unit membership function) magnitude of z is 0.4709. These results conform to ones derived by an analytical solution.

The fuzzy results obtained by α-level optimization procedure is attainable by the extension principle only for very dense discretizations of input fuzzy variables. In structural analysis α-level optimization is preferred, therefore. In this simple benchmark test the mapping model is defined explicitly as a function. In structural analysis it is possible only for very simple problems. Usually the mapping model gets the form of a numerical.

4. Stress-strain analysis in elastic-plastic kinematic hardening material model

Following simple benchmark test from the previous chapter the extension principle and α-level optimization are applied to the numerical analysis of stress in the bar subjected to uniaxial tension. The bar is made of elastic-plastic material. The strain increases linearly up to 1% at the end of the test. The goal of this fuzzy analysis is the prediction of maximum axial stress (fuzzy result variable) in the stretched bar while the uncertainty of material parameters is considered. Two types of kinematic hardening material models [4] are considered: Prager-Ziegler material model – linear hardening, and Armstrong-Frederick material model – non-linear hardening. Both models are defined by the following equations:

Stress-strain relation for elastic deformation:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}^{el} \tag{4}$$

Plastic flow law:

$$\dot{\varepsilon}_{ij}^{pl} = \frac{3}{2}\frac{\dot{\varepsilon}^{pl}}{\sigma_Y}\left(s_{ij} - x_{ij}\right) \tag{5}$$

Yield function:

$$\sqrt{\frac{3}{2}\left(s_{ij} - x_{ij}\right)\left(s_{ij} - x_{ij}\right)} - \sigma_Y = 0 \tag{6}$$

In equations (4)-(6)

$$\dot{\varepsilon}^{pl} = \sqrt{\frac{2}{3}\dot{\varepsilon}_{ij}^{pl}\dot{\varepsilon}_{ij}^{pl}} \tag{7}$$

is equivalent plastic strain,

$$s_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk} \tag{8}$$

is deviatoric stress,

$$dx_{ij} = \frac{2}{3}cd\varepsilon_{ij}^{pl} - \gamma\, x_{ij}d\varepsilon^{pl} \tag{9}$$

is the increment of the back stress and γ is the material constant (in Prager-Ziegler material model $\gamma = 0$).

For simple uniaxial loading the stress increment is related to strain increment as shown in (10)

$$d\sigma = E\left(1 - \frac{E}{E + c - \gamma x}\right)d\varepsilon \tag{10}$$

Numerically found hysteresis loops for both models are presented in Fig. 6. Here: E=2e5 MPa, σ_Y=300MPa, c=1e4 MPa, γ=100, $\varepsilon \in [-0.1\ ,\ 0.1]$.

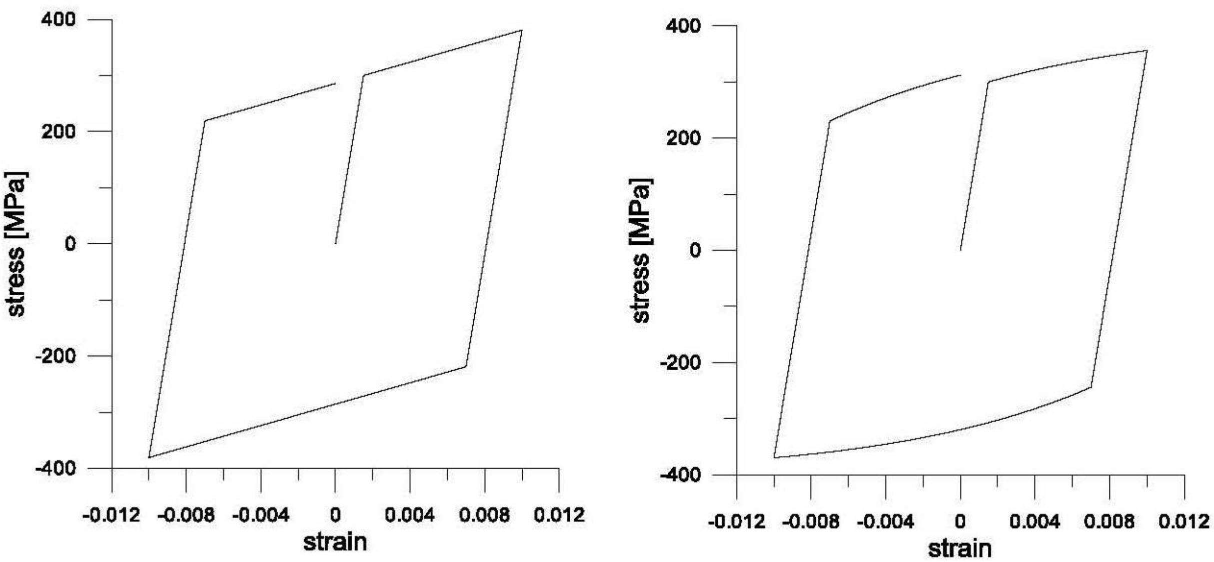

Fig. 6. Hysteresis loops for Prager-Ziegler (left) and Armstrong-Frederick (right) material models

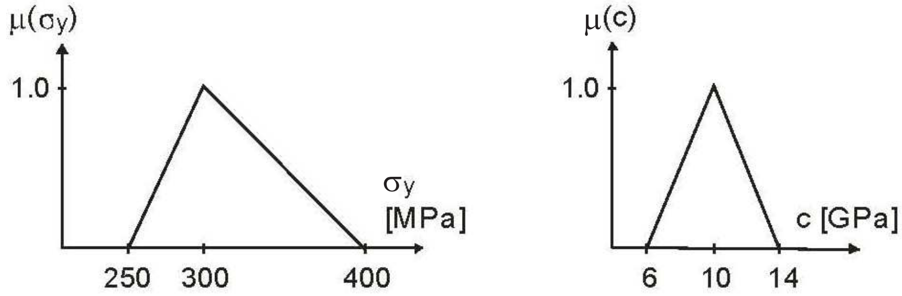

Fig. 7. Membership functions of fuzzy input variables

The yield stress and strain hardening modulus are fuzzy variables in this analysis. In this numerical simulation for testing purposes the range [250, 400] MPa of yield stress variation is assumed. Fig. 7 shows triangular membership functions associated with both fuzzy variables. This simulation answers the question what stress arises in the stretched bar up to 1% if yield stress σ_Y is about 300 MPa, and strain hardening modulus c is about 5% of Young's modulus.

The fuzzy result stresses in the stretched bar obtained by application of the extension principle and α-level optimization for Prager-Ziegler material model are shown in Fig. 8.

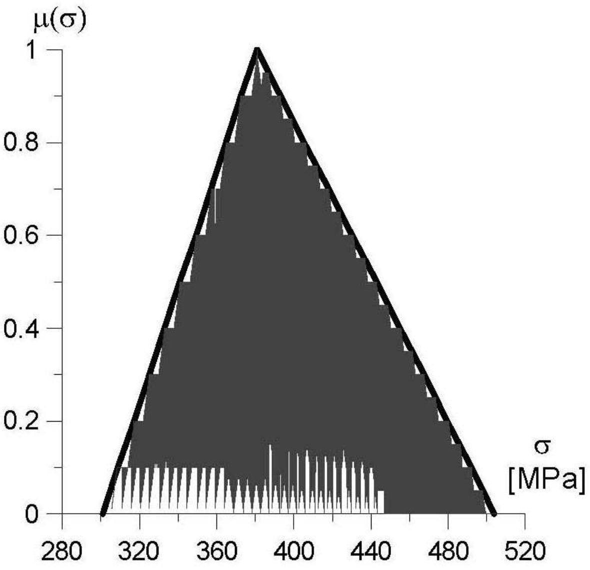

Fig. 8. The fuzzy result stress – Prager-Ziegler material model

The fuzzy result stress for Armstrong-Frederick material model (not presented here) is comparable to the above solution. Fig. 8 shows that the variation of axial stress is [301.0, 503.9] MPa. Both approaches give similar results, but the solution obtained by α-level optimization is smoother. The result membership function is almost linear. It is caused by the problem itself (mapping model) as well as by assumed shape of membership functions (linear).

One of the most important features in fuzzy set theory is the defuzzification. Defuzzification is the conversion of fuzzy result into a precise output. Defuzzification of the fuzzy variable represents the mapping of fuzzy variable into the crisp value for which the uncertainty is evaluated. Several defuzzification algorithms are available: height method, centroid method, level rank method, Jain and Chen defuzzification and others. In the height method known also as the *maximum membership principle* the number of maximum degree of membership is chosen. This method can be applied only if output contains the maximum peak. For numerical simulation presented here the maximum of membership function is reached for stress 380.95 MPa. This is the most reliable solution for stress in the stretched bar.

Very interesting results are obtained for the case when both input variables and the mapping model are uncertain. In the numerical algorithm of solving equations (4) to (9) the explicit integrations scheme is chosen. This type of integration is conditionally stable i.e. when the step increment is too large the convergence does not exist [5]. In such a case the fuzziness of mapping model is caused by the improper integration procedure. The fuzzy result stress obtained for the large step increment (Fig. 9) clearly shows the problems arising in the integration procedure.

The result membership function shown in Fig. 9 is nonlinear. The solutions obtained by the application of extension principle and α-level optimization differ significantly. Only 400 time increments are executed here in explicit integration of (4) to (9), while in the previous test (Fig. 8) 10000 integrations are made. Numerically found stress-strain curves (not presented here) for simple tension test obtained for 400 and 1000 integration increments are almost identical. It seems that 400 time increments explicit integration gives reliable results, therefore. However, fuzzy logic analysis clearly shows (Fig. 9) existence of convergence problems. The fuzzy set theory is then a very sensitive tool for detection of possible problems arising in mathematical models and applied numerical procedures.

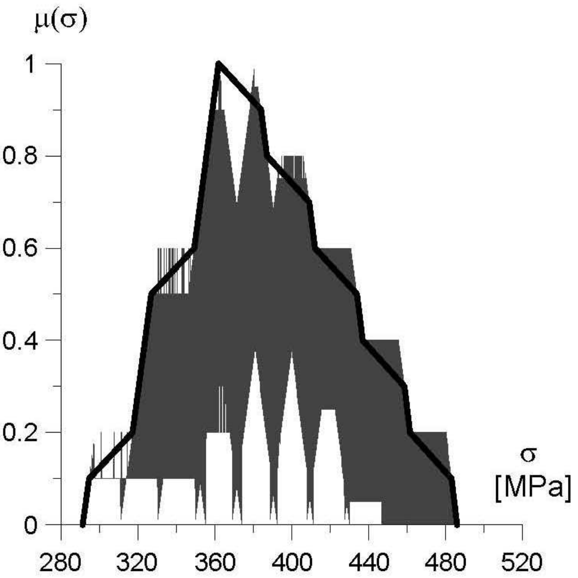

Fig. 9. The fuzzy result stress for the case of fuzzy mapping model caused by large integration step

5. Bodner-Partom material model

Many constitutive theories were proposed over the past years. Many plasticity models based on the second invariant of deviatoric stress have been found useful for practical applications. Better understanding of physics of deformation causes progress in theories of materials inelastic response. Unified theories of plasticity describe the time dependent inelastic behavior in formulations which do not rely on a yield criterion or loading and unloading conditions. Macroscopic equations represent the principle response properties such a strain rate sensitivity and temperature dependence for inelastic deformation, stress saturation under straining, isotropic and directional hardening for monotonic and reversed loadings, creep, thermal recovery and stress relaxation. To be useful, the equations should be reasonably simple and consistent with the principles of mechanics and thermodynamics. All the characteristics of inelastic deformations are usually represented in a single strain term. The evolution equation of this term includes internal variables whose number depends on the hardening mechanisms and the complexity of the loading history.

The Bodner-Partom material (BP) is an example of unified theory of plasticity. BP material model allows to take simultaneously into consideration elastic and plastic effects, isotropic and kinematic hardening, visco-plasticity, creep and relaxation for a wide range of temperature. For the elastic deformations the relation between stresses and strains is linear (generalized Hooke's law). The plastic flow law constitutes the relation between these variables for inelastic deformations. The description of plastic multiplier is sophisticated in order to include in the analysis all types of nonlinearities mentioned above [6].

The BP model does not use the yield condition. For increased load the participation of inelastic strains advantages the participation of elastic ones.

The Bodner-Partom material model is defined by the following equations:

Superposition of elastic and inelastic strains

$$\varepsilon_{ij} = \varepsilon_{ij}^{(e)} + \varepsilon_{ij}^{(ie)} \tag{11}$$

Incompressibility condition for inelastic deformations

$$\varepsilon_{kk}^{(ie)} = 0 \tag{12}$$

Plastic flow law

$$\dot{\varepsilon}_{ij}^{(ie)} = \lambda\, s_{ij} \tag{13}$$

where plastic multiplier is

$$\lambda = \sqrt{\frac{D_0^2}{J_2} \exp\left(-\left(\frac{Z^2}{3J_2}\right)^n\right)} \tag{14}$$

In (14) D_0 and n are BP material constants, J_2 is the second invariant of deviatoric stress. State variable $Z = Z^I + Z^D$ represents the resistance of material to inelastic deformations - both isotropic Z^I and directional Z^D.

The evolution of Z^I is defined as

$$\dot{Z}^I = m_1 \left(Z_1 - Z^I\right) \dot{W}_p - A_1 Z_1 \left(\frac{Z^I - Z_2}{Z_1}\right)^{r1} \tag{15}$$

Parameter Z^D depends on tensorial quantity β_{ij}

$$\dot{\beta}_{ij} = m_2 \left(Z_3 \frac{\sigma_{ij}}{\|\sigma_{ij}\|} - \beta_{ij}\right) \dot{W}_p - A_2 Z_1 \left(\frac{\|\beta_{ij}\|}{Z_1}\right)^{r2} \frac{\beta_{ij}}{\|\beta_{ij}\|} \tag{16}$$

$$Z^D = \beta_{ij} \frac{\sigma_{ij}}{\|\beta_{ij}\|} \tag{17}$$

Here: m_1, m_2, A_1, A_2, Z_1, Z_2, Z_3, r_1, $r_2$are BP material constants described in Table 1.

TABLE 1
Bodner-Partom material parameters

Constant	Unit	Description
E	MPa	Elastic modulus
ν	–	Poisson's ratio
D_0	s^{-1}	Limiting shear-strain rate
Z_0	MPa	Initial value of isotropic hardening variable
Z_1	MPa	Limiting value for isotropic hardening
Z_2	MPa	Fully recovered value for isotropic hardening
Z_3	MPa	Limiting value for kinematic hardening
m_1	$(MPa)^{-1}$	Hardening rate coefficient for isotropic hardening
m_2	$(MPa)^{-1}$	Hardening rate coefficient for kinematic hardening
n	–	Strain rate sensitivity parameter
A_1	s^{-1}	Recovery coefficient for isotropic hardening
A_2	s^{-1}	Recovery coefficient for kinematic hardening
r_1	–	Recovery exponent for isotropic hardening
r_2	–	Recovery exponent for isotropic hardening

Tensorial norms $\|\sigma_{ij}\| = \sqrt{\sigma_{ij}\sigma_{ij}}$ $\|\beta_{ij}\| = \sqrt{\beta_{ij}\beta_{ij}}$.

For inelastic strain rate $\dot{\varepsilon}_{ij}^{(ie)}$ derived from (13) the elastic stress rate $\dot{\sigma}_{ij}^{(e)}$ is determined from the generalized Hooke's law

$$\dot{\sigma}_{ij}^{(e)} = C_{ijkl}\left(\dot{\varepsilon}_{kl} - \dot{\varepsilon}_{kl}^{(ie)}\right). \tag{18}$$

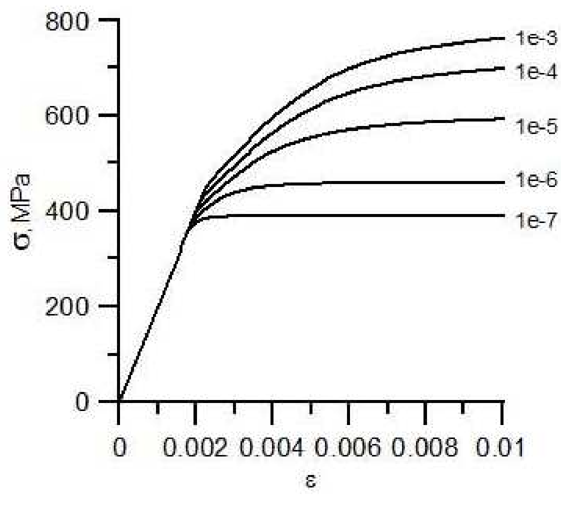

Fig. 10. Stress-strain plots for various strain rates

The response of BP material to external loading depends on initial magnitudes of material constants and loading history. Special analysis cases are: purely elastic solution, elastic-visco plastic solution, cyclic plasticity, creep under constant stress, relaxation under constant strain. The BP material response cannot be determined a' priori without integration of (15) and (16). In Fig. 10 presented is special case of stress-strain plot of BP material model which takes into consideration strain rate dependence.

6. Uncertain stress-strain analysis in Bodner-Partom material model

In this chapter numerical simulations of the truss subjected to uniaxial tension are preformed. Mechanical behavior of this bar is described by BP material model. For the strain increasing monotonically up to 1% the stress in the bar at the end of test is searched. The resulting stress depends on BP material constants magnitudes. The most of BP material constants may be determined [6] in uniaxial tension tests completed for various load rates. Some of them are determined from the $\eta - \sigma$ plot (Fig. 11) where:

$$\eta = \frac{1}{\sigma}\frac{d\sigma}{d\varepsilon^{(ie)}}. \tag{19}$$

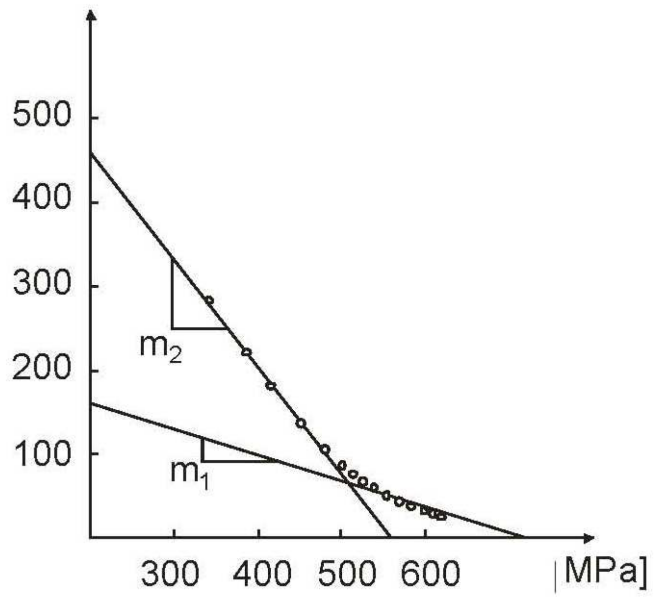

Fig. 11. $\eta - \sigma$ plot for identification of B-P parameters

From exemplary plot $\eta - \sigma$ shown in Fig. 11 the parameters m_1, m_2 are computed as the solution of two linear regression problems (tangent of two sections in Fig. 11), Z_1, Z_3 are found in an extrapolation of two straight sections to $\eta = 0$. The quality of BP parameters found in this way is rather poor. It is caused by ill-conditioned differentiation (19). The precision of determination of BP parameters may be improved by solving the optimization problem [7] or by the application of genetic algorithms [8]. In this paper for the first time the fuzzy set theory is applied to accomplish this task.

In presented simulation selected BP material constants (m_1, m_2, n, Z_1, Z_3) are treated as fuzzy variables. The initial magnitudes of BP parameters are taken from [6] for B1900 + Hf nickel based alloy. These are: $D_0 = 10^4$ s^{-1}, $m_1 = 0.27$ MPa^{-1}, $m_2 = 1.52$ MPa^{-1}, Z_1=3000 MPa, Z_2=1150 MPa, n=1.0

The range of BP parameters variation is chosen in comparison of Bodner and Chan results [6] with results obtained by other researches [7]. A pseudo normal membership functions are assumed. These functions unlike the Gauss distribution have finite support – range $\lambda_1 \div \lambda_2$ in (20). Infinite support is not accepted because in this case any magnitude of fuzzy variables is available. The modified Gauss distribution (20) chosen as the shape of membership functions is defined as:

$$\mu(x) = \exp \frac{4(\lambda_2 - x)(x - \lambda_1) - (\lambda_2 - \lambda_1)^2}{4(\lambda_2 - x)(x - \lambda_1)} \qquad x \in [\lambda_1, \lambda_2] \tag{20}$$

Normal and pseudo normal distributions of membership function are more adequate to reality than the triangular distribution. The function (20) is class C^∞ and the derivative in the central point is continuous. The disadvantage of use of the function (20) is that this function does not satisfy the partition of unity requirement.

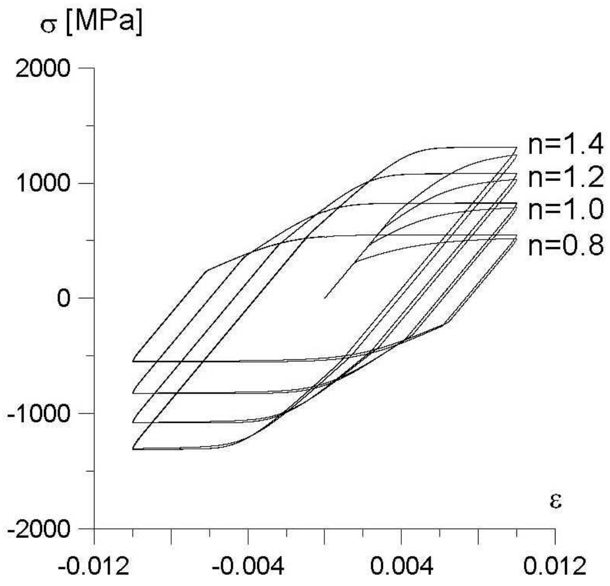

Fig. 12. Hysteresis loops for various n magnitudes

Fuzzy set theory is often called a method of computing with language. It uses linguistic variables like: tall and small, smooth and rough, thin and thick, wide and narrow. A linguistic variable is a fuzzy variable and is often associated with fuzzy set quantifiers like: very, more or less, slightly etc. Linguistic variable "concentrated" by quantifier requires proper definition of membership function. In this research the fuzzy variable n is assumed to be a linguistic variable. The property of this variable is based on operator knowledge and experience. Numerical simulations made for different n show that the this variable influences on the width of hysteresis loop in cyclic tension and compression test (Fig. 12).

The membership functions associated with fuzzy variable n comprises the terms: very narrow, narrow, medium, wide and very wide as shown in Fig. 13.

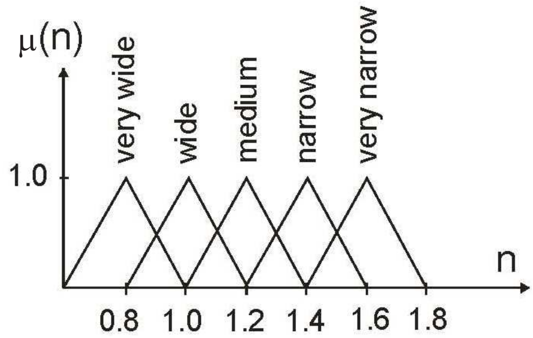

Fig. 13. Membership functions of the linguistic variable n

Assumed triangular distribution of membership functions satisfies the requirement of partition of unity.

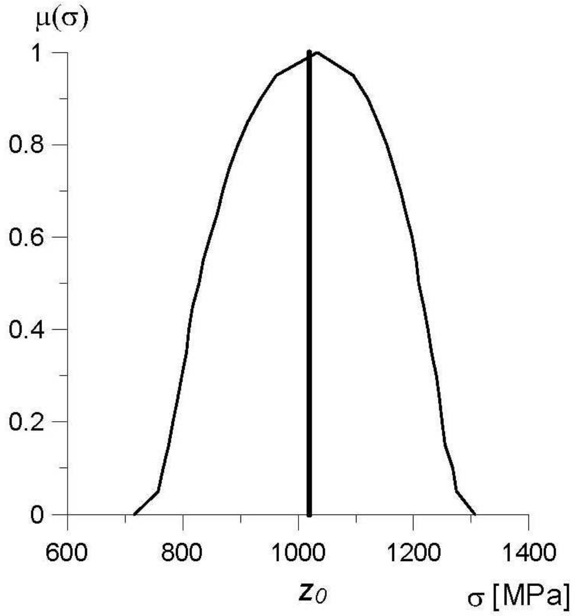

Fig. 14. Fuzzy result stress and the crisp value

Many numerical simulations of uniaxial tension test are carried out for different number of fuzzy Bodner-Partom material constants (from two to five fuzzy variables) and for different shapes of membership functions. In Fig. 14 presented is an exemplary solution of the fuzzy stress in the bar obtained for five m_1, m_2, n, Z_1, Z_3fuzzy variables. Non-fuzzy result stress gained by defuzzification procedure is also presented in this figure (vertical line).

The variation of maximum axial stress in the bar is [715.71, 1306.40] MPa. The crisp stress z_0 =1019.29 MPa represented in Fig. 14 by the vertical line is computed by centroid method (21)

$$z_0 = \int_z z\mu(z)\,dz \Big/ \int_z \mu(z)\,dz \qquad (21)$$

This crisp stress z_0 is the most reliable magnitude of maximum stress in the bar computed for uncertain BP material parameters. This result should be compared with the results of experimental investigations of specimen made of B1900 +Hf nickel based alloy. Compatibility criterion of numerical and experimental results allows for proper selection of magnitudes of BP material parameters. However, simulations of simple tension test with easy-to-interpret results (maximum stress) are not sufficient to accomplish this task. In numerical tests the crisp stress z_0 may be obtained for many different magnitudes of fuzzy variables. To obtain an unique solution the whole stress-strain curve should be analyzed and appropriate integral-form error norm should be assumed. In experimental investigations various strain rates should be considered. More advanced experimental investigations are also recommended. Such typical tests used in investigation of elastic-plastic problems are: bent beam and thick-walled cylinder under internal pressure.

7. Conclusions

Fuzzy logic is a powerful tool for the analysis of data and mathematical models which contain uncertainties. In an engineering practice nearly all problems are more or less uncertain. The fuzziness of input data e.g. yield stress, strain hardening modulus, may be determined on the basis of statistics or developer experience gained in the investigation of a particular problem or similar problems. The influence of the selected model parameters on randomness of the model response can be investigated. In the defuzzification procedure the most valuable result may be selected from the set of fuzzy result variable.

The Bodner-Partom material model is a very good example of fuzziness. Over ten material model constants are determined in the experimental tests enriched by numerical procedures. The quality of obtained BP parameters is questionable and strongly depends on the type of experimental tests and instruments precision. A good practice is the assumption of BP constants randomness and execution of fuzzy logic analysis.

In this paper only the simple tension test is considered. The maximum stress reached at the end of test is a simple and easy-to-interpret parameter. In order to investigate more precisely the influence of Bodner-Partom material constants on the model response, the whole stress-strain curve should be considered. The elastic-plastic beam bending test as well the thick-walled cylinder under internal pressure test are also welcome. The comparison of the results obtained in the experimental investigations and defuzzificated results of fuzzy logic analysis can help to select the optimal magnitudes of BP material model parameters.

Presented here numerical simulations show the potential of application of fuzzy logic analysis to solve inelastic problems. Parameters of elastic-plastic material models are always more or less imprecise. Fuzzy set theory allows to find the variation of material response to the external load when uncertainness of input parameters is taken into consideration. The advantage of fuzzy logic over deterministic methods is the ability of use linguistic variables. This way numerical procedures are related to human language.

Fuzzy set theory brings the new quality to many scientific and engineering problems. In particular fuzzy logic is very useful in the analysis metal forming processes and investigations of elastic-plastic material behaviors.

Received: 10 January 2011.

REFERENCES

[1] A. Piegat, Modelowanie i sterowanie rozmyte, Exit, Warszawa (2003).

[2] B. Möller, M. Beer, Fuzzy Randomness, Springer-Verlag, Berlin Heildelberg (2004).

[3] B. Möller, W. Graf, M. Beer, Fuzzy Structural analysis using a-level optimization, Computational Mechanics **26**, 547-565, Springer-Verlag (2000).

[4] F. Dunne, N. Petrick, Introduction to computational plasticity, Oxford University Press, Oxford New York (2005).

[5] W. Cecot, Adaptive FEM analysis of selected elastic-visco-plastic problems, Computer Methods in Applied Mechanics and Engineering **196**, 3859-3870, Elsevier (2007).

[6] S.R. Bodner, Unified plasticity for engineering applications, Kluwer Academic, New York Boston (2002).

[7] I.K. Senchenkov, G.A. Tabieva, Determination of the parameters of the Bodner-Partom model for thermoviscoplastic deformation of materials, Int. Applied Mechanics **32(2)**, 132-139, Springer (1996).

[8] R. Mahnken, E. Stein, Parameter identification for viscoplastic models based on analytical derivatives of a least-squares functional and stability investigations, Int. J. Plasticity **12(4)**, 451-479, Elsevier (1996).