Arch. Metall. Mater. **66** (2021), 2, 537-542

DOI: 10.24425/amm.2021.135890

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OPTIMIZATION OF A BANDGAP IN THE ULTRASONIC PHONONIC COATING

This work concerns the study of the coatings for the ultrasound frequency range as a quasi one-dimensional phononic crystal structure protecting a sea object against high resolution active sonar in the frequency range most commonly found for this type of equipment. The topology of the examined structure was optimized to obtain a band gap in the 2.2-2.3 MHz frequency band. For this purpose, a genetic algorithm was used, which allows for optimal distribution of individual elements of the ultrasound multilayer composite. By optimal distribution is meant to achieve a structure that will allow minimal reflectance in a given frequency range without height reflectance peaks with a small half width. Analysis of the wave propagation was made using the Transfer Matrix Method (TMM). As part of the research, 15 and 20-layer structures with reflectance at the level of 0.23% and 0.18%, respectively, were obtained. Increasing the number of layers in the analyzed structures resulted in finding such a distribution in which a narrow band of low reflectance was obtained, such distributions could also be used as bandpass filters. The use of a genetic algorithm for designing allows to obtain modern coatings, the characteristics of which result from the structure.

Keywords: reflectance coating, mechanical waves, phononic crystal, band gap, optimization

1. Introduction

Over the past two decades, many researchers have studied phononic structures (PnC phononic crystals) [1-6]. Due to the way these structures are constructed, a band gap phenomenon occurs in them (mechanical waves from a given frequency range do not propagate), which allows for a number of possible applications [7-13]. The band gap width depends on the crystal structure, mechanical wave phase velocity between and mass density of the materials used. Furthermore, the position and bandwidth of the band gap depend on the direction of wave propagation (angle of incidence) [14]. So far, only a few works have been created in which the angle of incidence of the wave and its effect on transmittance and reflectance of PnC has been considered. In 2019 Yu et al. at work [15] showed that the angle of incidence of the wave on metamaterial units and the periodicity of the grid significantly affect the frequency range of the band gap occurring. The authors of the work [16] investigated the influence of the acoustic wave angle of incidence on total transmission and reflection of a phononic structure.

One of the most interesting applications of phononic structures is the use of such a structure to build an acoustic coating. Research on acoustic coatings built based on phononic structures is currently being widely developed. The applications of the acoustic coating result from the need for sound insulation of the object in selected frequency bands. A very interesting review item in which the classification and research background of acoustic coatings is presented by Bai et al. [17]. This study explains the research significance of acoustic coatings for both military and civil applications. The work focuses in particular on discussing passive acoustic coatings in terms of their design and optimization. One of the important applications of acoustic coatings is their application in the marine environment. These coatings can absorb unnecessary sound waves to improve positioning accuracy when mapping the seabed [18], detecting fish stocks [19] or reducing noise pollution [20]. Such coating can also absorb sound waves emitted by active sonar. For this reason, acoustic coatings are widely used as ultrasound concealing equipment in the construction of submarines. In addition, coatings can also suppress hull vibration and isolate noise inside the boat.

Many authors investigating phononic structures optimize their topology to achieve a band gap in the specified frequency range. One of the most interesting works in this field is the work of Bendsøe and Sigmund [21]. In this work, the authors presented

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a method of topology optimization for the design of materials and structures with band gaps. Much of the work related to optimizing PnC topology is based on genetic algorithms (GA). The authors of the work [22] tested acoustic coupling between two acoustic coatings in cooperation with damping rubber. In this work, algorithms of differential evolution in combination with FEM were used to optimize two types of Alberich acoustic coatings. To maximize the band gap, Zhong et al. [23] used GA and the plane wave expansion method. Similar methods were used by Liu et al. to optimize the PnC topology in work [24]. Dong et al. [25] used the inverse scheme of a two-stage genetic algorithm to optimize topology. The optimization was designed to maximize the gap between adjacent two-dimensional PnC bands. The bi-directional genetic optimization algorithm was used by Li et al. [26] to obtain the maximum band gap of PnC. In [27] Gazonas et al. described the use of genetic algorithms (GA) for optimal phonon band design in periodic, flexible two-phase structures. Hussein et al. in [28] they carried out research using a multi-run genetic algorithm regarding the optimal topology of one-dimensional periodic elementary PnC cells.

This work concerns the study of the coatings for the ultrasound frequency range as a quasi one-dimensional phononic crystal structure protecting a sea object against high resolution active sonar in the frequency range most commonly found for this type of equipment. The topology of the examined structure was optimized to obtain a band gap in the 2.2-2.3 MHz frequency band. For this purpose, a genetic algorithm was used, which allows for optimal distribution of individual elements of the ultrasound multilayer composite. By optimal distribution is meant to achieve a structure that will allow minimal reflectance in a given frequency range without height reflectance peaks with a small half width. Analysis of the wave propagation was made using the Transfer Matrix Method (TMM).

The Thomson-Haskell method commonly known as Transfer Matrix Method (TMM) was used to analyze the wave transition through the studied phononic structures. The main advantage of this method is its speed. For this reason, it is widely used in physical modeling to mathematically describe the transition of a wave through a multi-layer medium. At the layer boundaries, wave propagation continuity conditions are taken into account, with each layer having an infinite range in lateral directions. The TMM method breaks down the waves in each layer into forward and backward waves. The mathematical description of wave propagation in a layer is done by matrix multiplication, and the physical sizes of different media can be combined using an interface matrix. The TMM is a method of predicting reflectance and transmission of multilayer structures for a given frequency. This method has been successfully used in many works. Dazel et al. [29] they determined the reflection coefficient and transmission of multilayer structures using the TMM method. The same algorithm was used to analyze three multilayer structures with a defect in the form of a piezoelectric layer Garus et al. [30]. Using the TMM method, Sigalas and Soukoulis [31] studied the propagation of elastic waves in disordered multi-layer structures made of two different materials. The TMM method was also used by Pop and Cretu [32] to analyze the longitudinal propagation of 1D elastic wave by multilayer media. The authors of the work [33] examined the phononic properties of Severin's quasi-one-dimensional aperiodic structure. Luan and Ye have analyzed by using TMM, the problem of sound wave propagation in a one-dimensional water channel containing many air blocks [34].

Genetic Algorithm (GA) developed by J. Holland [35] was used to find the optimal distribution of layers from given materials. The genetic algorithm was used in conjunction with the plane wave expansion method (PWE) to maximize the gap width gap by Zhong et al. [36] and by Han and Zhang in paper [37]. The PnC topology with square-shaped lattices was optimized with the use of GA by Liu et al. [38]. In the work [39] S. Garus and W. Sochacki analyzed the phase space of searching solutions state space while minimizing the transmission of the quasi one-dimensional structure.

2. Transfer Matrix Method

The transmission T for a given frequency can be determined directly from the characteristic matrix M of the structure as

$$T = \frac{Z_{out}\Theta_{in}}{Z_{in}\Theta_{out}}\left|\frac{1}{\mathrm{M}_{1,1}}\right|^2 \tag{1}$$

Lossless materials were analyzed in the paper to detail the impact of the structure on the minimization of reflectance R, which can be determined by

$$R = 1 - T \tag{2}$$

Mechanical wave propagation is described by the matrix equation (3) in which p_{in}^{+} is incident wave, p_{in}^{-} reflected and p_{out}^{+} transmitted and the characteristic matrix M depends on the structure topology, type and thickness d_i of materials used (Z_i – layer acoustic impedance and defined as $Z_i = v_i\rho_i$, where v_i is mechanical wave phase velocity, and ρ_i is mass density of the i layer).

$$\begin{bmatrix} p_{in}^{+} \\ p_{in}^{-} \end{bmatrix} = \mathrm{M}\begin{bmatrix} p_{out}^{+} \\ 0 \end{bmatrix} \tag{3}$$

The characteristic matrix M is defined as

$$\mathrm{M} = \Phi_{in,1}\left[\prod_{i=1}^{n-1}\Gamma_i\Phi_{i,i+1}\right]\Gamma_n\Phi_{n,out} \tag{4}$$

and consists of $\Phi_{i,i+1}$ which is transmission matrix on the border of i and $i + 1$ layers, and propagation matrix Γ_i. The $\Phi_{i,i+1}$ matrix is defined by

$$\Phi_{i,i+1} = \frac{1}{t_{i,i+1}}\begin{bmatrix} 1 & r_{i,i+1} \\ r_{i,i+1} & 1 \end{bmatrix} \tag{5}$$

where r and t are pressure reflection and transmission coefficient respectively, described as

$$r_{i,i+1} = \frac{Z_{i+1}\cos\Theta_i - Z_i\cos\Theta_{i+1}}{Z_{i+1}\cos\Theta_i + Z_i\cos\Theta_{i+1}} \tag{6}$$

$$t_{i,i+1} = \frac{2Z_{i+1}\cos\Theta_i}{Z_{i+1}\cos\Theta_i + Z_i\cos\Theta_{i+1}} \quad (7)$$

and the propagation matrix Γ_i, where Θ_i is angle of incidence of forward propagated mechanical wave in i layer, is given by

$$\Gamma_i = \begin{bmatrix} e^{jk_i d_i \cos\Theta_i} & 0 \\ 0 & e^{-jk_i d_i \cos\Theta_i} \end{bmatrix} \quad (8)$$

where the wave vector k_i of a given layer depends directly on the frequency f and phase velocity v_i through

$$k_i = \frac{2\pi f}{v_i} \quad (9)$$

3. Genetic Algorithm

At the beginning of the algorithm, its initialization occurs, and within it a population of randomly designated structures is determined. For each of them, transmission is determined using the TMM algorithm, and then the values of the objective function F_C are determined on its basis.

$$F_C = \left\| \int_{f_{\min}}^{f_{\max}} R(f)df \right\| \cdot \left\| \int_{f_{\min}}^{f_{\max}} \left| \frac{\partial R(f)}{\partial f} \right| df \right\| \quad (10)$$

Integrals normalization occurs within each population and ensures a balanced effect of reflectance minimization in the analyzed frequency area (first integral) and minimization of the chance of occurrence of high reflectance peaks with a small half-width (second integral). The function F'_C without normalization was used to compare structures between populations.

$$F'_C = \int_{f_{\min}}^{f_{\max}} R(f)df \cdot \int_{f_{\min}}^{f_{\max}} \left| \frac{\partial R(f)}{\partial f} \right| df \quad (11)$$

Then the table of multilayer structures is sorted according to the increasing value of the objective function. The two most favorable structures are left unchanged at this stage. The two structures with the highest value of the objective function are rejected and new randomly structures are designated. The remaining structures are subjected to mixing with a probability depending on the value of the objective function. The final step is the mutation process with low probability of each individual layer from all structures. This process is designed to reduce the probability of the algorithm being in the minimum local solution space. After determining the new population, the entire cycle (determining transmission for the population, determining the objective function, sorting, mixing, mutation) is repeated.

4. Results and Discussion

The materials collected in Table 1 were used as individual layers of composite structures. The materials were selected so that there were large differences in acoustic impedance between them, and the thickness of the layers for individual materials so as to increase the impact of destructive interference inside the layers. During the search of the space of the solution states, it was assumed that the angle of incidence of the wave to the analyzed structure was zero degrees.

TABLE 1

Material parameters used for analysis [12,40-43]

Material	Symbol	Mass density ρ [kg·m^{-3}]	Phase velocity v [m·s^{-1}]	Layer thickness d [10^{-6}·m]
Sea water 13°C	surrounding environment	1026	1500	—
$Fe_{64}Co_{10}Y_6B_{20}$ amorphous alloy	W	6829	1633	725.8
Polystyrene	X	42	512	227.6
Soft Rubber	Y	950	1050	466.7
Epoxy resin	Z	1180	2535	1126.7

There were 20 structures in a single population. The research were carried out for structures that had 15, 20, 40 and 60 layers, respectively. The mutation for each layer in the new population was 1% probability. The 400 steps of the genetic algorithm were performed for each case considered. Fig. 1 shows the value of the F'_C objective function for the best structures from each population. Fig. 2 shows how the transmission of the best structures of each generation has changed. Dark color means no reflectance, while white means full reflectance in a given frequency area.

It should be noted that the increasing number of layers in a single structure means that the genetic algorithm requires more steps to find a stable solution. For 15 layers, the most optimal structure was $YZY_3ZYZYZY_3ZY$, for 20 to $YZYZY_4Z_2Y_2ZY_3ZYZY$, for 40 to $W_3XYZW_2YZWY_2WYZYW_3X_2W_4XZ_2YWY_2ZY_2WZ_2W$, while for 60 it was $WXW_5YW_2X_2W_2Z_2YWZ_2WY_3ZWZ_3YZ_3YZWZXW_9XZ_2WYWYZWZ_2YZ$. The subscript in the notation means the number of repetitions of a given layer. In the 15 and 20 layer structures there were only layers made of soft rubber and epoxy resin. However, in the 40 and 60 layers structures, all available materials were used. Fig. 3 shows the reflectance in the analyzed frequency range for found structures.

For 15 and 20 layer structures, low reflectance (below 0.25%) was demonstrated in the whole range of frequencies tested. In contrast, 40 and 60 layered structures showed a narrow frequency band in which the reflectance was low.

5. Conclusions

The research showed that it is possible to use TMM and GA algorithms together with the proposed objective function to find structures with the lowest possible reflectance in a given frequency band. As shown, structures with 15 and 20 layers, thanks to working in a wider frequency range, correspond much

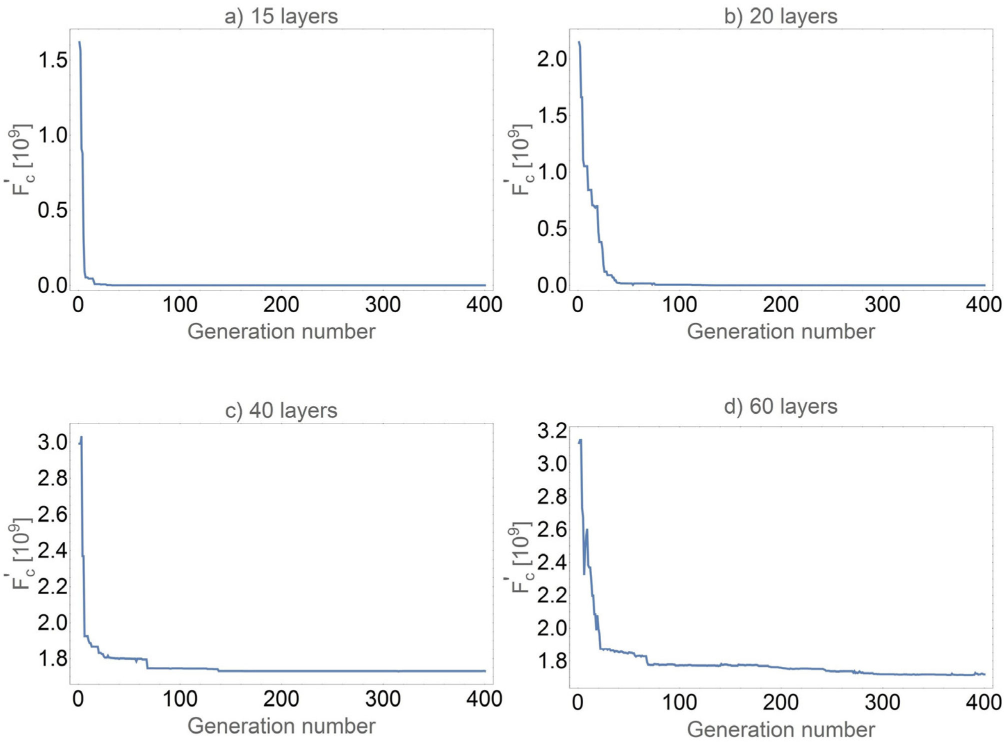

Fig. 1. The objective function values of the best individuals for each generation for different amounts of layers

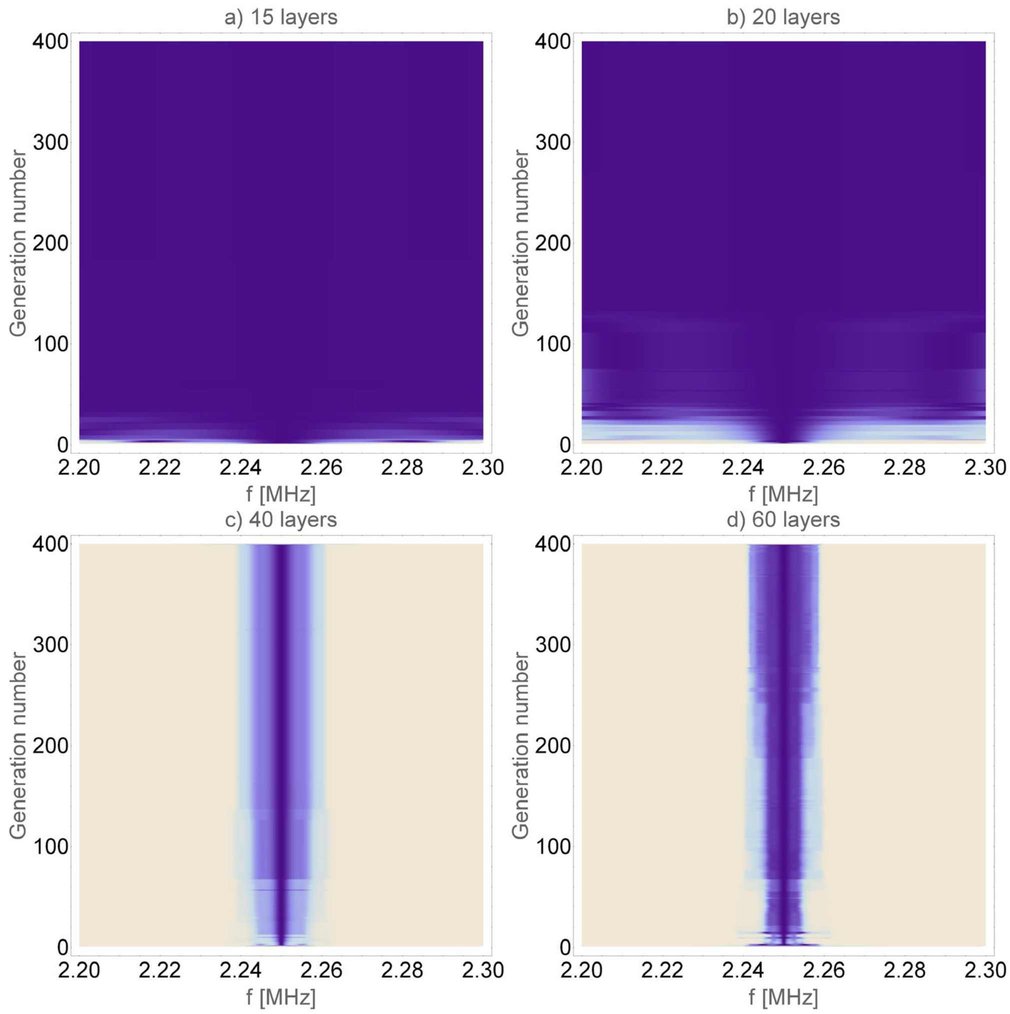

Fig. 2. The plot shows how the reflectance changed for the best individuals for each generation for different amounts of layers

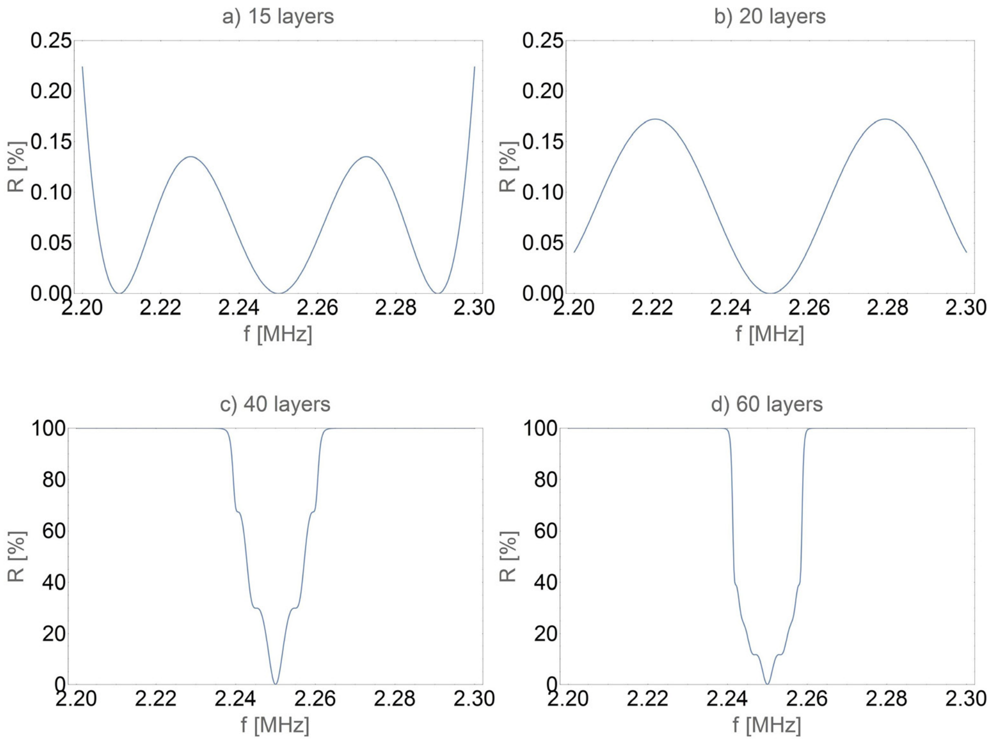

Fig. 3. The reflectance for the best found structure for different amounts of layers

more to the requirement for coatings used under water and the reflectance at the level of 0.23% and 0.18%, respectively, were obtained. Increasing the number of layers in the analyzed structures resulted in finding such a distribution in which a narrow band of low reflectance was obtained. However, the structures obtained for 40 and 60 layers can be used as band filters in devices generating mechanical wave beams. The use of a special type of objective function has eliminated the narrow peaks in the analyzed spectrum that appear as the number of layers increases.

The use of a genetic algorithm for designing allows to obtain modern coatings, the characteristics of which result from the structure.

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