A R C H I V E S O F M E T A L L U R G Y A N D M A T E R I A L S
Volume 52 2007 Issue 3

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MODELLING OF RHEOLOGICAL PROPERTIES OF SELECTED DISINTEGRATED MEDIA

MODELOWANIE WŁASNOŚCI REOLOGICZNYCH WYBRANYCH OŚRODKÓW ROZDROBNIONYCH

The basic pre-requisite for modelling the deformation process of a real material is knowing its rheological model. An analytical description of the material's rheological characteristics is very difficult and in many cases simply impossible, so it is suggested to apply the time characteristics method for identifying rheological models of disintegrated media. A rheological model can be identified on the ground of a step-like characteristics that represents time dependence of unit pressures in the material under examination, generated by step load. Parameters of the rheological model can be determined on the ground of experimental relationships. Approximation of the coefficients $k_C(\delta)$ and $k_T(\delta)$ that define elastic and viscous properties was performed using the $v_L = f(\delta)$ function that represents the relationship between the ultrasonic wave velocity in the medium and its densification degree, to be determined in ultrasonic tests. The comparative analysis of the simulation and experimental results has proved that the developed identification method makes it possible to determine a rheological model that precisely describes the rheological characteristics of disintegrated media, both in a quantitative and qualitative way.
Keywords: disintegrated medium; identification; time method; step-like characteristics; rheological model

Podstawowe wymaganie modelowania procesu odkształcania materiału rzeczywistego w czasie stanowi znajomość jego modelu reologicznego. Analityczny opis charakterystyki reologicznej materiału jest bardzo trudny a w wielu przypadkach wręcz niemożliwy, dlatego do identyfikacji modeli reologicznych ośrodków rozdrobnionych zaproponowano zastosowanie metody charakterystyk czasowych. Model reologiczny można zidentyfikować na podstawie charakterystyki skokowej, która przedstawia zależność zmian nacisków w badanym materiale w funkcji czasu, wywołanych obciążeniem zrealizowanym w sposób skokowy. Parametry modelu reologicznego można wyznaczyć na podstawie zależności określonych w sposób eksperymentalny. Do aproksymacji zależności przedstawiających współczynniki $k_C(\delta)$ i $k_T(\delta)$ określające właściwości sprężyste i lepkie zastosowano zależność $v_L = f(\delta)$, przedstawiającą zależność prędkości fali ultradźwiękowej w ośrodku w funkcji stopnia jego zagęszczenia, którą można wyznaczyć na podstawie badań ultradźwiękowych. Analiza porównawcza wyników badań symulacyjnych i eksperymentalnych wykazała, że opracowana metodyka identyfikacji umożliwia wyznaczenie modelu reologicznego, który dokładnie opisuje zarówno pod względem jakościowym jak również ilościowym, charakterystykę reologiczną ośrodków rozdrobnionych.

1. Introduction

Each field of applied mechanics of continuous media is based on the laws that describe the behaviour of materials under loading process and define their strength. These laws can be determined empirically only (Cytović et al. [1], Jaske et al. [2], Laube et al. [3], Wiłun et al. [4]). In simple experiments, the material samples are subject to homogenous stress and deformation and the relationships between stress and deformation are determined.

In basic considerations, the disintegrated media (e.g. soils) are treated as monophase and isotropic bodies [1]. Such an assumption involves the necessity of introducing two material constants: Young's modulus E and Poisson's ratio υ that describe the behaviour of the medium under loading process. Since the concepts of invariant quantities describing the state of stress and deformation are unknown, there is no simple way of determining the Young's modulus E and Poisson's ratio υ. As things are, it is necessary to introduce other, substitute material constants.

The deformation vs. stress relationship, e.g. in compression test of soils, is obtained using the edometer. The instrument measures normal stresses in the soil sample and respective vertical deformations. The stresses in horizontal directions are unknown. The Poisson's ratio is determined from the relationship:

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$$\upsilon = \frac{\mu}{1+\mu}, \tag{1}$$

where: μ – side pressure coefficient.

Knowing the value of the edometric compression modulus E_e and the Poisson's ratio permits calculation of the generalised compression modulus E_o being an equivalent of the Young's modulus:

$$E_o = E_e\left(1 - \frac{2\upsilon^2}{1-\upsilon}\right). \tag{2}$$

The real Young's modulus value is always larger than the E_o value. The difference results from the fact that for disintegrated media the deformation vs. stress relationship is almost always curvilinear. So, the disintegrated media are non-linear media showing elastic, viscous or viscoelastic features, and often all these features at the same time.

The real Young's modulus can be determined using the ultrasonic method [1, 5]. Velocity of longitudinal ultrasonic wave v_L in an elastic or viscoelastic medium is determined by the formula

$$v_L = \sqrt{\frac{E}{\rho}}. \tag{3}$$

Knowing the velocity of longitudinal ultrasonic wave propagation in a medium and density of the medium ρ, one can calculate its Young's modulus from the relationship (3).

The briefly presented questions concerning the description of mechanical properties of disintegrated materials (e.g. soils) refer also to such materials like e.g. moulding sands that can be treated as a kind of soil [2]. They are composed of sand (mostly high-silica) and 5–8% of moulder's loam (bentonite). This is why the laws developed by soil mechanics can be used in examination of mechanical properties of moulding sands (S a k w a et al. [6]).

Mathematical modelling of time behaviour of a material under load can be described by a rheological model. Such a model of a real material can be determined on the grounds of knowing the material structure, deformation process and its related changes of physical properties.

In most cases there is no sufficient database to build such models, so direct description of rheological characteristics of a real material is an extremely difficult and complex problem and thus, in practice, the rheological properties are described using the models that represent the material structure and the deformation process in a simplified way (P i n d e r a et al. [7]).

In spite of the simplifications accepted in the description of materials' rheological properties, mathematical modelling of this process in an analytical way is extraordinarily complex and, in many cases, simply impossible. Things being like this, identification of rheological models of materials and media can employ two identification methods of properties of dynamic physical systems: the time and the frequency methods that permit determination of time or frequency characteristics not only analytically but experimentally as well (Ż e l a z n y et al. [8], K a c z o r e k et al. [9], A w r e j c e w i c z et al. [10]).

This paper presents the application of the time method for identifying the rheological model of a disintegrated medium on an example of moulding sand, whereby the suggested method can be used for identification of other disintegrated materials, e.g. soils.

2. Identification of rheological model of moulding sand

2.1. Description methods of the properties of dynamic physical systems

The mathematical model of a physical system dynamics makes an equation or a system of differential equations that determine the time relationships between the input and output quantities of the system (Fig. 1):

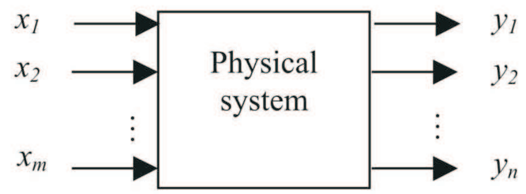

Fig. 1. Block diagram of a dynamic physical system

Dynamic properties of a physical system can be determined in an analytical or experimental way using the following methods:

- the time method or
- the frequency method.

When it is impossible to formulate the analytical description, the dynamics of a physical system (object) can be evaluated on the grounds of time or frequency characteristics determined experimentally.

The time characteristics determine the object's response $y(t)$ to standard input functions $x(t)$. The most common time characteristics are:

- the step characteristic $h(t)$ being the response to the unit input function $1(t)$ (Fig. 2a),
- the impulse characteristic $g(t)$ being the response to the impulse input function $\delta(t)$ (Dirac delta function) (Fig. 2b).

A lot of important information about the examined physical object can be obtained on the grounds of time characteristics. The knowledge of these permits defining the system as linear or non-linear and, in many cases, identifying the differential equation that describes the examined physical system.

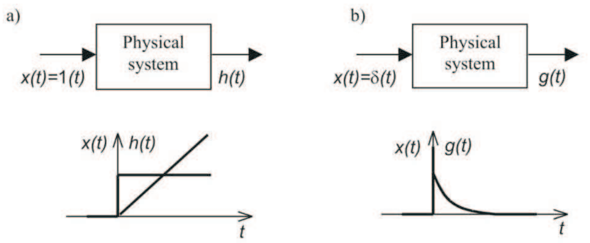

Fig. 2. Illustration of time characteristics of physical systems: step characteristic (a) and impulse characteristic (b)

The frequency characteristics describe the behaviour of a physical system subject to sinusoidal input function with variable frequency (Fig. 3). They represent the response to input amplitude ratio $M(\omega) = A_2/A_1$ and the phase displacement $\varphi(\omega)$ between the response and input as frequency functions.

The grounds for analytical determination of frequency characteristics is made by the spectral transmittance function $G(j\omega)$ obtained by transformation of the operational transmittance function $G(s)$ determined from the differential equation that describes the system dynamics. The characteristics concerned can be also determined in an experimental way.

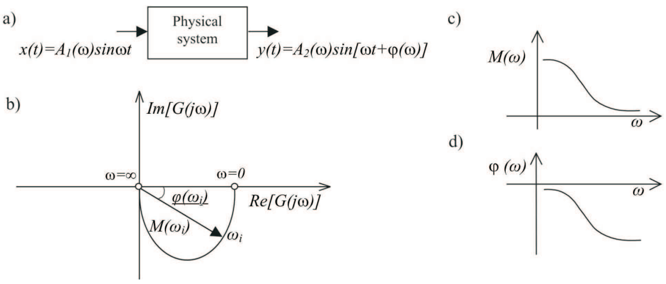

Fig. 3. Illustration of frequency characteristics: block diagram of the system (a), amplitude-phase characteristics (b), amplitude characteristics (c) and phase characteristics (d)

The frequency characteristics contain complete information on the physical system dynamics, so they are often used in research of dynamics, first of all in electrical and electronic systems.

Certainly, identification of rheological properties of disintegrated materials (soils, moulding sands) is easier and more convenient using the time method [11, 12]. It makes it possible to determine e.g. the time step characteristic in a simple way, with no use of complicated apparatus. On the basis of this, it is possible to define the nature of the physical system (to verify its linearity) and identify the equation that describes its dynamics.

2.2. Rheological model of moulding sand

The time-step characteristic of moulding sand was experimentally determined on the test stand whose layout is shown in Fig. 4. The stand consists of the following sub-assemblies:

- high-speed pneumatic drive (HPD),
- measurement sleeve,
- measuring system.

The HPD used on the test stand, thanks to the impulse valve (2), permits reaching the pressing foot velocity of the order of 8–10 m/s. Such speed of the material loading element allows treating the input function as a single impulse.

The step characteristic measuring system was equipped with circuits to measure pressure p_1 in the HPD working chamber (C_1-W_1) and unit pressures in lower layers of the examined sample (C_2-W_2). The sensing element C_2 was equipped with a special liquid adaptor that permits measurement of non-directional pressure inside the sample of the tested material.

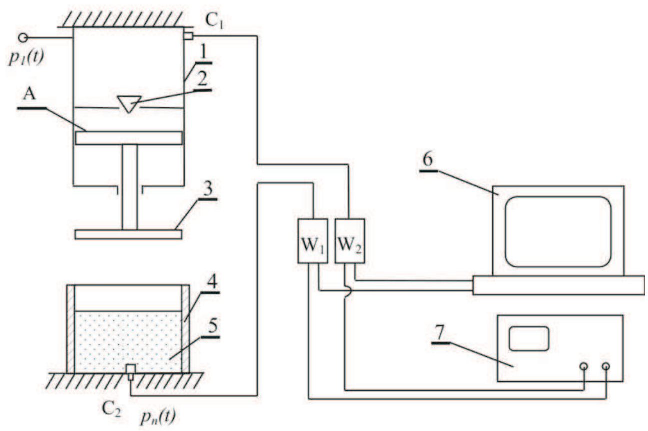

Fig. 4. Lay-out of the test stand: high-speed pneumatic drive (1), impulse valve (2), pressing foot (3), measurement sleeve (4), test material sample (5), PC with measurement card TAD 05 (6),oscilloscope (7), C_1 and C_2 – pressure gauges type 601 H (Kistler), W_1 and W_2 – charge amplifiers type 5001 (Kistler), A – HPD piston face

Figure 5 shows the step characteristics $h(t) = p_n(t)$ determined for moulding sand with 6% of bentonite and humidity W = 2.44%, representing the time function of unit pressures in the moulding sand under step-like load.

The analysis of the relationship $p_n = f(t)$ indicates that it represents highly damped decaying oscillation. This involves the conclusion that the moulding sand behaves as an oscillating system (object) and that its rheological properties can be modelled using the viscoelastic rheological shown schematically in Fig. 6.

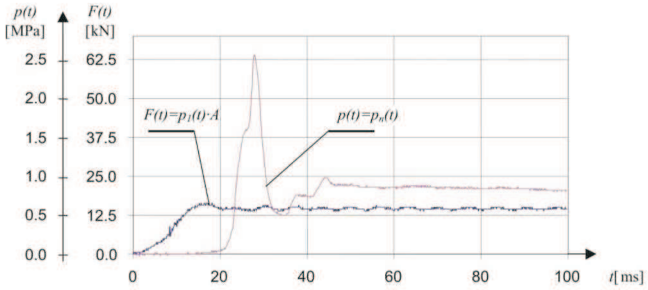

Fig. 5. Step characteristics of moulding sand with 6% of bentonite and humidity W = 2.44%

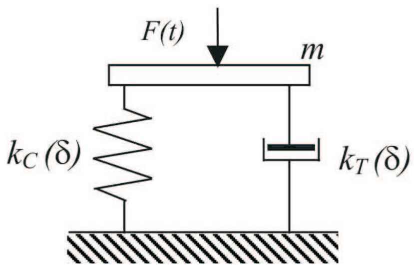

Fig. 6. Diagram of the viscoelastic rheological model of moulding sand: m – weighed sample of moulding sand, $k_C(\delta)$ – coefficient of elastic properties of moulding sand, $k_T(\delta)$ – coefficient of viscous properties of moulding sand, $\delta = \rho_0/\rho$ – consolidation degree of moulding sand (ρ_0 – apparent density, ρ – specific gravity)

2.3. Evaluation of the rheological model parameters

In order to apply the rheological model for describing the medium (material) loading process it is necessary to know the coefficients that characterise its rheological properties.

The coefficients of the viscoelastic rheological model describing the rheological properties of moulding sands, and also other disintegrated media like soils, can be determined in two ways:

- on the grounds of the published relationships by Scott et al. [13]

$$k_C(\delta) = 2\sqrt{\frac{A_p}{\pi}} \cdot E(\delta)(1-\upsilon^2)$$
$$k_T(\delta) = 0,6 \cdot A_p \cdot \rho \cdot v_L(\delta), \tag{4}$$

  where: $v_L = f(\delta)$ – velocity of ultrasonic longitudinal wave in the examined material, $E(\delta)$ –Young's modulus of the examined material, υ – Poisson's ratio, A_p – loaded material area, ρ – material density
- on the grounds of the experimentally determined relationships

$$k_T(\delta) = a_1 \cdot \exp[a_2 \cdot v_L(\delta)] \tag{5.1}$$

$$k_C(\delta) = b \cdot_1 \exp[b_2 \cdot v_L(\delta)], \tag{5.2}$$

where: a_i, b_i – coefficients.

The first of the above-mentioned ways can be used for determining the parameters of a rheological model of homogeneous media (materials) with, for instance, a linear relationship defining the influence of consolidation degree on mechanical properties. The other way, however, is advantageous when applied to evaluating the parameters of a model of multicomponent mixtures with complex relationships between properties and structure.

It should be noted that, irrespective of the accepted way of evaluating the coefficients that characterise the medium's rheological properties, knowing the ultrasonic testing results is necessary.

The $v_L = f(\delta)$ relationship, required for approximation of the coefficients $k_C(\delta)$ and $k_T(\delta)$, can be determined on the grounds of the ultrasonic testing results obtained on the test stand shown schematically in Fig. 7.

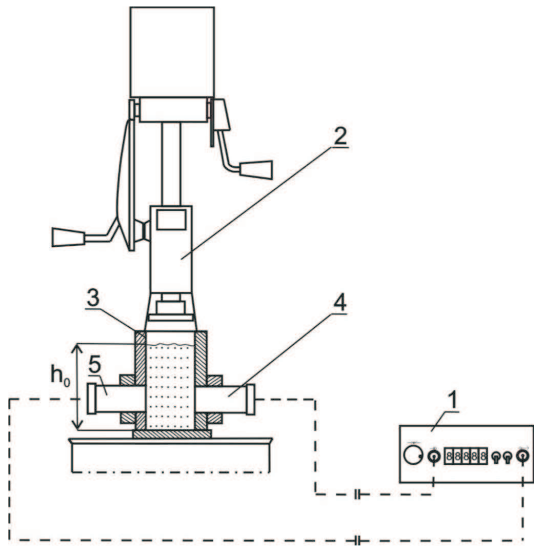

Fig. 7. Layout of the test stand for measurements of longitudinal ultrasonic wave propagation velocity: Tester 543 (1), rammer LU (2), measurement chamber (3)

The test stand is composed of: material tester type 543 (Unipan Poland) (1), laboratory rammer LU (Multiserw Poland) (2), measurement chamber (3), ultrasonic transmitter (4) and receiver (5) heads.

The transmitting and the receiving ultrasonic heads are rigidly fitted to the opposite walls of the rectangular measurement chamber. Such an arrangement assures fixed length of the test sample and good contact with the heads, which guarantees good repeatability of the test results.

Ultrasonic testing of moulding sands consisted in measuring the time delay between the transmitted and the received impulses of the longitudinal ultrasonic wave propagating in a properly densened sample of the moulding sand. The measurement procedure was as follows.

The sample of the moulding sand (m = 350 g) was dosed into the measurement chamber and densened with the rammer. Then, the longitudinal ultrasonic wave was generated by a single stroke of the rammer's weight and the time of its propagation through the sand sample height h_1 was read on the Tester 543 counter. Measurements of the propagation time t_i and the sample height h_i were repeated several times, each time with the moulding sand increasingly densened with the laboratory rammer.

The sample height and propagation time measurements made the ground for calculation of the densening degree δ and the correlation $v_L = f(\delta)$ that can be approximated by an exponential function:

$$v_L = c_1 \cdot \exp(c_2 \cdot \delta), \tag{6}$$

where: c_1, c_2 – coefficients.

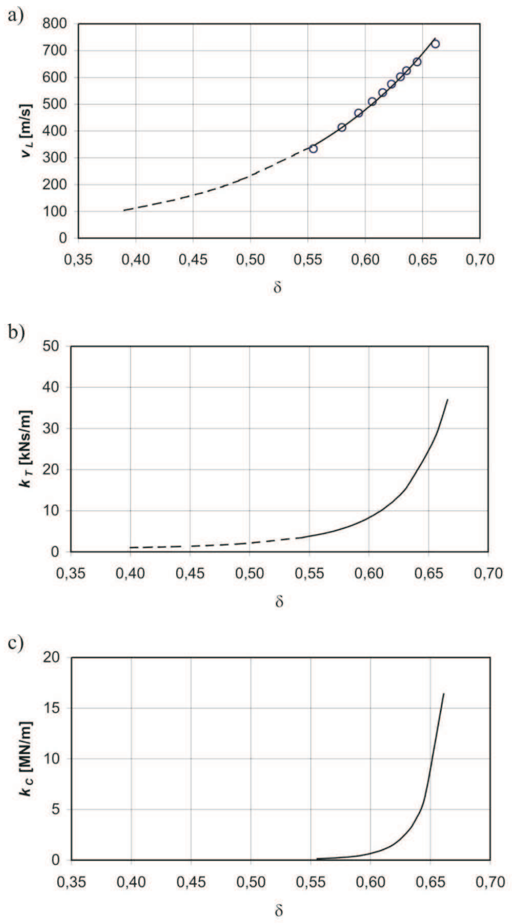

Fig. 8. Experimental results of longitudinal ultrasonic wave propagation velocity $v_L = f(\delta)$ (a) and the relationships $k_C = f(\delta)$ and $k_T = f(\delta)$ for the moulding sand with 6% of bentonite and moisture content W = 2.44%

The relationship (6) is valid in the range of densening degree variation $\delta = \delta_1 \div \delta_n$. The coefficients $k_C = f(\delta)$ and $k_T = f(\delta)$ are obtained by introducing an approximated function $v_L = f(\delta)$ to the relationships (5.1) and (5.2).

Knowledge of the rheological model parameters makes it possible to apply it to describing the medium deformation process as a function of time. In the case of moulding sands, the rheological model can be used e.g. for mathematical description of the dynamic densening process.

Figure 8 shows the relationships $v_L = f(\delta)$, $k_C = f(\delta)$ and $k_T = f(\delta)$ for the moulding sand with 6% of bentonite and moisture content W = 2.44%.

2.4. Experimental verification of the rheological model of moulding sand

The experimentally identified rheological model of moulding sand made a basis for developing a mathematical model for the process of dynamic densening of moulding sands, described by Ganczarek [14].

The simulation research of the mathematical model developed in the Laboratory of Basic Automation of the Institute of Machine Engineering and Automation of Wroclaw University of Technology was performed in the Matlab-Simulink environment. The experiments were carried out on the test stand shown in Fig. 4.

Fig. 9 illustrates the results of simulation and experimental research of the process of dynamic densening of moulding sand with 6% of bentonite and moisture content W = 2.44%.

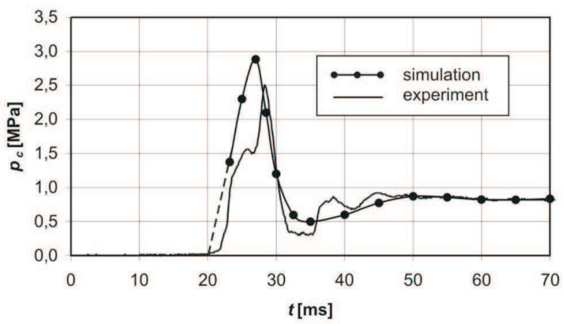

Fig. 9. Results of simulation and experimental research of the process of dynamic densening of moulding sand with 6% of bentonite and moisture content W = 2.44%

It can be found on the grounds of the presented results that the developed mathematical model of the moulding sand pressing process describes its dynamics very well, both qualitatively and quantitatively.

The above statement leads to the following conclusion: The identified rheological model of moulding sand describes its rheological properties very well.

The obtained results have proved explicitly that the time method used for identifying dynamical properties of physical systems can be successfully applied for identifying rheological properties (rheological model) of disintegrated media, e.g. soils, moulding sands etc.

3. Summary

A rheological model of real materials can be determined on the grounds of knowing the material structure, deformation process and its related changes of physical properties. In practice however, a direct description of rheological characteristics is a very complex and difficult question, and in many cases even impossible. In this situation, the rheological model can only be determined experimentally.

Such a model can be identified by one of two methods used for describing dynamic properties of physical systems: the time method and the frequency method. As far as identifying rheological characteristics of disintegrated media is concerned, the time method is easier to apply. Identification can be made on the grounds of the time step characteristics determined experimentally.

The time method was applied to identification of the model of moulding sand that can be treated as a kind of soil.

An analysis of the time step characteristics of the moulding sand proved that the mix can be represented by the viscoelastic rheological model.

Applying the rheological model to the description of the deformation process requires knowing the model's parameters, which in the case of moulding sand means the coefficients $k_C = f(\delta)$ and $k_T = f(\delta)$ that determine its elastic and viscous properties.

The parameters of a rheological model can be determined on the grounds of the results of ultrasonic testing of a specific medium (moulding sand, in this case). Knowing the $v_L = f(\delta)$ relationship permits approximation of the parameters $k_C(\delta)$ and $k_T(\delta)$ that characterise elastic and viscous properties of the material concerned.

The method of identifying a rheological model and determining its parameters has been verified experimentally. The obtained results of simulation and experimental research lead to the conclusion that the time method offers precise identification of a rheological model, and the experimentally determined relationships $k_C(\delta)$ and $k_T(\delta)$ exactly represent the elastic and viscous properties of the viscoelastic rheological model of moulding sands.

The presented methodology of identifying and evaluating the parameters of a rheological model can be used for modelling of rheological properties of disintegrated media.

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Received: 18 Junw 2007.