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CLASSIFICATION OF PHASE TRAJECTORY PORTRAITS IN THE PROCESS OF RECOGNITION THE CHANGES IN TECHNICAL CONDITION OF MONITORED MACHINES AND CONSTRUCTIONS

KLASYFIKACJA OBRAZÓW TRAJEKTORII FAZOWYCH W PROCESIE ROZPOZNAWANIA ZMIAN STANU MONITOROWANYCH MASZYN I KONSTRUKCJI

In the article, a methodology of the functioning correctness control of the foundry machines and devices was described. The new approach to construction of vibration-based monitoring system was showed. The discussion of the different solution variants together with their controlling algorithms is described. In detail, the methodology based on phase trajectory observation is presented. The discussion of functional features of the proposed procedure is described. For the proposed solution of the monitoring system, the solution of the classification phase trajectory portraits problem was introduced. This classification can be used as a automatic system recognition of technical condition of the object. The damage index, which fulfill the function of the damage indicator was defined. The efficiency of the proposed procedure was evaluated on experimental data for a unbalanced machine and damage of a construction element.
Keywords: diagnostics, monitoring, phase trajectory, foundry machines

W artykule opisano metodykę nadzoru poprawności funkcjonowania konstrukcji maszyn oraz urządzeń odlewniczych, bazująca na systemach monitoringu drganiowego. Przedstawiono dyskusję różnych wariantów rozwiązań z właściwymi ich realizacjami algorytmów kontrolnych. W szczególności zaprezentowano metodykę opartą na procedurze obserwacji zmian trajektorii fazowych monitorowanych przebiegów drganiowych. Przeprowadzono dyskusję jej walorów funkcjonalnych. Dla omówionego rozwiązania wskazano rozwiązanie systemu klasyfikacji obrazów trajektorii fazowych stanowiące narzędzie automatycznego rozpoznawania zmiany stanu monitorowanych obiektów. Zdefiniowano miarę spełniającą funkcje identyfikatora rozpoznawanych w tym systemie zaburzeń. Rozważania zilustrowano przykładami potwierdzającymi skuteczność proponowanych rozwiązań.

1. Introduction

When reviewing practically functioning monitoring systems for machinery and/or construction technical condition, and the results of currently held research it can be briefly concluded, in several general thoughts, that:

– Watching the machine and/or construction technical condition is made by monitoring systems where several quantities are observed: particular values of numeric estimates (*e.g. rms, peak and mean values, natural frequency, damping coefficient or their combination*), or particular functional images of monitored diagnostic signals (*movement trajectory of shaft neck in a bearing, spectral density function, correlation, coherence, cepstrum, envelope, mode shape etc*).
– Criteria values for monitored diagnostic symptoms (defining particular states of the object) are described with relevant standards, regulations and agreements emerging out of exploitation experience or assuming acceptable scenarios of faults of the object or from statistical processing of controlled diagnostic signals.
– There are only few monitoring system solutions which functions are based on partial relation-binding monitored state of the objects with the changes of the signal observed.
– In construction rules of monitoring system there is no consistent theory enforcing logical relations of conditions of lack of safe functioning capabilities in monitored object to the choice rules for diagnostic symptoms for state change observation of the monitored objects and the conditions of their undisturbed estimation.
– Often the construction and exploitation features of the object are not well enough taken into consideration in the process of monitoring system building.

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The above assessment does not tend to name all the problematic questions that appear during the development of the monitoring system or describe known research and experimental results. Yet it might be an inspiration to look for new methodological guidelines for construction process of monitoring system without the limitations presented above.

The present paper aims to describe the potential use of phase trajectories, for fault detection in machines and structural components, the attractor of which is a static equilibrium point of the element being diagnosed (static deflection of the component).

The effectiveness of the proposed method was, at first, tested on a simple model with two dof. The damage was modelled by changing the stiffness and damping coefficients connecting the dofs.

After testing the effectiveness of the proposed method on a simple model with two dof, it was tested with two experimental data recorded for: unbalanced machine during the start-up process and for a constructional element which damage is the change in the wall thickness.

With the diagnostic method based on quantitative analysing the phase trajectory, the fault can be quickly and effectively detected. That theory will be the basis for the framework of research actions and their algorithms management dedicated to search for new diagnostic symptoms and the choice of the quantification levels allowing diagnostic decision making.

2. System dynamics in the phase space

Phase space of a dynamic system is the mathematical space of orthogonal coordinates representing all variables essential to determine momentary state of the system.

Each dynamic system can be described with a system of differential equations:

$$\frac{dx}{dt} = f(x(t)) + u(t) \tag{1}$$

where: $\dot{x}$ – velocity vector, x – vector of displacement $u(t)$ – input (forcing) vector.

Unfortunately, for most of dynamic systems, complete information regarding the velocity and displacement of all points (degrees of freedom) is not available.

Exhaustive description of the system in phase space can be obtained also if system attractors are known. The attractor is a certain set in the phase space, to which trajectories beginning in various areas of the phase space (i.e. trajectories for various initial conditions) evolves after a long enough time.

The attractor can be a point, closed curve (limit cycle) or fractal (strange atractor). Attractor is one of the key terms used in chaos theory [2]. Each attractor has its attraction area, known as the attraction basin (a set of such initial conditions for which the trajectory evolves to the attractor).

The potential use of the limit cycle quantitative analysis for a structural components diagnostics has been described in other papers of the present authors, e.g. [3,4,7] and Trendafilova [12]. The present paper focuses on the potential uses of the quantitative analysis of a trajectory in case when the attractor is a point.

Phase trajectory in phase space is a multidimensional curve (each degree of freedom is represented as its separate dimension). With no harm to the general nature of divagations, trajectory projection on certain plane, formed by two perpendicular axes of the phase space, can be analysed [1,2].

In the next section various coordinates of axes forming the projection plane for the phase trajectory are described. In other words, those are the variables (e.g. acceleration, velocity, displacement) of one point of the component or diagnosed structure, which can be used for determining the trajectory projection plane.

3. Methods for trajectory determining

The most apparent coordinates of the plane on which the phase trajectory can be projected, are used for topologic analysis of vibrations: these are the velocity and displacement. Determining a trajectory on such plane involves the determination, with the aid of an accelerometer, a time series of vibration acceleration, and integrating it afterwards. Each point on the trajectory has the coordinates corresponding to displacement and velocity values, on the horizontal and vertical axis respectively. The values are determined in the same moment of time.

In lieu of velocity as the function of displacement, the acceleration as the function of displacement, or the acceleration versus velocity can be analysed. In addition to obvious benefits from a smaller number of time series integrations (time and cost of calculations), another advantage of the trajectory of the velocity-acceleration coordinates is that it can be determined by measurement, directly on the diagnosed structure.

Other method for constructing projection plane, known from chaos theory, is the use of the method of delays. With that method, it is sufficient to determine one time series (e.g. vibration acceleration) and, based on that, determine the phase space, including the plane sought, on which the phase trajectory is projected.

The reconstruction of phase space is based on the fact that the time series contains some information of non-observed states of the system generating it (diagnosed component) and dynamics of such states.

Reconstruction of a trajectory based on a single time series requires formation of additional variables. Takens theorem [1] is useful while seeking new variables, whereby each point in the phase space $a(n)$ is represented by a series of subsequent values of the time series:

$$\begin{aligned} a(n) &= [y(n), y(n+\tau), \ldots, y(n+(m-1)\tau)] \\ a(n+1) &= [y(n+1), y(n+1+\tau), \ldots, \\ &\qquad y(n+1+(m-1)\tau)] \end{aligned} \tag{2}$$

where: m – dimension of the phase space, τ – time delay.

Reconstruction of the space with the method of delays requires its dimensional parameter m and the time delay τ to be adopted a priori.

The most commonly used procedure for the selection of space dimension is the False Nearest Neighbours method (FNN). The method is based in the fact that for sufficiently large m, in the reconstructed attractor, the proportions of distance between states no longer change significantly, i.e. close states in m-dimensional space remain close, also after the addition of $m+1$ coordinate [11].

Due to the use (for diagnostics purposes) only the projection of so reconstructed trajectory on any plane, determined by the coordinates, e.g. $y(n)$ and $y(n+\tau)$, the knowledge of the dimension of phase space is dispensable.

The value of the time delay τ can be determined as a result of the auto-correlation function analysis, as the moment in which the function gets the zero value for the first time. A major drawback of the auto-correlation based method are linear dependencies assumed between observations [8]. The criterion for selection of delay, which also uses non-linear dependencies, is the Mutual Information (MI) method. The amount of mutual information $I(x_i, T)$, is obtained from the equation [9]:

$$\begin{aligned} I(x_i, T) = \frac{1}{N}\sum_n \{\log_2[\, p(x_i(n), x_i(n+T)]- \\ -\log_2[p(x_i(n)] - \log_2[p\ \ (x_i(n+T))]\} \end{aligned} \tag{3}$$

where: $p(x_i(n))$ is the marginal probability distribution function of the analysed series, T time delay, $p(x_i(n), x_i(n+T))$ is the joint probability distribution function, N – number of samples in a time series.

According to that method, as the value of delay τ the lowest T value in equation (3) should be assumed, for which the function of mutual information has the local minimum.

The method of delays is the most popular, but not the only one, used for reconstruction of phase space. The most commonly known methods are: factor analysis [5] and Pakard's method of derivatives [10].

4. Quantitative analysis of the system phase trajectory

This section of the paper describes the potential use of phase trajectories, for damage detection of structural components, the attractor of which is a static equilibrium point of the part being diagnosed (static component deflection). In other words, it is the trajectory of the mechanical system "reaching" the equilibrium state after change of the initial conditions or after its impulse excitation.

To move to the quantitative analysis of the phase trajectory, repeatability of initial conditions should be ensured during vibration analysis. In case of numerical calculations, this is done by assuming zero initial conditions and exciting vibrations with an impulse of finite (but not zero) duration. In experiments, the excitation was done with a steel ball falling freely. In each case, the initial height of the ball was kept the same and bouncing was eliminated.

The effectiveness of the proposed method was, at first, tested on a simple model with two dof, as shown in Fig. 1. The damage was modelled by percent change of spring coefficient k_2 (decreasing) and damping coefficient c_2 (increasing). Spring and damping coefficients k_1 and c_1 have remained constant. For such model, selection of damage index was proposed.

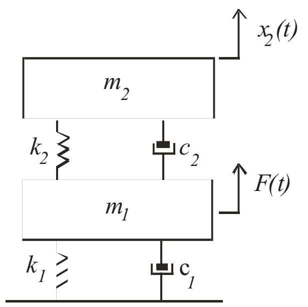

Fig. 1. System with two degrees of freedom

Figure 2 shows example of phase trajectories determined numerically for the undamaged (continuous line) and damaged (dashed line) system.

In Fig. 2, the initial and final parts of vibration were omitted, as otherwise such samples would make the figure illegible.

The authors propose to carry out quantitative analysis of the trajectory using damage index, which refers to variation of the distance between a point on a trajectory and the point which is the attractor of the trajectory. For that purpose the location of each point on the trajectory should be described by polar coordinates (r, ϕ).

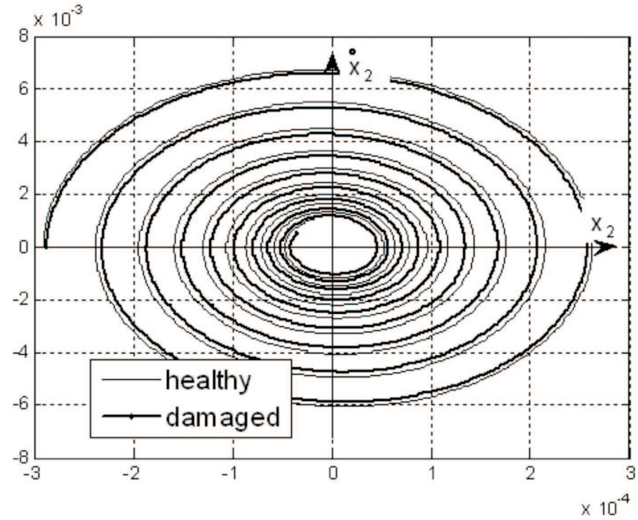

Fig. 2. Phase trajectory variation due to damage

Figure 3 shows fragments of example of $r(t)$ vector for an undamaged (continuous line) and damaged component (dashed line).

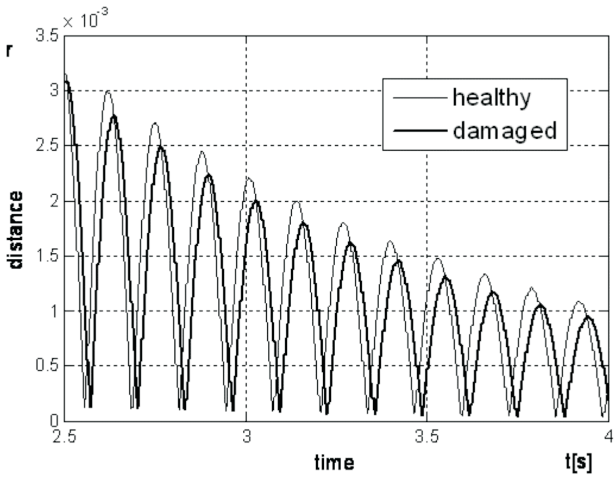

Fig. 3. Distance variation between the point on phase trajectory and the attractor

It is proposed that a sum o relative vector differences r for each moment (each n sample) is used as the damage index. Such damage index DI_r is obtained from the formula:

$$\mathrm{DI_r} = \frac{1}{N}\sum_{n}\frac{r_d(n) - r_h(n)}{r_h(n)} \qquad (4)$$

where: r_d – vector of distance between points on trajectory and the attractor, determined for the damaged element, r_h – vector of a healthy element, N – number of samples.

Figure 4 shows the proposed damage index as the function of damage for the 2 dof system (Fig. 1).

Fig. 4. Changes of the proposed damage indexes as function of damage severity

After testing the effectiveness of the method proposed on a simple model of two freedom degrees, its effectiveness was verified on an experimental data.

5. Analysing trajectory of an unbalanced machine

Phase trajectory have been determined by a measure of the vibration velocity and its integration (the Fig. 5. Test stand in fig is showed)

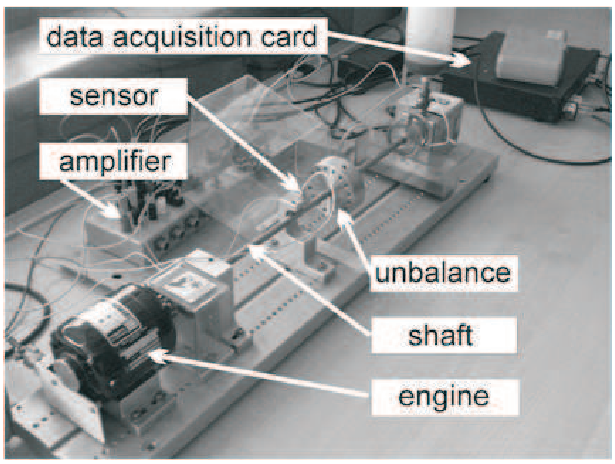

Fig. 5. Test stand

During the experiment data for five different value of machine unbalance was recorded i.e. 15.2, 30.5, 45.7, 61.0 and 91.5 gmm (gram×millimetre).

Figure 6 shows an example of the recorder velocity time history during the run-up (after trend subtraction).

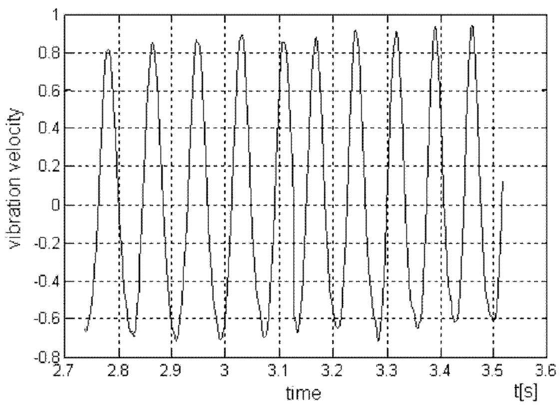

Fig. 6. An example of the vibration velocity

All the phase trajectories are determined in reverse time and the trajectory for lowest unbalance (15.2 gmm) as a relative one was adopted. Fig. 7 shows the proposed damage index versus relative increase of unbalance.

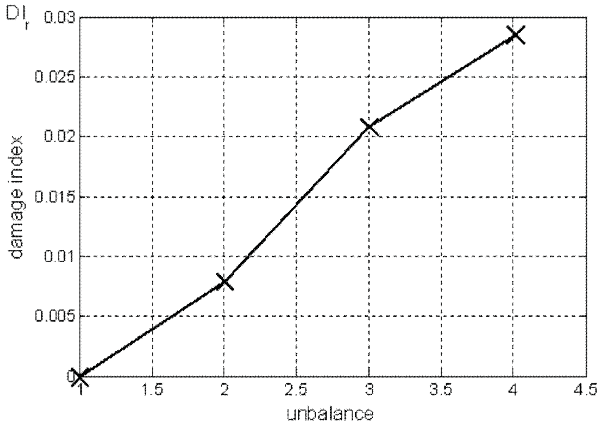

Fig. 7. Damage index as a function of unbalance

It is easy to notice, that observation of the proposed damage index allow to determine the level of rotational machine unbalance during start-up process. Such early of the unbalance detection and recognition allow to forced downtime before it reach the nominal (high) rotational speed and associated with it large value of the inertia force and dynamic shaft deflection.

6. Analysing trajectory of a constructional component

The evaluation of the usefulness of the phase trajectory portraits to the diagnostics of the structural component have been carried out on an example of mine shaft component i.e. cage guide. The change of the wall thickness is damage of such element, that is why the proposed method can be applied as the diagnostic method and the method of the product quality assessment.

Phase trajectories have been determined based on measure of a vibration acceleration and Takens theorem. Time delay as a result of the auto-correlation function analysis was determined.

The proposed damage index as the function of wall thickness reduction in Fig. 8 was shown.

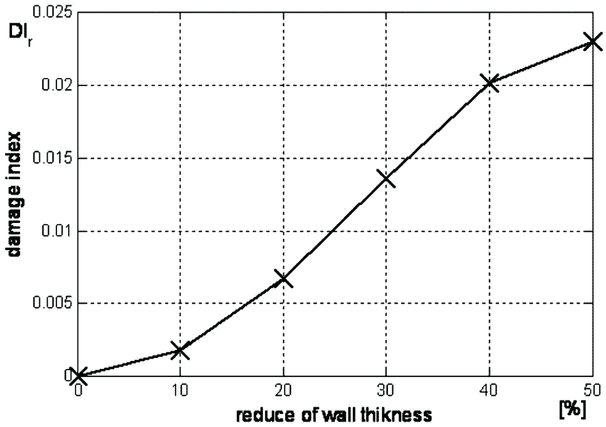

Fig. 8. Damage index versus relative change of wall thickness

Analysing the course of the proposed index in the function of the proportional change of the wall thickness, it is easily to notice that the analysis of the index value changes of the object impulse response can be used to the diagnostics of the constructional elements.

The proposed method can be adopted also to the control of the quality of casting products and monitoring of foundry processes.

7. Summary and conclusions

With the diagnostic method based on analysing the phase trajectory, the fault or damage can be quickly and effectively detected.

Analysis of the damage indexes in function of damage shown in Figs. 7 and 8 demonstrated high sensitivity of the proposed diagnostic method. Early detection of faults allows to: optimise repair activities (as to the scope and necessity), avoid loss related to forced downtime, reduction of the cost of undesired storage of spare parts and cost incurred through unexpected breakdowns.

The method does not filter non-linear effects and variations of the frequency structure in the diagnostic signals, related to the progress of damage, which can be considered as its special advantage.

Practical application of the trajectory variation tracking method appears a very useful tool for identifying the process of damage initiation and progress. It can serve as its major indicator and is easily adaptable in practical applications.

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