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STATE OF STRESS IDENTIFICATION IN NUMERICAL MODELING OF 3D ISSUES

IDENTYFIKACJA STANU NAPRĘŻENIA W NUMERYCZNYM MODELOWANIU ZAGADNIEŃ 3 D

The purpose of the stress triaxiality k is an unequivocal identification via single numerical value of the state of stress in the point (FEM node) for individual shaping process phases. The paper presents the analysis of the stress triaxiality k both for planar and tree-dimensional cases. The imperfections of currently used stress triaxiality k have been indicated and a new factor k_n had been proposed. This new factor unequivocally allows to determine the state of stress both for 2D (two-dimensional) and 3D (three-dimensional) problems.
Keywords: numerical simulation, stress triaxiality

W mechaniczno-matematycznym podejściu do modelowania odkształcenie i pękanie materiału silnie zależy od stanu naprężenia panującego w kształtowanym materiale. Praktyczne zastosowanie funkcji odkształcalności granicznej, czy też znanych z literatury licznych kryteriów do prognozowania pękania materiału w procesach technologicznych, wiąże się z koniecznością jednoznacznego określenia stanu naprężenia panującego w poszczególnych obszarach kształtowanego materiału. Posługiwanie się w takich przypadkach tensorem naprężenia jest niepraktyczne i trudne. Z tego względu stan naprężenia obecnie najczęściej opisuje się za pomocą wskaźnika stanu naprężenia stanowiącego stosunek naprężeń średnich do intensywności naprężeń. Zadaniem wskaźnika stanu naprężenia k jest za pomocą jednej wartości liczbowej jednoznacznie identyfikować stan naprężenia w punkcie (węźle – MES) na poszczególnych etapach procesu kształtowania. W pracy przeprowadzono analizę wskaźnika stanu naprężenia k zarówno dla przypadków rozważanych jako płaskie jak i przestrzenne. Wskazano na niedoskonałości obecnie stosowanego wskaźnika k i zaproponowano nowy wskaźnik k_n, pozwalający jednoznacznie określić stan naprężenia zarówno w modelowaniu zagadnień płaskich 2-D jak i przestrzennych 3-D. Jego praktyczne zastosowanie w modelowaniu numerycznym procesów plastycznego kształtowania materiałów wiąże się jedynie z koniecznością napisania prostego podprogramu działającego według przedstawionego w pracy algorytmu.

1. Introduction

In mechanical-mathematical modeling approach, the material's strain and ductile fracture strongly depends on the existing state of stress in individual areas of the material being strained. In the most general case, the state of stress in the strained material is determined by nine stresses: three normal stresses (σ_x, σ_y, σ_z) and six shearing stresses (τ_{xy}, τ_{xz}, τ_{yz}, τ_{yx}, τ_{zx}, τ_{zy}). Those stresses may be denoted in form of a symmetrical matrix – stress tensor:

$$T_\sigma = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}, \tag{1}$$

Each state of stress may be expressed as principal stress tensor:

$$T_\sigma = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}. \tag{2}$$

In this case, the principal stresses (σ_1, σ_2, σ_3) are present in principal axes directions, but shear stress components equal zero.

In several cases, using the state of stress tensor to identify unequivocally individual stresses during shaping process is really inconvenient and awkward. As a practice approach, the stress triaxiality is used to identify the state of stress and is defined as follows:

$$k = \frac{\sigma_m}{\sigma_i}. \tag{3}$$

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Where, in general case:

$$\sigma_m = \frac{\sigma_x + \sigma_y + \sigma_z}{3}. \tag{4}$$

$$\sigma_i = \frac{1}{\sqrt{2}} \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)}. \tag{5}$$

σ_m – mean normal stress, σ_i – effective stress

Triaxiality *k* in the plane state of stress

In general case, the plane state of stress may be expressed as follows:

$$T_\sigma = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix}. \tag{6}$$

Therefore, the stress triaxiality in the plane state of stress may be expressed as follows:

$$k = \frac{\sigma_m}{\sigma_i} = \frac{\frac{1}{3}(\sigma_x + \sigma_{y)}}{\frac{1}{\sqrt{2}}\sqrt{(\sigma_x - \sigma_y)^2 + \sigma_y^2 + \sigma_x^2 + 6\tau_{xy}^2}}. \tag{7}$$

Triaxiality *k* in the plane state of strain

In general case, strains in the plane state of strain may be expressed as follows:

$$T_\sigma = \begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}. \tag{8}$$

The primary strain z on the third direction is calculated from the equation:

$$\sigma_z = \frac{\sigma_x + \sigma_y}{2}. \tag{9}$$

Therefore, the stress triaxiality in the plane state of strain may be expressed as follows:

$$k = \frac{\sigma_m}{\sigma_i} = \frac{\frac{1}{2}(\sigma_x + \sigma_{y)}}{\frac{1}{\sqrt{2}}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - (\frac{\sigma_x+\sigma_y}{2}))^2 + ((\frac{\sigma_x+\sigma_y}{2}) - \sigma_x)^2 + 6\tau_{xy}^2}}. \tag{10}$$

Triaxiality *k* in the three-dimensional state of stress

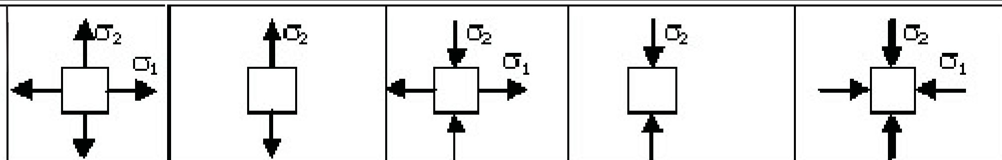
For the three-dimensional state of stress, stresses σ_m and σ_i are calculated from equations (4) and (5).

The stress triaxiality k is used practically in several scientific and research projects, mainly to describe the forming limit function e.g. [1-6], and also several ductile fracture criteria e.g. [7-12].

Even advanced capabilities of numerical methods enabled common use of the ductile fracture modeling for two-dimensional states, and currently such analyses are performed also for three-dimensional models. Actually, the forming limit curves are determined for simple cases of stresses, analyzed usually in an axisymmetric and two-dimensional states. For example, the formability limit function $\varepsilon_p = f(k)$, determined experimentally for two-dimensional state of stress, may be used to forecast the fracture with FEM in such a state, however cannot be used for two-dimensional strain modeling, as in this case values of the stress triaxiality *k* differ (see Table 1). Currently FEM allows determination of the state of stress not only for special cases, but almost for all state of stress cases. Variety of such cases in three-dimensional modeling makes that the stress triaxiality k takes identical values for different states of stress (see Table 2). Using this stress triaxiality for three-dimensional modeling may lead to significant errors due to improper state of stress identification. Such a state become a basis to perform the relevant experiments.

TABLE 1

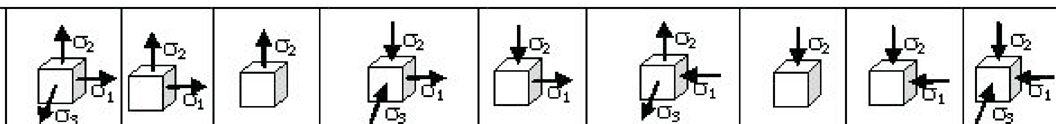
The stress triaxiality k values for individual cases analyzed in the plane state of stress and the plane state of strain



two-dimensional state of stress		$\sigma_1 = \sigma_2$	$\sigma_1 = 0,\ \sigma_2 > 0$	$\sigma_1 = -\sigma_2$	$\sigma_1 = 0,\ \sigma_2 < 0$	$-\sigma_1 = -\sigma_2$
stress triaxiality value	plane stress (7)	0,66	0,33	0	-0,33	-0,66
	plane strain (10)	∞	0,57	0	-0,57	$-\infty$

TABLE 2

The stress triaxiality k (recently used) values and factor k_n (new proposed one) for 3D analyzed cases



tree-dimensional state of stress		$\sigma_1=\sigma_2==\sigma_3>0$	$\sigma_1=\sigma_2,\ \sigma_3=0$	$\sigma_2>0\ \sigma_1=\sigma_3=0$	$\sigma_1>0,\ \sigma_2=\sigma_3=-\frac{\sigma_1}{2}$	$\sigma_1=-\sigma_2\ \sigma_3=0$	$\sigma_1<0,\ \sigma_2=\sigma_3=-\frac{\sigma_1}{2}$	$\sigma_2<0\ \sigma_1=\sigma_3=0$	$-\sigma_1=-\sigma_2,\ \sigma_3=0$	$\sigma_1=\sigma_2==\sigma_3<0$
stress triaxiality value	so far used k (3)	∞	0,66	0,33	0	0	0	-0,33	-0,66	$-\infty$
	suggest new k_n (11)	∞	3	2	0,33	0	-0,33	-2	-3	$-\infty$

The actual application of the forming limit function or other commonly recognized criteria for metal working process fracture forecasting requires also unequivocal state of stress determination in individual areas of processed material. As demonstrated, applying the stress triaxiality used in three-dimensional modeling may lead to ambiguous state of stress identification. That is why it's purposeful to perform research works for new indicator, which fits its purpose.

2. Proposition of a new stress triaxiality k_n

In order to solve the problem of state of stress' unequivocal identification in three-dimensional problems, the software application has been developed based on the algorithm as shown on Figure 1.

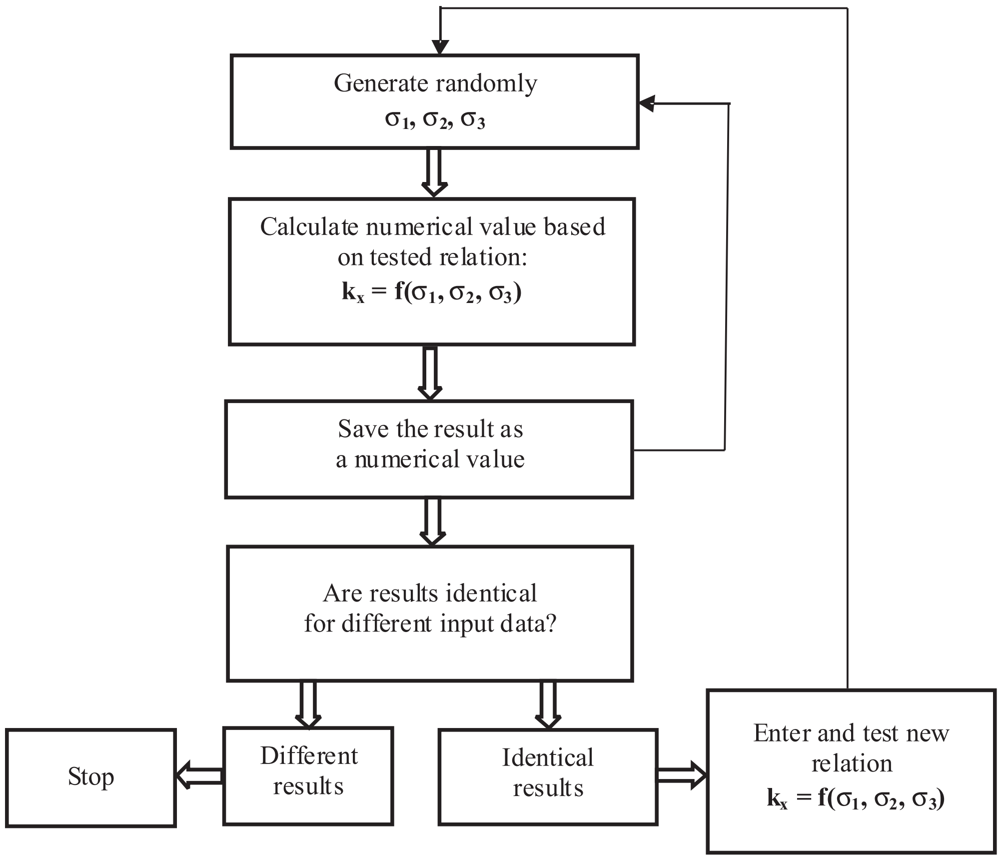


Fig. 1. The algorithm of testing program

The purpose of this program is testing various mathematical equations expressed as $k_x = f(\sigma_1, \sigma_2, \sigma_3)$ to eliminate the disadvantage of recently used stress triaxiality. The numeric values has been entered randomly. Once the calculations have been performed, it has been checked whether their numerical results for various input data (different states of stress) match, as for the recently used stress triaxiality (see Tab. 2). Where results matched for different states of stress, the examined relation has been modified and then the re-testing has been performed. After multiple strenuous and unsuccessful attempts, the relation has been formulated as follows:

$$k_n = \frac{2(\sigma_{\max} + \sigma_{\min}) + \sigma_{\mathrm{mid}}}{\sigma_i}, \qquad (11)$$

where: $\sigma_{\max}$ – the highest principal stress selected from $(\sigma_1, \sigma_2, \sigma_3)$, σ_{mid} – medium principal stress selected from $(\sigma_1, \sigma_2, \sigma_3)$, min - the lowest principal stress selected from $(\sigma_1, \sigma_2, \sigma_3)$.

The state of stress k_n determined based on proposed relation (11) unequivocally identifies individual states of stress comparing to recently used stress triaxiality (see Tab. 2). The equation as referred to the above has been tested for various states of stress and it has not been stated within tested stress range that two different states are identified with identical value of the stress triaxiality. Moreover, the span and numerical values of proposed stress triaxiality are higher than the recently used (see Tab. 2), and it is quite convenient and also simplifies analyzing achieved results.

Significant improvement of the proposed stress triaxiality is visible in the state of stress ranges, which are close to shearing state. For example, within five selected stress states (see Tab. 3) recently used factor k distinguishes none of them despite the different stress intensity σ_i for each of them. This may be also assumed that the fracture strain will be different for each of the stated states. New proposed stress triaxiality k_n is able to distinguish the presented stress states (see Tab. 3), enabling their identification unequivocally.

TABLE 3

The stress triaxiality k (recently used) values and factor k_n (new proposed one) for example five stress states

Lp.	σ_1	σ_2	σ_3	σ_i	k (3)	k_n (11)
1.	100	-100	0	173	**0**	**0**
2.	100	-50	-50	150	**0**	**0.33**
3.	-100	50	50	150	**0**	**-0.33**
4.	100	-25	-75	156	**0**	**0.16**
5.	-100	95	5	169	**0**	**-0.02**

Similarly to factor k, the proposed stress triaxiality k_n is the stress tensor invariant, and its values are not changed along with the reference system change.

Using new stress triaxiality k_n in order to identify the state of stress in computer simulations with e.g. finite element method (FEM), combines also using the simple algorithm (Fig. 2).

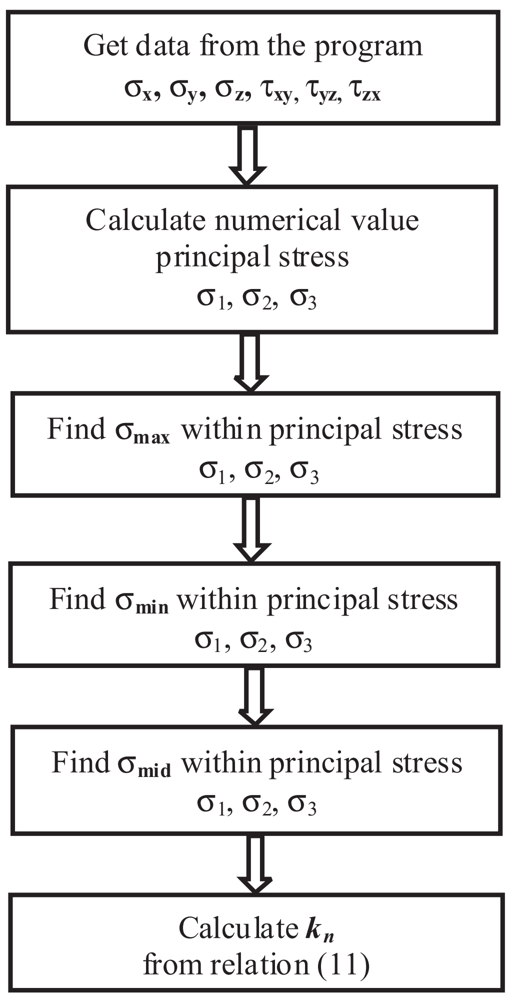


Fig. 2. The k_n calculation algorithm using the computer software

3. Conclusions

As presented, new stress triaxiality kn is much more useful in context of an identification of three-dimensional state of stress than the recently stress triaxiality k. New stress triaxiality may be used both for three-dimensional and two-dimensional stress analysis. The only requirement of its practical application in numerical modeling of plastic material processing is to develop simple routine based on the presented algorithm.

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