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J. MICHALCZYK* , P. CZUBAK*

METHODS OF DETERMINATION OF MAXIMUM AMPLITUDES IN THE TRANSIENT RESONANCE OF VIBRATORY MACHINES

METODY WYZNACZANIA AMPLITUD MAKSYMALNYCH MASZYN WIBRACYJNYCH W REZONANSIE PRZEJŚCIOWYM

The development of methods of the determination of maximum amplitudes in a transient resonance of over-resonance vibratory machines and vibroinsulating systems is presented in the hereby paper.

The classic approach as well as the modern investigations of these problems are discussed and the estimation of their practical suitability – on the basis of the authors own investigations – is performed.

A uselessness – in practical applications – of methods based on the rotor angular velocity assumed in advance was indicated in the paper. It was also pointed out, that during the transient resonance of vibratory machines with a drive utilizing the self-synchronisation effect, a disphasing of vibrators occurs, which is related to the character change of the excitation forces and challenges the assumed in references models of this effect.

Keywords:

W pracy przedstawiono rozwój metod wyznaczania amplitud maksymalnych w rezonansie przejściowym nadrezonansowych maszyn o ruchu drgającym i układów wibroizolacji.

Omówiono zarówno podejście klasyczne jak i najnowsze badania w tym zakresie i dokonano oceny przydatności praktycznej poszczególnych rodzajów metod opierając się na oryginalnych badaniach autorów.

W szczególności w pracy wskazano na nieprzydatność w praktyce metod opartych na założonym z góry przebiegu prędkości kątowej wirnika. Wykazano również, że w trakcie rezonansu przejściowego maszyn wibracyjnych o napędzie z wykorzystaniem zjawiska samosynchronizacji, zachodzi rozfazowanie wibratorów, co wiąże się ze zmianą charakteru sił wymuszających i podważa przyjmowane w literaturze modele tego zjawiska.

1. Introduction

Vibratory machines applied in metallurgical industry, for a classification of loose materials (vibratory screens), short-distance transport of hot materials (vibratory conveyers), crushing operations or loose materials compaction etc., constitute often problems due to high values of maximum amplitudes during start-up and rundown operations, which can be exceeding, by several dozen times, the working amplitudes values. This results from the fact, that over-resonance machines (the most commonly used) are excited to resonance vibrations at their start-up and rundown since at that time the vibrator rotations frequency passes through the natural frequency of machine on its system of elastic suspension. Such situation can cause loosing of feed, collisions with machines cooperating at the same technological line, or an excessive load on a floor or frame of building or supporting structures on which they are installed.

On account of the problem significance, it was analysed many times in references and in effect there are several calculation formulas or nomograms for the determination of maximum amplitudes in the transient resonance. However, these methods provide various results, and in the most disadvantageous case (it means for a free rundown of a machine) usually very different from the actual values [2].

Currently known estimation methods of maximum amplitudes in transient states of vibratory machines can be divided into some categories due to the assumed model and the applied determination method.

a) Methods based on the harmonic oscillator model of the excitation performed by the a priori given quasi harmonic force of a linearly variable frequency and a constant or variable amplitude;
b) Methods of energy balance;
c) Nomogram methods based on numerical calcu-

* AGH UNIVERSITY OF SCIENCE AND TECHNOLOGY, FACULTY OF MECHANICAL ENGINEERING AND ROBOTICS,30-059 KRAKÓW, 30 MICKIEWICZA AV., POLAND

lations of machine motion equations of a rectilinear or circular trajectory;

d) Simulation methods for systems of several degrees of freedom.

2. Methods based on the harmonic oscillator model

Historically, the first solution of the transient resonance problem was obtained by F. Lewis in 1932 [7]. He analysed the system shown in Figure 1, at the excitation by a quasi harmonic force of a linearly variable angular velocity ω(t) and a constant amplitude P.

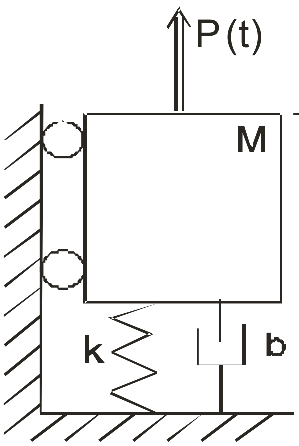

Fig. 1. Calculation model

He solved the equation of motion:

$$M\ddot{x} + b\dot{x} + kx = P\sin(\varphi_0 + \omega_0 t \pm \frac{\varepsilon \cdot t^2}{2}) \quad (1)$$

for the case of start-up (+) and rundown (-) by the impulse transfer function method, obtaining the solution envelope and determining the maximum value of the vibration amplitude.

The solution of the problem, being nearer to practice, when the amplitude of the exciting force is not constant, but proportional to the rotational velocity square $P = c\omega^2(t)$, as in the case of machines excited for vibrations by an unbalance of a rotor or piston, was given by Kac [8] in 1947.

His solutions enabled the preparation of nomograms (Fig. 2) for the determination of maximum amplitudes [10], which are currently the most broadly applied tools for this type of determination [15].

The presented model was analysed by other methods also (details can be found in [5]). The paper of R. Markert and M. Seidler [9] is quite interesting, since they provided solution of a more general case, enabling to take into account a tangent component of an unbalanced rotor force of inertia.

The similar model, in respect of describing exciting forces operations, but more resembling practical situations, was analysed by J. Goliński [15], who introduced a machine body plane motion as well as some forms of a general motion in a linear perspective.

The problem of a transient resonance of undamped system in a flat motion was also analysed by W. Bogusz [14], who uncoupled vibrations into their main forms and applied an equivalence of a differential equation and a relevant integral of the Volterra equation. This allowed to estimate the amplitude in any time section.

Simpler dependencies for machines in a flat motion and uncoupled forms of progressive and rotational vibrations, obtained without taking into account damping, are given for start-up and rundown of vibratory machines by T. Banaszewski and W. Turkiewicz [12, 13]. In a similar fashion as all discussed previously approaches, these authors' solution requires the *a priori* determination of the vibrator angular velocity during passage through a circum-resonant range. However, such determination, in practice, is only possible on the basis of the analysis of the vibrator rotational motion under the influence of the driving moment and the anti-torque moment in bearings.

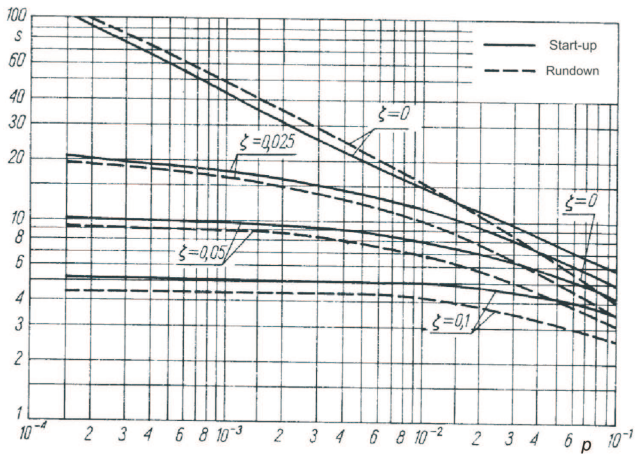

Fig. 2. Nomogram for the determination of the ratio of the maximum amplitudes to the far over-resonance amplitudes

$$\mathrm{p} = \varepsilon\frac{M}{k}, \xi = \frac{b}{2\sqrt{kM}}, \mathrm{s} = \frac{A_{\max}}{A_{\omega=\infty,\xi=0}} \quad (2)$$

A common feature of the described above analyses is also the dependence of the maximum amplitude in the transient resonance on the expected value of angular acceleration ε of the unbalanced rotor, constituting a source of excitation forces. For weakly damped systems this dependence corresponds, in approximation, to formula:

$$A_{\max} \approx a\frac{1}{\sqrt{\varepsilon}} \quad (3)$$

where: a – constant value.

Drawbacks of these group of methods.

It was shown in reference [2], that all those methods demonstrate a systematic error, which can be equal to 300% and even higher percentage for a free rundown. The fundamental cause of such errors constitute – according to [2] – an incorrect estimation of the angular acceleration on the grounds of the driving and anti-torque moments acting on the unbalanced rotor. Actually this rotor motion depends mainly on - intensifying in resonance – machine vibrations, which are the source of an additional moment (the so-called *vibratory moment* M_v [1]) influencing the rotor.

In order to explain this effect, the model – in which instead of the a priori applied exciting force the physical way of generating this force in unbalanced vibratory machines is taken into account – will be considered (Fig. 3).

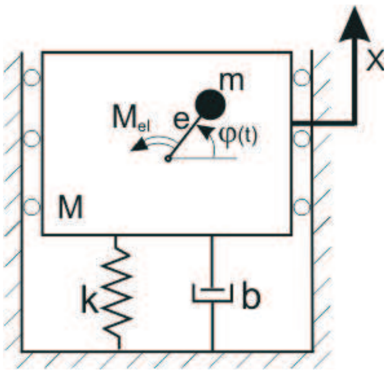

Fig. 3. Model of the system

where: M – mass of a machine body,

m, e – mass and eccentric of an unbalanced rotor, respectively,

Mel – moment acting on a rotor from the side of a driving motor,

k, b – constants of elasticity and damping of the machine supporting system, respectively,

Mo – resistance to motion moment.

Dynamics of this system is determined by equations (4):

$$(M+m)\ddot{x} + b\dot{x} + kx = \dot{\varphi}^2 me \sin\varphi - \ddot{\varphi} me \cos\varphi \quad (4a)$$

$$J\ddot{\varphi} = M_{el} - M_o - \ddot{x} me \cos\varphi \quad (4b)$$

where: expression $-\ddot{x}me\cos\varphi$ – describes the *vibratory moment*,

J – moment of inertia of a vibrator together with engine, calculated versus the axis of rotation,

An example of the time-history of the unbalanced mass angular velocity determined for the free rundown is shown in Figure 4:

a) When the vibratory moment was omitted,

b) When the machine vibration influence on the angular motion of the rotor was taken into account.

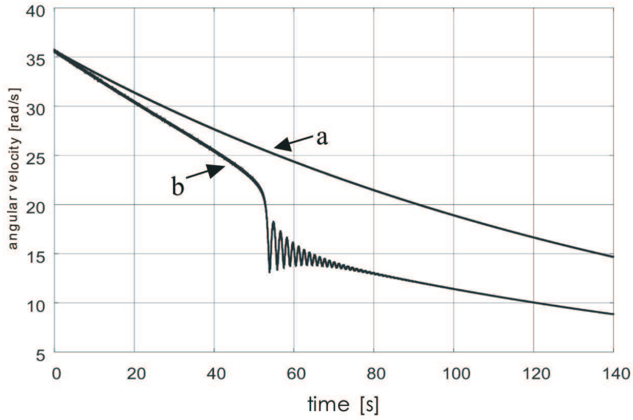

Fig. 4. Vibratory moment influence on the angular motion of the unbalanced rotor during the free rundown

As can be clearly seen, coupling in between body vibrations and the rotational motion of the rotor causes in a circum-resonant zone a sudden collapse of the rotor angular velocity, which can not be taken into account by methods based on the model of the system with one degree of freedom. This collapse is decisive for the transient resonance and the obtained amplitudes.

3. Methods of energy balance

First works taking into consideration the vibratory moment influence were based on the energy balance of the system directly before the resonance and in the moment of upmost vibrations appearance, obtaining 'top estimations' of maximum amplitudes. Thus, e.g. in papers [2, 3] the kinetic energy of the vibrators set in the moment of entering the resonance zone of the ith form of vibrations was compared with the machine vibration energy, which was vibrating in accordance with this form of vibrations. In a general case this leads to equation:

$$n \cdot \frac{1}{2} J_{zr} \omega_{0i}^2 = \frac{1}{2} \dot{\mathbf{q}}_{\max i}^{\mathbf{T}} \cdot \mathbf{M} \cdot \dot{\mathbf{q}}_{\max i} \quad (5)$$

where: n – number of identical, synchronous driving systems,

J_{zr} – moment of inertia of an unbalanced vibrator reduced on an engine shaft,

ω_{0i} – angular velocity, at which an exchange of energy occurs (in general, only in a small range different from the ith frequency of natural vibrations of a machine body on an elastic suspension system),

$\mathbf{q} = \mathrm{col} x_s, y_s, z_s, \varphi_x, \varphi_y, \varphi_z$ – coordinates vector in the central co-ordinate system describing small body vibrations versus a static equilibrium position,

$\mathbf{M}$ – inertia matrix

$$
\mathrm{M} = \begin{bmatrix} m_\Sigma & 0 & 0 & 0 & 0 & 0 \\ & m_\Sigma & 0 & 0 & 0 & 0 \\ & & m_\Sigma & 0 & 0 & 0 \\ & & & J_{xx} & -J_{xy} & -J_{xz} \\ & sym. & & & J_{yy} & -J_{yz} \\ & & & & & J_{zz} \end{bmatrix} \tag{6}
$$

m_Σ – mass of a body with vibrators,

J_{ij} – relevant elements of tensor of inertia in the central system Sxyz.

This allowed to obtain the formula for **the maximum amplitude of the k$^{\text{th}}$ coordinate during the system passage through resonance with the i$^{\text{th}}$ frequency of natural vibrations:**

$$
q_{\max ki} = \sqrt{\frac{n \cdot J_{zr}}{\mathbf{a}_{ki}^{\mathbf{T}} \cdot \mathbf{M} \cdot \mathbf{a}_{ki}}} \tag{7}
$$

where: $\mathbf{a}_{ki}$ – normalized, in consideration of the k$^{\text{th}}$ coordinate, modal vector of the i$^{\text{th}}$ vibration form.

In special cases, formula (7) leads to simple equations of maximum amplitudes [2], such as e.g. for the system shown in Figure 3:

$$
A_{\max} = \sqrt{\frac{J_{zr}}{M + m}} \tag{8}
$$

The energy balance was also applied in paper [11], however, the vibrator kinetic energy was not compared to the body kinetic energy but to the maximum potential energy of the suspension system.

The problem, in this type of energy method, constitutes the necessity of the determination of the vibrator angular velocity, at which an exchange of energy occurs, while this velocity does not comply directly with the natural vibrations frequency of the system [15] and the method does not explain how it can be determined.

Drawbacks of energy methods

Energy methods can be successfully applied for various structural schemes of machines. However, they provide 'top estimation' of real values, which can – in special cases – lead to over-estimating of the real values of the machine vibrations amplitudes in the transient resonance.

4. Nomogram methods based on numerical solutions of equations of machine motion of rectilinear or circular trajectory

G. Cieplok [5] considered the transient resonance for the symmetric system shown in Figure 5.

On the bases of the transformation of equations of symmetric machine motion into the coordinate system rotating with the velocity of unbalanced mass it was possible to define the relative units and to decrease two times the number of parameters describing the system motion.

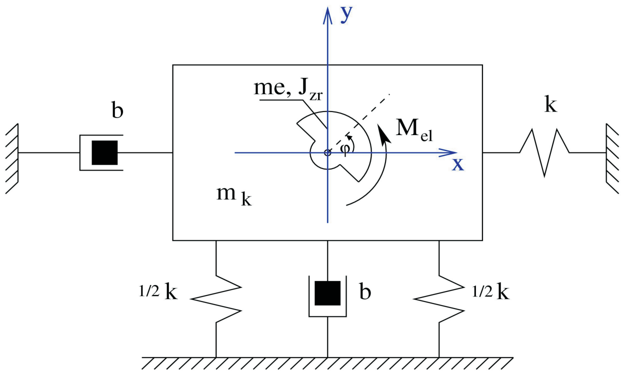

Fig. 5. Vibratory machine model

Thus, the **set of six physical parameters** $\boldsymbol{m_c, me, J_{zr}, M_{el}, k, b}$ needed for presenting the machine dynamics in natural coordinates **was reduced to three parameters** $\boldsymbol{\sigma, q, \gamma}$ when the relative parameters were applied:

$$
\sigma = \frac{m^2 e^2}{(m_k + m) J_{zr}} \tag{9}
$$

$$
q = \frac{M_{el}}{J_{zr}} \frac{1}{\omega_0^2} \tag{10}
$$

$$
\gamma = \frac{b}{2\sqrt{(m_k + m)k}} \tag{11}
$$

where: $m_c = m_k + m$ – total mass,

m_k – mass of the machine body,

me – static unbalance of the unbalanced mass m,

J_{zr} – moment of inertia of the driving system and the unbalanced mass, reduced on the vibrator shaft of the unbalanced mass

k – coefficient of elasticity of the machine support,

b – coefficient of the viscositic damping of the machine support,

M_{el} – driving moment of the constant value.

$$
\omega_o = \sqrt{\frac{k}{m_k + m}} \tag{12}
$$

Forms of equation obtained in the new notation allow for the clear graphical presentation of a large number of numerical solutions of the system motion equations. The nomogram obtained in such way for the rundown phase is presented in Figure 6.

There is also a possibility of adaptation the nomogram **for the machine of a lineary trajectory** of the body motion. In this case it is enough to use – at reading the nomogram – two times lower value of parameter σ than it results from the formula (9).

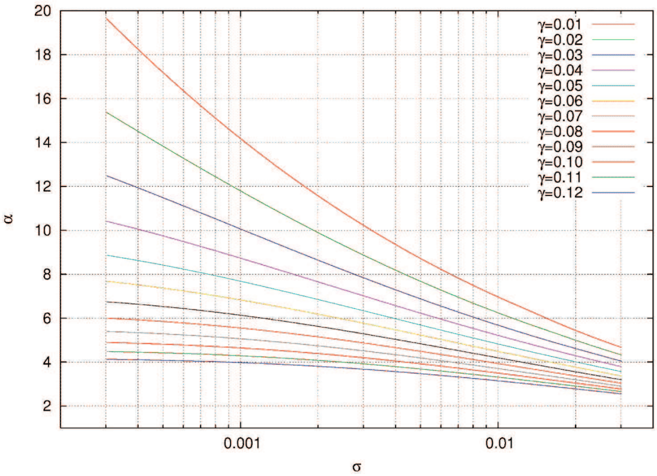

Fig. 6. Coefficient of the amplitude multiplication of the machine body vibration at vibrator rundown, $\alpha = \frac{A_{\max}}{me/m_c}$

Drawbacks of the method

In cases when the analysed real system corresponds to one of the two schemes, assumed in this method (a symmetric machine – Fig.5, or a machine of a rectilinear motion – Fig.3), this method provides accurate estimations both for the start-up and rundown as well as in the expanded version [16] where it is possible to take also into account the resistance to motion during rundowns.

However, there are two causes limiting the range and accuracy of this method:

1° Real objects of a flat motion usually do not comply with the scheme shown in Fig. 5, since they have different elastic constants in x and y direction. The numerical experiments indicate that the system motion is highly sensitive to the diversification of these constants and due to this – the model of a symmetric machine is not suitable.

2° In cases of machines of a rectilinear trajectory, the excitation rectilinearly oriented is obtained by the application of two counter running inertial vibrators, which achieve the proper cophasal on the grounds of a *free synchronization* [1].

As it was pointed out in paper [3], the cophasal running ceases to be stable at the rundown when the oscillatory vibrations frequency of the body is exceeded. Since, as can be proofed, this frequency is – in the real systems – the highest, vibrators have a tendency to disphasal when entering the resonance range of translatory motions. Thus the scheme of the system is fundamentally changed in relation to the one assumed in the discussed method.

The above problem will be investigated below by the numerical simulation method.

5. Numerical simulation method

In order to indicate that the effect of changing the exciting force character – in the circum-resonant zone – occurs in the real systems the vibratory machine with two vibrators will be discussed (Fig.7). This scheme corresponds to several over-resonance vibratory machines such as screens and conveyers, self-discharge grating shake-out machines etc.

The mathematical model of the system, which will be applied for the numerical simulation of the machine motion, is presented in paper [4].

On the grounds of this model the transient resonance course, including time history, will be investigated:

a) body displacement in the direction of working vibrations μ,

b) body displacement in the direction ν perpendicular to μ,

c) body angular oscillations α,

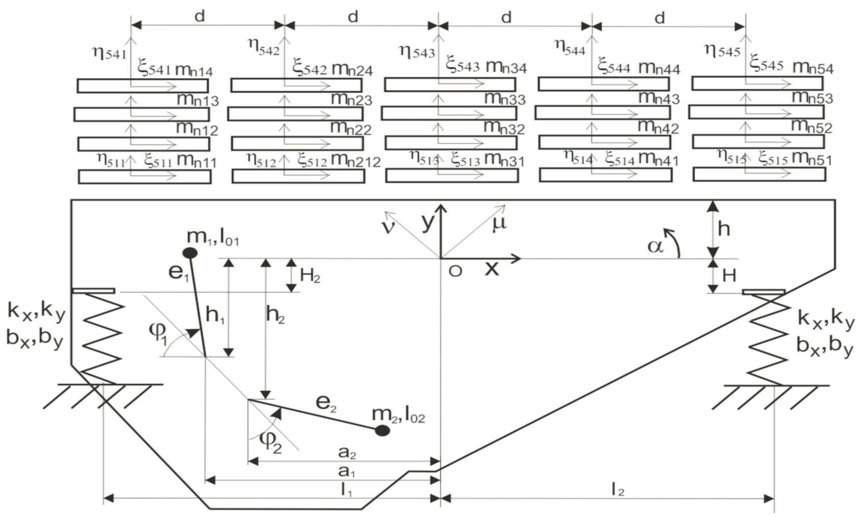

Fig. 7. Calculation model of the over-resonance two-vibrator vibratory machine with loose feed

d) Angle of a mutual dissynchronising of vibrators $\varphi_1 - \varphi_2$ during the free coasting.
Simulations were performed in the following variants:
A.Courses obtained when analysing the machine motion without taking into account the feed influence and without the gravity forces influence on unbalanced masses are presented in Figures 8, 9.

Diagrams showing body angular oscillations α, without taking into account the influence of feed and the gravity forces and angle of a mutual dissynchronising of vibrators $\varphi_1 - \varphi_2$, without taking into account the influence of feed and the gravity forces are not included, since these values – during the simulation – are at the zero level, in a similar fashion as displacement in the ν direction (Fig. 9).

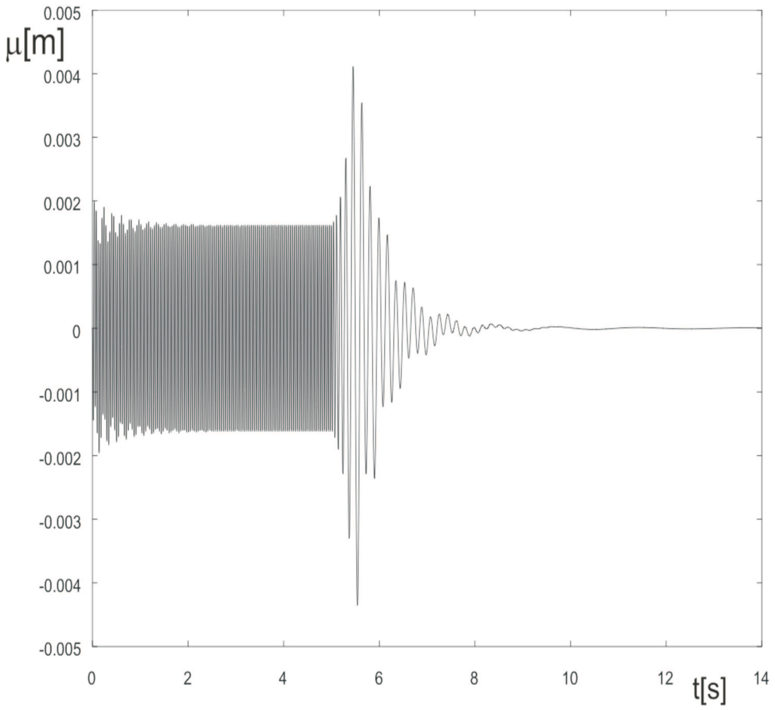

Fig. 8. Body displacement in the direction of working vibrations μ, without taking into account the influence of feed and the gravity forces

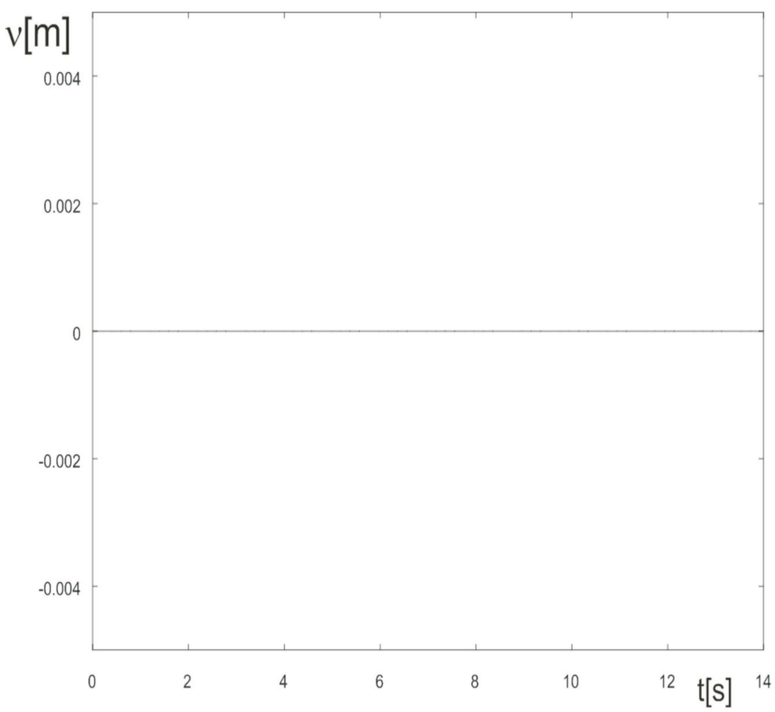

Fig. 9. Body displacement in the direction ν perpendicular to μ, without taking into account the influence of feed and the gravity forces

B.Courses obtained when analysing the machine motion without taking into account the feed influence but with taking into consideration the gravity forces influence on unbalanced vibrators masses are presented in Figures 10, 11, 12 and 13.

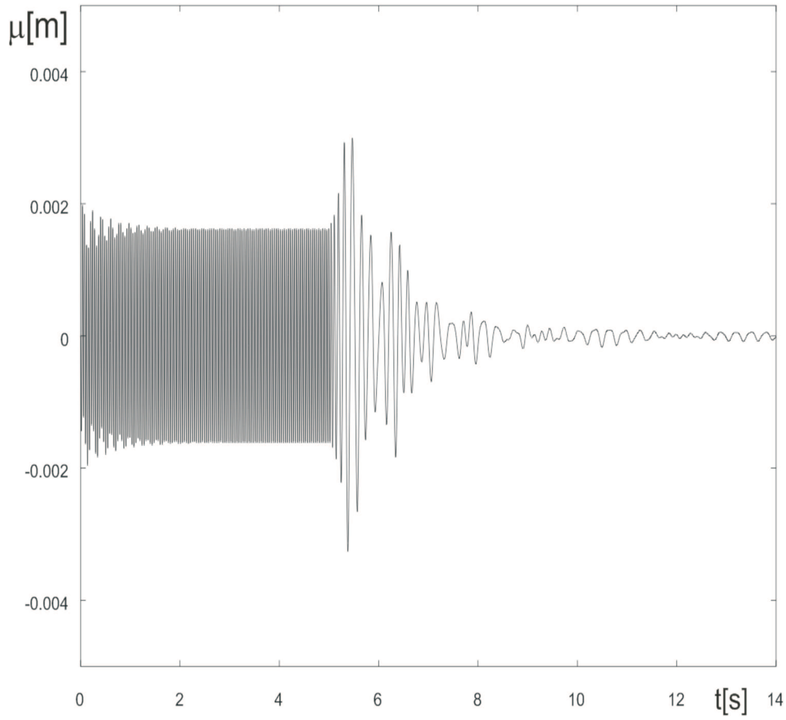

Fig. 10. Body displacement in the direction of working vibrations μ, without taking into account the influence of feed but with taking into account the gravity forces

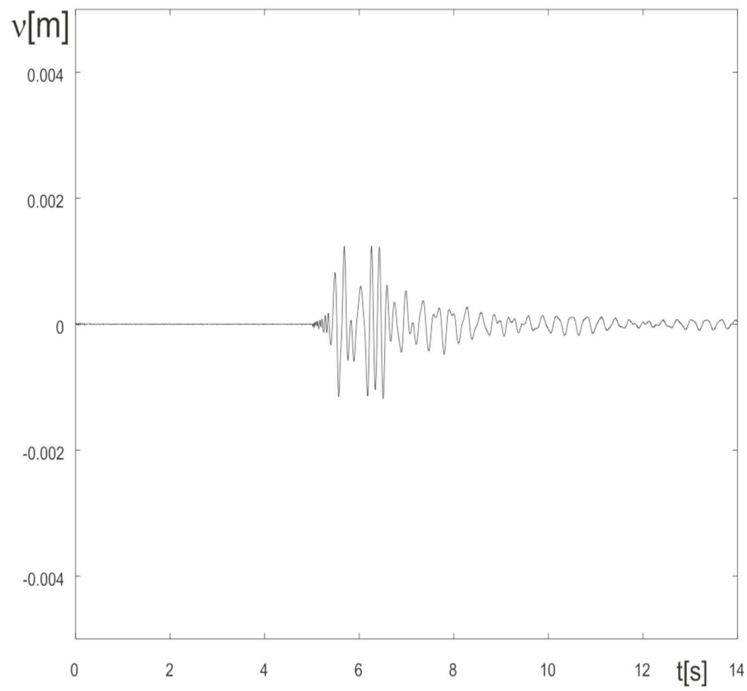

Fig. 11. Body displacement in the direction ν perpendicular to μ, without taking into account the influence of feed but with taking into account the gravity forces

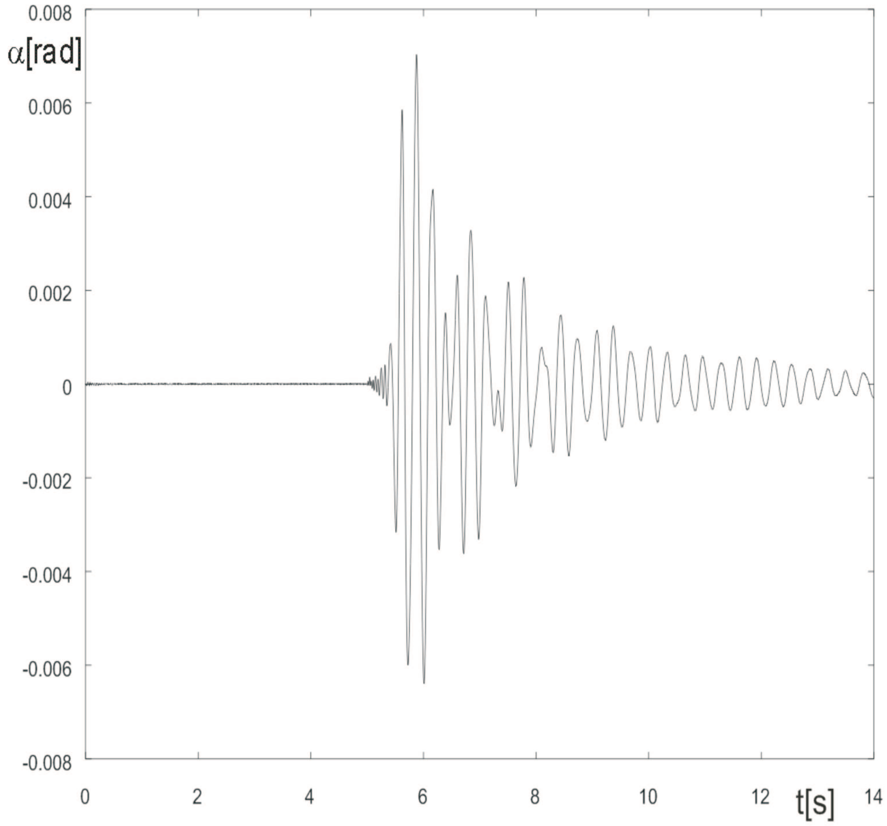

Fig. 12. Body angular oscillations α, without taking into account the influence of feed but with taking into account the gravity forces

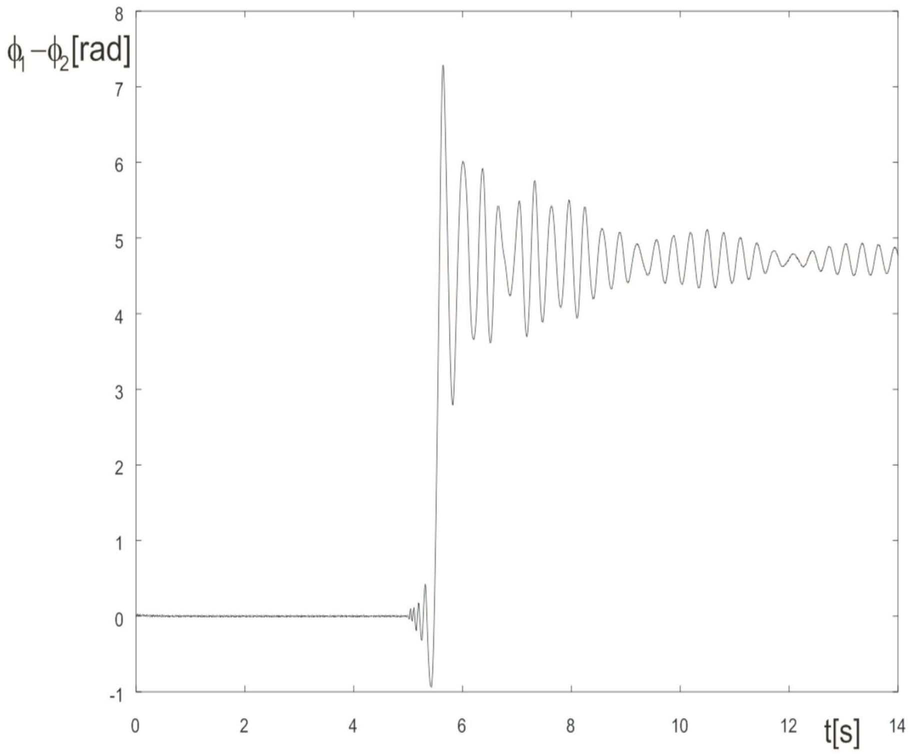

Fig. 13. Angle of a mutual dissynchronising of vibrators $\varphi_1 - \varphi_2$, without taking into account the influence of feed but with taking into account the gravity forces

C.The same courses – as above – for the machine loaded with feed of a mass being 0.5 of a body mass and with taking into account the gravity forces of unbalanced masses are shown in Figures 14, 15, 16 and 17.

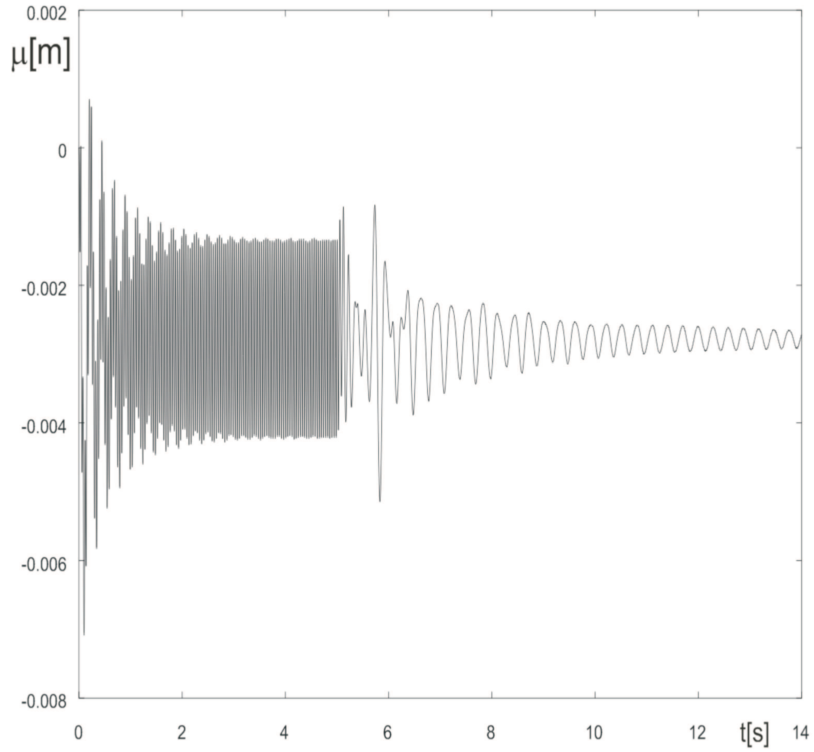

Fig. 14. Body displacement in the direction of working vibrations μ, when taking into account the influence of feed and the gravity forces

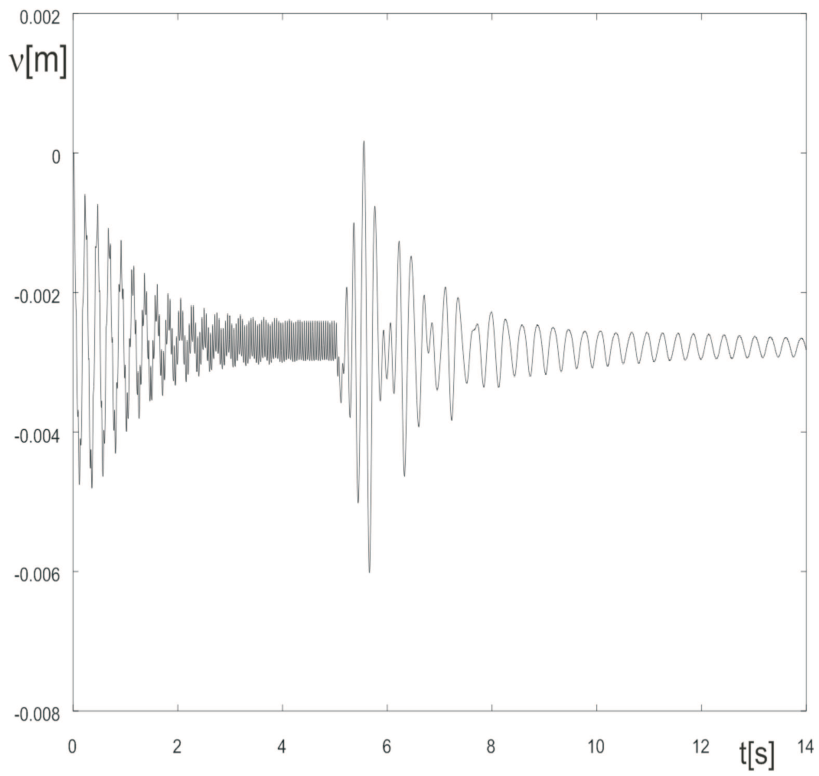

Fig. 15. Body displacement in the direction ν perpendicular to μ, when taking into account the influence of feed and the gravity forces

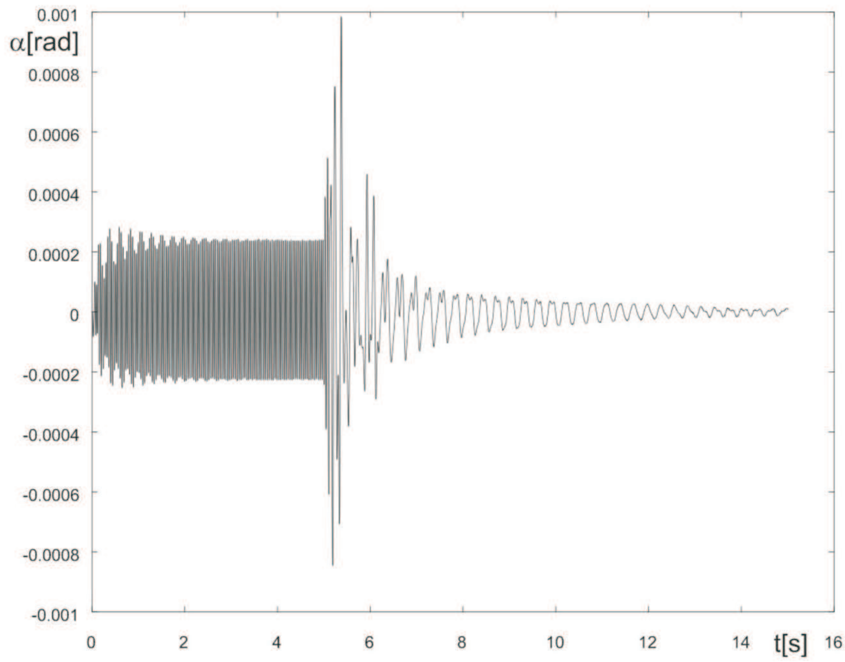

Fig. 16. Body angular oscillations α, when taking into account the influence of feed and the gravity forces

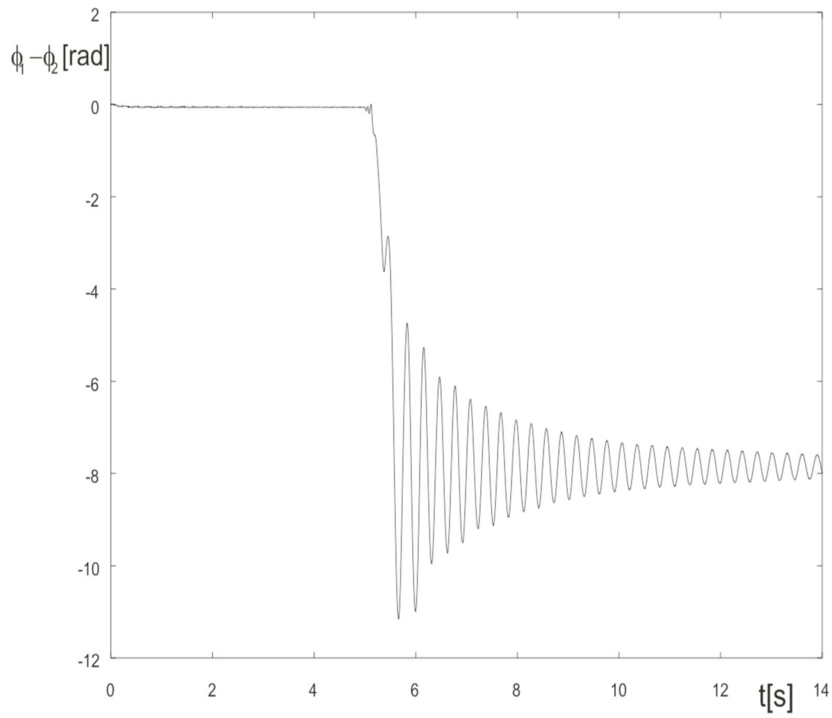

Fig. 17. Angle of a mutual dissynchronising of vibrators $\varphi_1 - \varphi_2$, when taking into account the influence of feed and the gravity forces

The analysis of the enclosed results of simulation investigations leads to the following **conclusions**:

1° In the case of a total symmetry of driving systems and when there are no disturbances caused by the different influence of feed and the gravity forces on both vibrators, the machine in the transient resonance behaves in accordance with the model assumed in paper [5], which allows to obtain the accurate results on the bases of nomograms included in this elaboration.

2° In real cases, when such factors as gravitation, influence of feed and the diversification of anti-torques of both driving systems, cause a certain asymmetry of the system, vibrators are loosing synchronization as was of their cophasal configuration, expected in paper [3]. This decreases amplitudes in the transient resonance in the direction of working vibrations and causes the appearance of intensive resonance rotational vibrations of the machine as well as vibrations in the direction perpendicular to the working motion.

3° The method described in paragraph 4 can be recommended, for the determination of maximum amplitudes in the transient resonance, for the real systems corresponding with the schemes shown in Fig. 3 and 5. For systems with more degrees of freedom or driven by two or more vibrators operating on the basis of self-synchronisation, estimation of amplitudes at circum-resonant various vibration forms can be obtained on the grounds of energy dependencies (as described in paragraph 3).

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