A R C H I V E S O F M E T A L L U R G Y A N D M A T E R I A L S
Volume 52 2007 Issue 3

W. BATKO*, L. MAJKUT*

THE PHASE TRAJECTORIES AS THE NEW DIAGNOSTIC DISCRIMINANTS OF FOUNDRY MACHINES AND DEVICES USABILITY

TRAJEKTORIE FAZOWE – NOWE WYRÓŻNIKI DIAGNOSTYCZNE ZDATNOŚCI MASZYN I URZĄDZEŃ ODLEWNICZYCH

In work, the new approach to construction of monitoring system was showed. The proposed system is based on analysis of phase trajectories and theory of technical stability. The tasks of stability theory, its mathematical formalism, have tight connections to the performance of the building process of diagnostic machine state change recognition conditions, including the matter of diagnostic symptoms and criteria values choice describing process of qualification of the changes happening in the monitored object. State changes of the monitored object are tightly connected with the changes of construction parameters of its parts, kinematic pairs or the conditions of their cooperation. They are related to relevant ratios changes: mass, elasticity, damping, which generate disturbances in characteristic movements of the initial states, and the changes of which might be the subject of control assessment in the monitoring system.

It shows purposefulness in controlling the phase images of the tested vibration signals, giving them the value of a useful tool for fault development process identification in the monitored object. Technical stability allows also determining the area of allowable solutions with regard to object load and external perturbations. About perturbation, it is assumed only that they are limited.

Keywords: phase trajectory, diagnostics, monitoring, technical stability

W pracy pokazano nowe podejście do konstrukcji systemu monitorującego zamian stanu maszyn, opartego na analizie trajektorii fazowych i teorii stateczności technicznej. Zadania teorii stateczności jej formalizm matematyczny mają ścisłe odniesienia do realizacji procesu budowy warunków diagnostycznego rozpoznawania zmian stanu maszyny, w tym zagadnień wyboru symptomów diagnostycznych jak i doboru wartości kryterialnych określających proces kwalifikacji zmian zachodzących w monitorowanym obiekcie. Zmiany stanu monitorowanego obiektu są, bowiem ściśle powiązane z zmianami wartości parametrów konstrukcyjnych jego elementów, par kinematycznych, czy warunków wzajemnej ich współpracy. Mają one odniesienia do związanych z nimi zmian współczynników: mas, sprężystości, tłumienia, które generują zaburzenia w ruchach charakterystycznych dla stanów początkowych, i których odchyłki mogą być przedmiotem ocen kontrolnych w systemie monitorującym.

W pracy wskazano na celowość kontroli zmian obrazów fazowych kontrolowanych sygnałów drganiowych, przypisując im walor użytecznego narzędzia identyfikacji procesu powstawania i rozwoju uszkodzeń monitorowanego obiektu. Stateczność techniczna pozwala również na konstrukcję obszarów trajektorii dopuszczanych z uwzględnieniem obciążenia maszyny oraz zakłóceń zewnętrznych, o których zakłada się tylko to, że są ograniczone.

1. Introduction

When reviewing practically functioning monitoring systems for machinery state, and the results of currently held research it can be briefly concluded, in several general thoughts, that:

- Watching the machine state is made by monitoring systems where several quantities are observed: particular values of numeric estimates (*e.g. rms, peak and mean values or their combination*), or particular functional images of monitored diagnostic signals (*movement trajectory of shaft neck in a bearing, spectral density function, correlation, coherence, cepstrum, envelope etc.*)
- Criteria values for monitored diagnostic symptoms (defining particular states of the object) are described with relevant standards, regulations and agreements emerging out of exploitation experience or assuming acceptable scenarios of faults of the object or from statistical processing of controlled diagnostic signals.

* DEPARTMENT OF MECHANICS AND VIBROACOUSTICS AGH UNIVERSITY OF SCIENCE AND TECHNOLOGY, 30-059 KRAKÓW, 30 MICKIEWICZA AV., POLAND

- There are only few monitoring system solutions which functions are based on partial relation-binding monitored state of the objects with the changes of the signal observed.
- In construction rules of monitoring system there is no consistent theory enforcing logical relations of conditions of lack of safe functioning capabilities in monitored object to the choice rules for diagnostic symptoms for state change observation of the monitored objects and the conditions of their undisturbed estimation.
- Often the construction and exploitation features of the object are not well enough taken into consideration in the process of monitoring system building.

The above assessment does not tend to name all the problematic questions that appear during the development of the monitoring system or describe known research and experimental results. Yet it might be an inspiration to look for new methodological guidelines for construction process of monitoring system without the limitations presented above.

The purpose of the paper is to show some possibilities. It seems that a good tool to perform such a task might be a theory of technical stability [3], which allows to meet and solve a number of tasks in the process of monitoring system development. That theory will be the basis for the framework of research actions and their algorithms management dedicated to search for new, related to the dynamics of the monitored object, diagnostic symptoms and the choice of the quantification levels allowing diagnostic decision making.

2. Adaptation of technical stability to diagnostics

The aim of technical stability [3] is determining the stability of solution the problem, which is described by the system of non-linear differential equations. Such approach differs from Lapunov approach because it permits to consider any (well-known) excitation (describing for example by: the technological process, different states of work and any external load distribution of machine or construction), and constantly acting external perturbations. These perturbations can also contain the uncertainty regarding to value and of some material constants and its distribution (density, Young modulus, Poisson ratio, ...). Mentioned above, excitation and perturbation, about which is assumed only that they are limited, influence on size the area of allowable solutions.

The definition of technical stability tells, that: "If for every initial conditions belonging to set ω, the phase trajectory stays in area Ω for every $t > t_0$ and with limited function of perturbations then the set of equations is technically stable with respect to areas ω and Ω [3].

In case of diagnostics, the phase trajectories can be determined by measurement on diagnosed object, however the problem is with determination the areas of allowable solutions Ω. The object, for which phase trajectory goes out outside area Ω can be declared as "unfit", with regard to condition for which this area was determined.

Such condition can be for example appearing the plastic strain in element. Let's assume that diagnosed element was designed to transfer some static loads. These loads cause the elastic stains in element. Modelling the damage in analysed element we check, for what value of damage the designed load will cause the plastic strains. For so definite value of damage the vibration amplitude the damaged element is determined in point where the position of sensor in diagnostic experiment is planed (by e.g. the Finite Element Method). In computer simulation, the force has to have the same amplitude, frequency and point of applying as it was planned in diagnostic experiment. Assuming that the analysed system is a linear system the amplitude of vibration velocity equals the product of vibration amplitude and frequency of excitation. Calculated in such way values of vibration and velocity amplitudes determines the rectangular area of allowable solution.

3. Numerical examples

In this section of work the changes of phase trajectories, caused by damages in few different elements was shown. In every analysed case the conditions needed to determine the areas of allowable solution are given. These areas are marked on every figure.

3.1. Diagnostics of foundry elements

The different damages of beam like element, in this part of work were analysed. As a physical model of element the simply supported Bernoulli - Euler was accepted. Such model in Fig. 1 was showed.

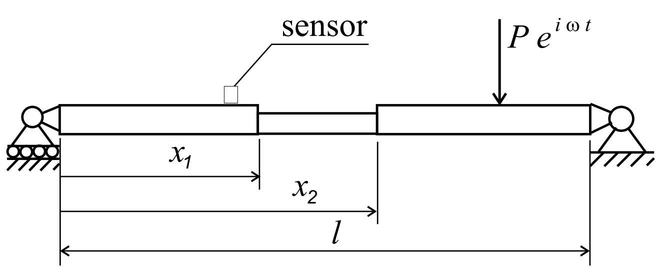

Fig. 1. Diagnosed beam

Proposed diagnostics method base on phase trajectories analysis that is determination displacement and velocity in arbitrary chosen point of beam (sensor in

Fig. 1). In this work, modelled excitation was the harmonic excitation with frequency less than the first natural frequency of element.

Three kind of damage was considered: two kind material decrement e.g. as result of friction and sectioned change of bending stiffness as result a corrosion.

The frictional decrements of material were divided in to two kinds: such, in which the depth of decrement is solid and the length $(x_2 - x_1)$ changes as well as such, in which the length of section $(x_2 - x_1)$ it is solid and the depth of decrement changes.

– stepped beam 1

Analysed damage was: the change relative length with decrease in cross-section $(x_2 - x_1)/l$. On whole sector the 5%, decrease in height of beam was analysed. The area of allowable solutions Ω was determined by stress analysing in normal state of work. As the limited value of damage, the value that will cause plastic strains was accepted.

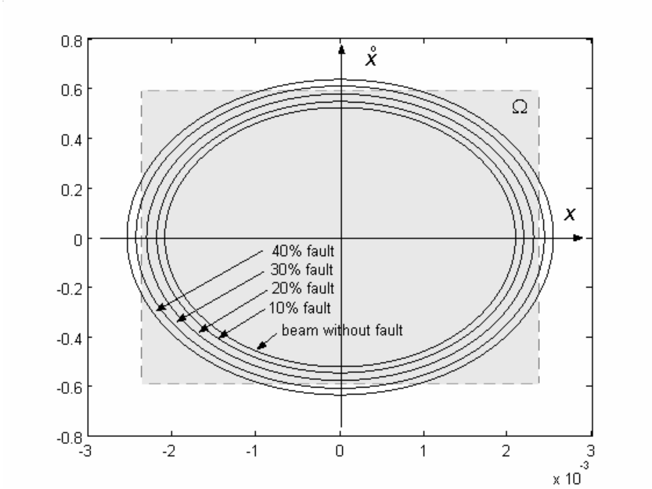

Fig. 2. Phase trajectories of stepped beam

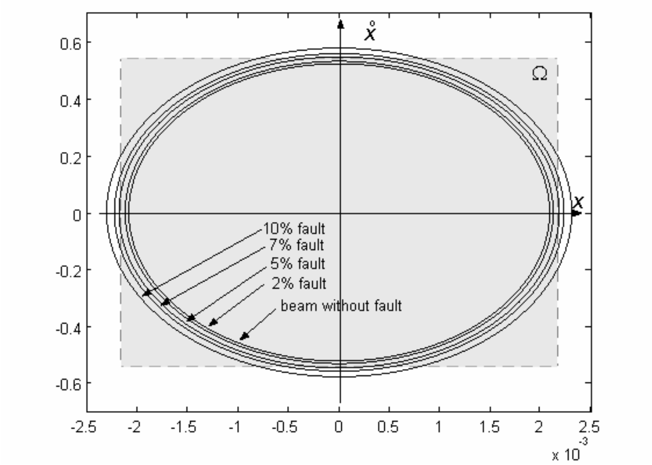

Fig. 3. Phase trajectories of stepped beam

– stepped beam 2

In this case it was assumed that the frictional decrement of material take place on section of length $(x_2-x_1)/l$ = 1/10 of beam length. The analysed damage was: the change (decrease) of relative height of beam. The area of allowable solutions Ω was determined by stress analysing in normal state of work. As the limited value of damage, the value, which will cause plastic strains, was accepted.

– beam with sectional change of bending stiffness

Such damage of construction comes from corrosion of constructional material. It was assumed that mass of beam is not changing (rust have not dropped off yet). The analysed damage was: the change of relative length with decrease in bending stiffness. On whole sector the 5% decrease in bending stiffness was analysed. The area of allowable solutions Ω was determined by stress analysing in normal state of work. As the limited value of damage, the value that will cause plastic strains was accepted.

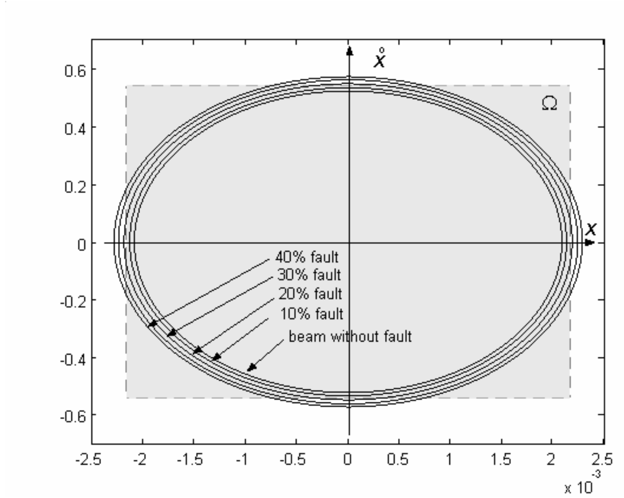

Fig. 4. Phase trajectories of beam with sectional change of bending stiffness

Such diagnostics can be used to investigation of usability to future work also two- dimensional elements that is plates. As examples of damage in such elements the weld crack of cantilever plate and two-dimensional crack in plate (scratch on parallel direction to weld) were analysed

– cantilever plate (plate fixed along one edge).

The analysed damage was: the relative length of weld crack. The area of allowable solutions Ω was determined by stress analysing in normal state of work (stress under designed static load). As the limited value of damage, the value, which will cause plastic strains, was accepted.

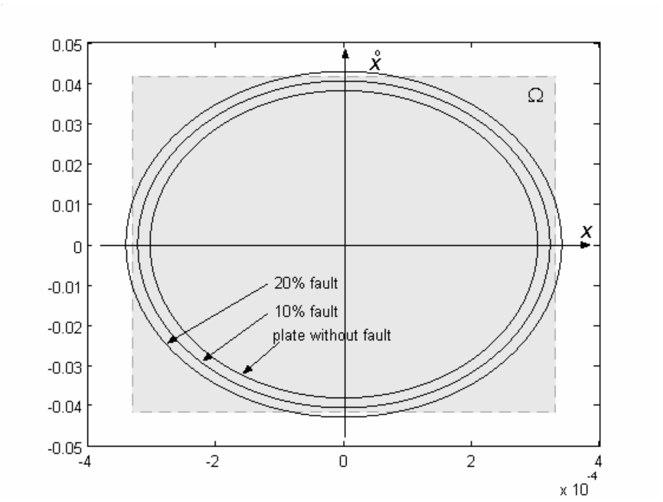

Fig. 5. Phase trajectories of cantilever plate

– scratched cantilever plate

The analysed damage was: the relative length of two-dimensional crack. The modelled depth of crack was accepted as 10% plate height. The area of allowable solutions Ω was determined by analysing stress analysing in normal state of work (stress under designed static load). As the limited value of damage, the value, which will cause plastic strains, was accepted.

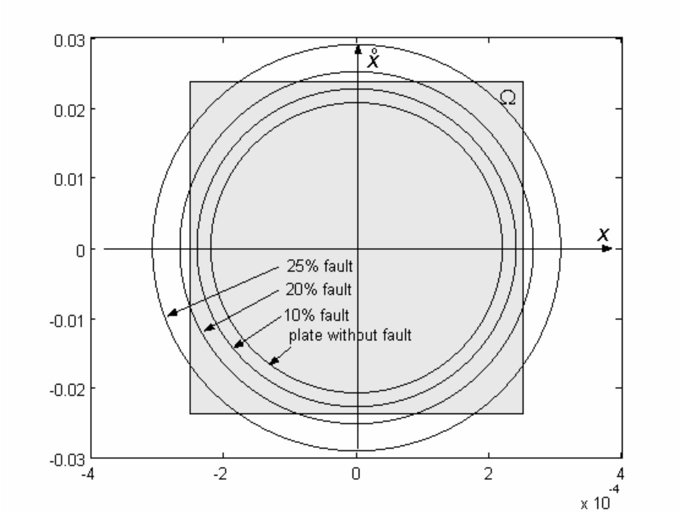

Fig. 6. Phase trajectories of cracked cantilever plate

3.2. Diagnostics of rotating machine

In this part of work, the influence of crack on dynamic behaviour of rotating machines shaft was analysed. The method of crack modelling in appendix A was described. The symptom of crack by observation of phase trajectories was searched. The change of phase trajectories in Fig. 7 was showed, which was determined for one co-ordinate and different length of crack. The area of allowable solutions Ω was determined for a certain crack length. The value of this length was defined so that crack growth rate per cycle (d*a*/d*N*) (calculated from Paris relationship) has chosen value. Such choice the condition for area Ω, allows to calculation a time to failure of analysed machine.

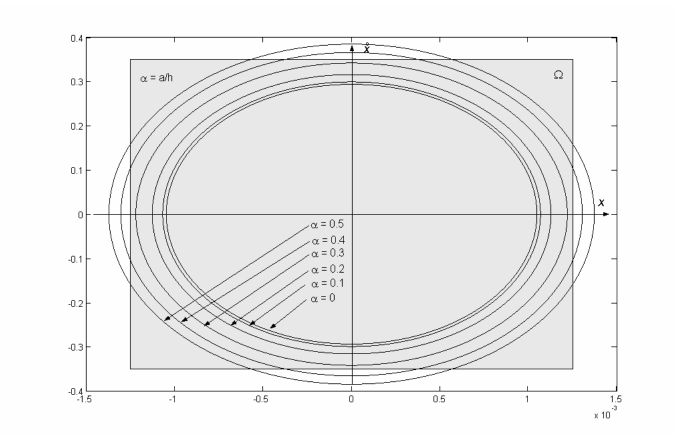

Fig. 7. Phase trajectories changing as result of crack

In fig. 8 shows the phase trajectories for shaft with one length but different location of crack. The area of allowable solutions Ω was determined in the same way as in previous example. Area Ω depends on stress intensity factor, so not only on length of crack but also on stresses in crack location.

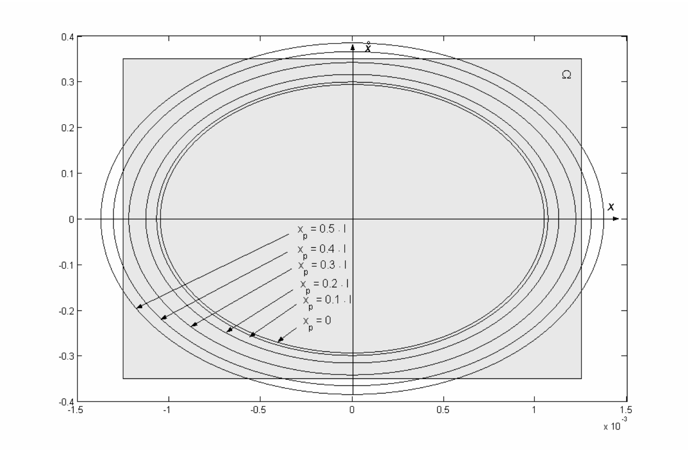

Fig. 8. Phase trajectories changing as result of crack

4. Final notes

Presented results of calculations, directed towards search of new research tools dedicated to better recognition of the monitored object state change seems to be promising.

Proposed approach based on technical stability of controlled object gives a good definition of the process of object's transition into functional disability. It relates fully to the non-linear physics of the phenomenon describing the process. It ties realized recognition with the dynamic state of the monitored object and with its constructional and exploitation parameter changes, which makes it universal.

The practical application of the phase trajectory change control method seems to be very useful tool of fault appearing and development process identification. It might be its main quality factor and is easily adoptable to practical application.

Appendix A

In order to estimate the effect of crack on shaft vibration it has been simulated as a set of flexibilities. The crack has been modelled as [2×2] flexibility matrix containing c_g, c_w and c_s coefficients on the main diagonal, and flexibility coefficients c_{gw} and c_{wg} outside the diagonal. Relation between displacements (longitudinal $u(x)$, lateral $y(x)$ and torsional $\varphi(x)$) from the right and left hand side of the cross-section with crack and longitudinal force $P_w(x_p)$, bending moment $M_g(x_p)$ and torsional moment $M_s(x_p)$ in this cross-section is given by matrix relation [5, 6]:

$$\begin{bmatrix} c_g & c_{gw} & 0 \\ c_{wg} & c_w & 0 \\ 0 & 0 & c_s \end{bmatrix} \begin{bmatrix} M_g(x_p) \\ P_w(x_p) \\ M_s(x_p) \end{bmatrix} = \begin{bmatrix} y'(x_p^+) - y'(x_p^-) \\ u(x_p^+) - u(x_p^-) \\ \varphi(x_p^+) - \varphi(x_p^-) \end{bmatrix}. \quad (A1)$$

The equation binding the every flexibility and crack depth based on fracture mechanic and Castigliano theorem are determined.

The fracture mechanics studies [8] allow to find relations between global quantity G – Energy Release Rate determining the increase in elastic strain energy for infinitesimal crack surface increase:

$$G = \frac{\partial U}{\partial A_p},$$

and local quantity K – Stress Intensity Factor (SIF), which is function of crack depth a:

$$G = \frac{1-\nu^2}{E} \cdot K_I^2 + \frac{1-\nu^2}{E} \cdot K_{II}^2 + \frac{1+\nu}{E} K_{III}^2,$$

where: G – energy release rate represents the elastic energy per unit crack surface area, A_p – area of crack, ν – Poisson ratio, E – Young modulus, $K_I, K_I I, K_I II$ – Stress Intensity Factor (SIF) of mode I, II and III of crack surface displacements.

In case of element with arbitrary loads one have to take under consideration fact that the normal stress come from both bending moment and the longitudinal force, that is

$$K_I = K_{Ig} + K_{Iw},$$

K_{Ig} – Stress Intensity Factor of mode I for bending moment M_g,

$$K_{Ig} = \sigma_g \cdot \sqrt{\pi \cdot a} \cdot F_{Ig}\left(\frac{a}{h}\right),$$

where: σ_g – normal stress from bending moment a-depth of crack F_{Ig} – correction function [7]

K_{Iw} Stress Intensity Factor of mode I for axial force P_w,

$$K_{Iw} = \sigma_w \cdot \sqrt{\pi \cdot a} \cdot F_{Iw}\left(\frac{a}{h}\right),$$

where: σ_w – normal stress from axial force, F_{Iw} – correction function [7]

The others Factors are given by:

$$K_{II} = \tau_{II} \sqrt{\pi a} \cdot F_{II}\left(\frac{a}{h}\right),$$

where: τ_{II} – this part of shearing stress, which is simultaneously parallel to the surface of crack and the perpendicular to its edge

$$K_{III} = \tau_{III} \sqrt{\pi a} \cdot \sqrt{\frac{2}{\pi a} \cdot tg \frac{\pi a}{2}},$$

τ_{III} – this part of shearing stress, which is parallel simultaneously to the surface of crack and to his edge

Total increase the elastic strain energy due to the crack has form

$$U = \int_{A_p} G \, dA_p.$$

Individual flexibilities included in the flexibility matrix (A1) can be calculated using the Castigliano theorem:

$$c_g = \frac{\partial^2 U}{\partial M_g^2(x_p)}; \quad c_w = \frac{\partial^2 U}{\partial P_w^2(x_p)}; \quad c_s = \frac{\partial^2 U}{\partial M_s^2(x_p)};$$

$$c_{gw} = \frac{\partial^2 U}{\partial M_g(x_p)\, \partial P_w(x_p)}; \quad c_{wg} = \frac{\partial^2 U}{\partial P_w(x_p)\, \partial M_g(x_p)}.$$

According to the Schwarz's theorem the sequence of differentiation has no effect on the final result, which means that $c_{gw} = c_{wg}$.

Relation (A1) described the continuity conditions in location with crack.

This work was done as a part of research project N 504 042 32/3443

REFERENCES

[1] W. B a t k o, Stateczność techniczna - nowa perspektywa modelowa dla budowy rozwiązań systemów monitorujących zmiany stanu maszyn. Mat. XXXIV Ogólnopolskiego Sympozjum Diagnostyka Maszyn. Wyd. Wydz. Transportu Politechnika Śląska CD-Węgierska Górka (2007).

[2] W. B a t k o, L. M a j k u t, Damage identification in prestressed structures using phase trajectories, Diagnostyka (2007).

[3] W. B o g u s z, Stateczność techniczna. PWN Warszawa (1972).

[4] L. M a j k u t, Identyfikacja pęknięcia w belkach o znanych warunkach brzegowych. Diagnostyka **32**, 107-116 (2004).

[5] L. M a j k u t, Modelowanie pęknięcia w wibroakustycznej diagnostyce według modelu. Diagnostyka **34**, 15-22 (2005).

[6] L. M a j k u t, Wibroakustyczne symptomy pęknięcia belki, Zagadnienia Eksploatacji Maszyn **40**, 4, 165-181.

[7] Y. M u r a k a m i, Stress Intensity Factors Handbook, Pergamon Press Oxford (1987).

[8] A. N e i m i t z, Mechanika pękania, Warszawa, PWN (1998).

Received: 18 June 2007.