THIN WALL DUCTILE IRON CASTINGS AS A GRADIENT MATERIAL

CIENKOŚCIENNE ODLEWY Z ŻELIWA SFEROIDALNEGO Jako MATERIAL GRADIENTOWY

In this paper a theoretical structural analysis of super thin walled casting made of ductile iron is presented. Analysis shows that such castings have inhomogeneous gradient structure, manifested by different: number of graphite nodules and ferrite and cementite fraction as a result of altered cooling rate along casting length. Critical distance above which the chills will be present and gradient of graphite nodule count is introduced. Theoretical investigations were experimentally verified for a casting with 1.6 mm wall thickness where graphite nodule count N, ferrite f_{f} and cementite eutectic f_{c} fraction and gradients G_{N}, G_{f}, G_{c} as a function of distance from runner were estimated. Finally rather a good agreement was found between the theoretical predictions and the experimental outcome.

W pracy przedstawiono teoretyczną analizę strukturalną super cienkościennej odlewów z żelwa sferoidalnego. Analiza wykazuje, że takie odlewów mają niejednorodną strukturę gradientową, przejawiającą się: różną liczbą kulek grafitu, udziałem ferrytu oraz eutektyki cementytowej jako efekt zmiennej szybkości ocieplenia. W pracy przedstawiono wyrażenie na krytyczną odległość, powyżej której występują zabiegi w odlewie oraz gradient liczby kulek grafitu. Badania teoretyczne zostały zweryfikowane dla odlewów o grubości ścianki 1.6 mm gdzie określono liczbę kulek grafitu, udział ferrytu f_{f} i cementytu f_{c}, oraz gradienty G_{N}, G_{f}, G_{c} w funkcji odległości od belki wlewowej. Wykazano, że przedstawiona teoria dobrze opisuje wyniki eksperymentalne.

1. Introduction

Advanced construction application require multifunctional working elements having more then one custom properties. As a result new materials were developed which are called Functionally Gradient Materials (FGM) [1–3]. To such type of materials ductile iron can be credited which characterize high sensitivity on cooling rate what in consequence leads to structural gradients G_{i} that is constant changes of structural features of cast iron (properties accordingly) along one of direction. In general, in recent years there has been an increasing demand for thin-wall ductile iron castings with wall thickness below 3 mm (TWDI) with high strength-weight ratios [4, 5].

In TWDI, there are numerous studies on the solidification morphology [6], microstructural characterisation [7, 8], mechanical properties [9–14], carbide formation factors [15, 16], production [17, 18]), solid state transformations [19, 21] and mold filling [21]. Moreover, according to the literature, there are different experimental relationships between the chemical composition [7, 15, 18, 19], pouring temperature [15], spheroidization and inoculation practice [15, 17, 22], casting geometry [11], plate thickness [7, 15, 21], mold material and nodule count [23], critical nodule count and critical plate thickness below which the chills develops (see Fig. 1).

These experimental relationships are very useful but they are limited in their physical meaning and in describing the mechanisms involved during the chill development of cast iron and the nodule formation. Hence, it can be said that the experimental relationships reported in the literature are just a type of “black box” which is fed on one side with the casting input parameters and the resultant solidification microstructure on the other side of the box, as shown schematically in Fig. 1. Accordingly, in this work an attempt is made to elucidate “what happens” in this black box by considering the solidification process of cast iron. In particular, in this work analytical expressions are derived for the explanation of formation mechanism of primary gradient structure of TWDI castings.
2. Experimental

Experimental melts were obtained in an electric induction furnace with a 15 kg capacity crucible. The raw materials were sulp-metal and commercially pure silicon. The metal was preheated at 1500°C and then poured into the mold which was equipped with a reaction chamber connected to a pouring basin and containing a mixture of 53 g of spheroidizer (Elmag 5800; 44-48% Si, 5.5-6.2% Mg, 0.8-1.2 RE, 1.0% max. Al) and 30 g of inoculant (FoundrySil; 73-78% Si, 0.75-1.25% Ca, 0.75-1.25% Ba, 0.75-1.25% Al). After filling the pouring basin, a graphite plug was removed allowing metal flow into the mold cavity. The chemical composition of cast iron was C = 3.74%; Si = 3.02%; Mn = 0.12%; P = 0.02%; S = 0.01%; Mg = 0.025%. The cast iron was poured into plate shaped mold cavity with wall thickness, s = 0.16 cm and length 20.0 cm (see Fig. 2). The thermocouples Pt-PtRh10 were placed in cross-runner in order to determined cooling curve. From cooling curve, temperature $T_p = 1280°C$ of cast iron entered into mold cavity were recorded. Length of obtained samples was shorter than length of mold cavity and amounted about 17.5 cm.

Metallographic evaluations of the nodule count, as well as cementite and ferrite fractions were made on samples cut from the plates. The average nodule count (average number of graphite nodules per unit area), $N_F$ was measured using a Leica QWin quantitative analyzer. Settings for measurements were as follows: magnification 200 $x$, calibration and detection threshold was set to 1 mm. In ductile iron the graphite nodules are characterized by a Raleigh distribution [24]. In this case the volumetric nodule count $N$ (the average number of graphite nodules per unit volume) can be related to $N_F$ using the Wientzke equation [25]:

$$ N = \sqrt{\frac{N_F}{f_g}}, $$

where $f_g$ is the volume fraction of graphite, in hypereutectic-TWDI cast iron $f_g \approx 0.14$.

3. Results and analysis

The results of metallographic evaluations are given in Table 1. Experimental results (Fig. 3) are approximated by the following equations:

Graphite nodule count (correlation coefficient $R = 0.97$)

$$ N_F = 2451 + 4.75 \exp(0.36x) $$

(2)

Gradient of Graphite nodule count

$$ G_{N_F} = \frac{dN_F}{dx} = 1.71 \exp(0.36x). $$

(3)

Ferrite fraction (correlation coefficient $R = 0.98$)

$$ f_f = 24.94 - 2.98x + 0.31x^2 - 0.01x^3. $$

(4)

Gradient of ferrite fraction

$$ G_{f_f} = \frac{df_f}{dx} = -2.98 + 0.62x - 0.03x^2. $$

(5)

Cementite fraction (correlation coefficient $R = 0.98$)

$$ f_c = 21.0798x - 341.1. $$

(6)

Gradient of cementite fraction

$$ G_{f_c} = \frac{df_c}{dx} = 21.08. $$

(7)
Fig. 3. Influence of distance x on nodule count and gradient of nodule count (a), ferrite fraction and gradient of ferrite fraction (b) and cementite fraction (c) along with regression curves.
Nodule count, cementite and ferrite fraction measured at distance x from cross-runner

<table>
<thead>
<tr>
<th>Distance, x cm</th>
<th>1.4</th>
<th>5.3</th>
<th>7.2</th>
<th>10.3</th>
<th>11.9</th>
<th>13.3</th>
<th>14.0</th>
<th>15.0</th>
<th>16.2</th>
<th>16.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nodule count, ( N_p ) 1/mm²</td>
<td>2461</td>
<td>2574</td>
<td>2588</td>
<td>2620</td>
<td>2772</td>
<td>2886</td>
<td>3300</td>
<td>3722</td>
<td>3966</td>
<td>3877</td>
</tr>
<tr>
<td>Distance, x cm</td>
<td>1.3</td>
<td>2.0</td>
<td>3.0</td>
<td>4.0</td>
<td>4.7</td>
<td>5.7</td>
<td>12.5</td>
<td>13.5</td>
<td>14.0</td>
<td>15.5</td>
</tr>
<tr>
<td>Ferrite fraction, ( f_f ) %</td>
<td>28</td>
<td>20</td>
<td>18</td>
<td>18</td>
<td>17</td>
<td>16</td>
<td>15</td>
<td>14</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>Distance, x cm</td>
<td>16.3</td>
<td>16.4</td>
<td>16.5</td>
<td>16.6</td>
<td>16.7</td>
<td>16.8</td>
<td>16.9</td>
<td>17.0</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Cementite fraction, ( f_c ) %</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td>12</td>
<td>12</td>
<td>15</td>
<td>13</td>
<td>16</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

From experimental result (Fig. 3 and Table 1) it follows that there exists critical distance in casting \( x_{cr} = 16.2 \) cm when there is grey to mottled structure transition. In grey zone nodule count \( N_p \) increases and ferrite fraction \( f_f \) decreases with distance \( x \). In mottled zone as distance \( x \) increases cementite fraction increases.

**Temperature drop during mold filling**

As a result of heat transfer between moving stream of liquid iron and walls of the mold temperature of the stream decreases. It can be proved (see appendix) that if tip of the stream is at the distance \( Z \) then temperature of liquid metal \( T_i \) inside stream at the distance \( x \leq Z \) (Fig. 7) can be given by Eq. (8)

\[
T_i = T_p e^{\left[\frac{4a}{\sqrt{\pi \alpha s}} \left(\sqrt{\frac{Z-x}{u}} - \sqrt{\frac{Z}{u}}\right)\right]}, \tag{8}
\]

where: \( T_p \) is the temperature of metal at the entrance to the channel \( (x = 0) \), \( a \) is the material mold ability to absorb heat, \( c \) is the volumetric specific heat, \( s \) is the wall thickness of the plate shaped channel, \( u \) is the flow velocity for metal in the mold channel cavity, \( Z \) and \( x \) are defined in figure 4.

From equation (8) and Fig. 4 it follows that the highest temperature drop \( T_i \) occurs in metal at the tip of the stream. Therefore emission time of the superheat \( L_p = c (T_i - T_e) \) from the tip of the stream is the shortest and it solidify in the first instance causing stopping of the stream tip at the distance \( Z \). Cooling curves of cast iron for a few points from \( x \leq Z \) range is shown in Fig. 4. Appropriate for these points cooling curves shows that as distance \( x \) decreases temperature of eutectic transformation \( T_m \) increases as a result of necessity of carrying away increasing superheat. As it was mentioned before at distance \( Z \) tip of the stream stops as a result of the solidification process. As a first approximation it can be assumed that at distance \( Z \), temperature \( T_i = (T_s + T_e)/2 \) (where \( T_s \) and \( T_e \) are given in Table 2), and from Eq. (8) the metal flow velocity is given by

\[
u = \frac{16a^2Z}{\pi c^2 s^2 \left(\ln \frac{x}{x_p}\right)^2}.	ag{9}
\]

**Temperature \( T_i \) and cooling rate \( Q \)**

Thin-walled castings are cooled down with high rate and their solidification temperature is close to temperature \( T_c \) [26]. The cooling rate of liquid cast iron at a given point of a casting can be given by [27]:

\[
Q = \frac{8T_c a^2}{\pi B c^2 s^2}, \tag{10}
\]
where:
\[
B = \ln \frac{T_1}{T_c}.
\] (11)

From eqs. (8), (10) and (11) it is evident that the cooling rate, among other things also depends on the initial liquid metal temperature, \(T_1\). Figure 5 show cooling rate \(Q\) along distance \(x \leq Z\), calculated according to equations (8)–(11) and using data from table 2. From this figure it follows that as distance \(x\) increases cooling rate \(Q\) increases especially at the tip of the sample.

![Fig. 5. Influence of distance from runner, \(x\) on the cooling rate \(Q\) (b) of castings with wall thickness \(s = 0.16\) cm](image)

**Selected thermophysical data**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values and units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material mold ability to absorb heat</td>
<td>(a = 0.09 - 0.11; \text{ J/cm}^2\text{s}^{1/2} \text{ C}^{-1/2})</td>
</tr>
<tr>
<td>Equilibrium temperature of graphite eutectic</td>
<td>(T_s = 1153.9 + 5.25) Si, °C</td>
</tr>
<tr>
<td>Solidification temperature of the of cementite eutectic</td>
<td>(T_c = 1130 + 4.1(\text{C-3.3 Si}), °C)</td>
</tr>
<tr>
<td>Latent heat of graphite eutectic</td>
<td>(L_s = 2028.8; \text{ J/cm}^3)</td>
</tr>
<tr>
<td>Diffusion coefficient of carbon in austenite</td>
<td>(D = 3.9 \times 10^{-6}; \text{ cm}^2/\text{s})</td>
</tr>
<tr>
<td>Specific heat of cast iron</td>
<td>(c = 5.95; \text{ J/cm}^3 \text{ °C}^{-1})</td>
</tr>
<tr>
<td>Coefficient related to the slopes of the solubility lines (J^E), (E^S) and (BC) in the Fe-C system</td>
<td>(\beta = 0.00155; °C^{-1})</td>
</tr>
<tr>
<td>Nucleation coefficients of graphite</td>
<td>(N_s = 6.8 \times 10^{10}; \text{ cm}^{-3})</td>
</tr>
<tr>
<td>C, Si – are the carbon and silicon contents in cast iron, respectively.</td>
<td>(b = 285.4; °C)</td>
</tr>
</tbody>
</table>

![Fig. 6. Function \(x = \text{ProductLog}[y]\)](image)

**Nodule count and gradient of nodule count**

A theoretical analysis for the solidification of ductile cast iron [27] indicates that the nodule count \(N\) can be related to the maximum undercooling \(\Delta T_m = T_s - T_m\), (where \(T_m\) is the minimum temperature at the onset of the eutectic solidification) by the equation:

\[
N = \frac{c B^{1/2}}{4\pi z L_c} \left( \frac{C^3}{2D^3\beta^3\Delta T_m^4} \right)^{1/2},
\] (12)

where:
\[
z = 0.41 + 0.93B.
\] (13)

In above equations, \(c, D, L_c, and \beta\) are defined in Table 2.

From Eq. (12), for a given cooling rate, results that \(Q\) the nodule count increases as the maximum undercooling \(\Delta T_m\) decreases what is in agreement with the experimental outcome.

According to the literature [28, 29], a relation between the maximum degree of undercooling \(\Delta T_m\) and the nodule count \(N\) can be described by

\[
N = N_s \exp \left( \frac{b}{\Delta T_m} \right).
\] (14)

where \(N_s\) and \(b\) – are the nucleation coefficients (\(N_s\) is the density of nucleation sites for graphite nucleation while \(b\) depends on the size of nucleation sites).

From Eqs. (10) and (12) \(\Delta T_m\) can be calculated, which upon substitution into Eq. (14) yields

\[
N = \frac{N_s}{\exp \left( 2\text{ProductLog}[y] \right)}.
\] (15)

where:
\[
y = \frac{b}{4} \left( \frac{C B L_c N_s s^3}{a^3} \right)^{1/2} \left( \frac{\pi^5 D^3 \beta^3}{T_c^4} \right)^{1/4}.
\] (16)

The Product Log[y] = x is the Lambert function, also known as the omega function and it is graphically shown in Fig. 6. This function can be easily calculated by means of the instruction ProductLog[y] in the Mathematica™ programme.
complicated function of $N_F = f(x)$ can be calculated. Curve in Figure 7 shows plot of this function for data given in Table 2. From this figure it can be found that the nodule count increases as the distance $x$ from the runner increases. Notice from this figure rather a good agreement between the theory and the experimental outcome.

![Fig. 7. Influence of distance $x$ on nodule count, points - experimental results, $Z = 17.5$ cm, $x_{cr} = 16.2$ cm, $T_p = 1280^\circ$C](image)

Fig. 7. Influence of distance $x$ on nodule count, points - experimental results, $Z = 17.5$ cm, $x_{cr} = 16.2$ cm, $T_p = 1280^\circ$C

After substituting of Eqs. (1), (8), (9), (11), (16) into Eq.(15) and after differenting towards $x$ it is obtained gradient $G_{N_F} = dN_F/dx$ of nodule count along distance $x$ of sample. Expression of $G_{N_F}$ is very complicated and thus only numerical results are show as curve 1 in figure 8. For comparison in the same plot is shown gradient $G_{N_F}$ estimated on the basis on experimental outcome (Eq.(3)). From figure it follows that gradient $G_{N_F}$ of nodule count reaches the highest values at distance $x_{cr}$ when zone of mottled structure started.

Minimal temperature for graphite eutectic solidification

From Table 1 follows that in sample with $17.5$ cm in length (see Fig. 2) cementite eutectic occure (chill) at the distance $Z = 16.2$ cm. It means that at the distance $Z$ temperature $T_m$ of the solidification of graphite eutectic equals to the solidification of the cementite eutectic $T_c$ while undercooling $\Delta T_m = T_s - T_m$ reaches $\Delta T_{ce} = T_s - T_c$. The minimal temperature $T_m$ for graphite eutectic can be calculated from Eqs. (10) and (12).

$$T_m = T_s - p \left( \frac{64}{N^2 g^3 d^3 s^6} \right)^{1/4}, \quad (17)$$

where $p$ is the coefficient contains the parameters related to the cast iron cooling rate,

$$p = \left( \frac{a^{3/2} c^{3/2}}{2\pi^{3/2} z B L c^{2} \beta^{1/2}} \right)^{1/2}. \quad (18)$$

![Fig. 8. Influence of distance $x$ on gradient of nodule count, curve 1 - theoretical prediction, curve 2 - experimental results](image)

Fig. 8. Influence of distance $x$ on gradient of nodule count, curve 1 - theoretical prediction, curve 2 - experimental results

Taking into account experimental results of $N_F$ (Table 1) and data from Table 2 as well as equations (1), (8), (9), (17) and (18) the minimal temperature, $T_m$ for graphite eutectic solidification as a function of distance, $x$ can be calculated. Results of these calculations by means of points are shown in figure 9. After substituting Eq. (14) into Eq. (17) and its solving it is obtained

$$T_m = T_s - \frac{b}{2 \text{ProductLog}[y]} \quad (19)$$

Equation (19) enables to calculate $T_m$ knowing nucleation parameters $N_s$ and $b$. Results of such calculation after taking into consideration Eqs. (1), (8), (9) and (19) and data from Table 2 is shown as a solid line in Fig. 9. Notice that differences in results between calculated nodule count and using nucleation coefficients $N_s$ and $b$ are rather small.
**Chilling tendency and critical distance \( x_{cr} \)**

It is well known, that as \( T_m = T_c \) or \( \Delta T_m = T_e - T_c = \Delta T_{sc} \) the cementite eutectic begins to solidify. By taking into account this condition and Eqs. (8), (11), (13), (17) and (18) it can be obtained

\[
x_{cr} = \frac{1}{8a} c \sqrt{\frac{4a}{u} + \frac{1}{2} c \frac{1}{\sqrt{\pi s}} \left( \ln \left( \frac{T_e}{T_p} \right) - A \right)}
\]

(20)

where

\[
A = 0.22 - 0.54 \sqrt{0.17 + \frac{0.85 a^3 T_{sc}^{3/2}}{c^2 L_e s^3 T_{sc}^{1/2}}} \cdot CT^3.
\]

(21)

CT is chilling tendency, given by

\[
CT = \left( \frac{1}{N_e^2 \beta^2 D^3 \Delta T_{sc}^4} \right)^{1/6} \exp \left( \frac{b}{3 \Delta T_{sc}} \right)
\]

(22)

or knowing the critical nodule count (critical nuclei density) \( N_e \) at temperature \( T_c \)

\[
CT = \left( \frac{1}{N_e^2 \beta^2 D^3 \Delta T_{sc}^4} \right)^{1/6}.
\]

(23)

N\(_F\) = 3966 mm\(^{-2}\) chills occure in sample. After recalculation N\(_F\) into N (Eq.(1)) for data from Table 2, according to equations (21), (22) and (23) it is obtained CT = 0.32 s\(^{1/2}\)C\(^{1/3}\) and critical length of sample \( x_{cr} \) = 16.4 cm. It is worth mentioning that both calculated values of \( x_{cr} \), are close to an experimental value \( x_{cr} \) = 16.2 cm. In foundry practice inoculation process strongly decreases chilling tendency CT. Calculated influence of CT on critical sample length \( x_{cr} \) is shown in figure 10 from it is seen that as CT increases also increases chilled zone up to 17.5 cm

4. Concluding remarks

1. It has been shown that despite of the constant wall thickness, the microstructural features of TWCD can be inhomogeneous (nodule count, chill, ferrite fraction) along its length as a result of changes in the cooling rates.

2. From the proposed theory new expressions are derived for the critical distance \( x_{cr} \) and gradient of nodule count \( G_{N_e} \) for ductile cast iron.

3. Theoretical calculations for the critical distance \( x_{cr} \) and nodule count were made and compared with the experimental outcome and rather a good agreement was found between the theoretical predictions and the experimental outcome.

5. Appendix

It is assumed (see Fig. 11) that liquid metal flows through a molding sand channel of constant cross section at constant velocity, \( u \). At the stream front where \( x = Z \), the convective heat transfer coefficient \( \alpha \) is relatively large due to contact of the metal stream with the cold mold, while at the channel entrance (\( x = 0 \), warm mold) is small. Under these conditions, heat transfer occurs normal to the metal flow direction. The convective heat transfer coefficient can be described by \[30]:

\[
\alpha = \frac{a}{\sqrt{\pi \left( \frac{Z - x}{a} \right)}}.
\]

(A1)

From heat balance equation results:

\[
dQ_m = dQ_a,
\]

(A2)

where \( Q_m \), and \( Q_a \) are the heat going towards the mold, and the heat accumulated in the metal, respectively. Considering a volume element of liquid metal, \( V \), the heat dissipated, \( dQ_m \) can be described by

\[
dQ_m = \alpha T_i F dt,
\]

(A3)
where $F$ is the surface area of liquid metal in contact with the mold channel, $T_i$ is the temperature of metal at any location $x$ away from the entrance.

Moreover, $dQ_a$ can be described by

$$dQ_a = -Vc dt,$$

where $c$ is the metal heat capacity. Considering constant $u$, that is $dt = dx/u$ a comparison between Eqs. (A3) and (A4) yields after integration

$$\ln T_i = \frac{2aF}{c \sqrt{\pi V}} \sqrt{\frac{Z-x}{u}} + C.$$  \hspace{1cm} (A5)

In the above expression, $C$ can be found from the condition: at $x = 0$, (at the entrance to the channel), the liquid metal temperature $T_i = T_p$, thus:

$$\ln T_p - \frac{2aF}{c \sqrt{\pi V}} \sqrt{\frac{Z}{u}} = C.$$  \hspace{1cm} (A6)

Substituting Eq. (A6) into Eq. (A5) and taking into account the liquid metal modulus $M = V/F \approx s/2$, where $s$ is the wall thickness of the plate shaped channel, the temperature of the metal can be described by:

$$T_i = T_p \exp \left[ \frac{4a}{\sqrt{\pi cs}} \left( \sqrt{\frac{Z-x}{u}} - \sqrt{\frac{Z}{u}} \right) \right].$$  \hspace{1cm} (A6)

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