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LOGARITHMICONORMAL DISTRIBUTION IN QUANTITATIVE METALLOGRAPHY OF DISPERSE CARBIDES IN STEELS

ROZKŁAD LOGARYTMICZNO-NORMALNY W METALOGRAFII ILOŚCIOWEJ DYSPERSYJNYCH WĘGLIKÓW W STALACH

Stereology for spheres, whose diameters distribution corresponds to the logarithmiconormal (LN) distribution is presented. A method for stereological estimation of parameters (α, β) of the LN probability density function is proposed. The size distribution of Fe_3C-particles in spheroidized cementite of some steels is of the LN type. In such a case the size distribution measurement may be reduced to stereological estimation of the α, β parameters.
Keywords: disperse carbide in steels, size distribution, stereology

Przedstawiono stereologię dla kul, których średnice mają rozkład logarytmiczno-normalny (LN). Zaproponowano stereologiczną metodę estymacji parametrów (α, β) funkcji gęstości prawdopodobieństwa rozkładu LN. Rozkład rozmiarów cząstek Fe_3C w sferoidycie niektórych stali jest rozkładem LN. W takiej sytuacji pomiar rozkładu rozmiarów cząstek sprowadza się do stereologicznej estymacji parametrów α, β.

1. Introduction

A typical disperse carbide phase in steel microstructure is formed by isolated particles distributed in a statistically uniform manner in the material space. Particle size may be interpreted as the value of a continuous random variable whose distribution is determined by the probability density function (PDF). In quantitative metallography the PDF is measured by stereological methods [1, 2]. In a first approximation particles are considered as spheres. If the spheres diameter distribution type is known (i.e., a family of PDFs), measurement of the PDF may be reduced to its parameters (the so-called parametric method [3]). For disperse carbides in steels, only unimodal and two parameter PDFs (e.g. the PDF of Weibull, gamma, logarithmiconormal distribution, etc.) are of practical meaning [2]. The spheres stereology with a Weibull distribution of diameters was analysed in [4]. The spheres stereology for the logarithmiconormal (LN) distribution of diameters was first formulated by Saltykow and DeHoff [2, 5]. The present study follows DeHoff's ideas and takes into cosideration testing of logarithmico-normality, using the formalism given in [4].

2. General stereology of spheres

In the $\mathbf{R}^3$ space a random set of non-overlapping spheres is given. Sphere diameter D is a random variable of PDF $f_3(D)$ for $D \in (0, \infty)$. The mean

$$\langle D^r \rangle = \int_0^\infty D^r f_3(D)\, dD \qquad (r = 0, 1, 2, ...), \quad (1)$$

is the r order diameter moment (for $r = 1$, $\langle D^r \rangle = \langle D \rangle$ is the mean diameter).
A circle (profile) is a planar section of sphere. Planar section of the sphere system forms random circles in the $\mathbf{R}^2$ plane. Circle diameter d is a random variable of PDF $f_2(d)$ for $d \in (0, \infty)$. The mean

$$\langle d^r \rangle = \int_0^\infty d^r f_2(d)\, dd \quad (r = -1, 0, 1, \ldots), \quad (2)$$

is the r order circle diameter moment. The diameter moments, $\langle D^r \rangle$ and $\langle d^r \rangle$, satisfy the equation

$$\langle d^r \rangle = \frac{\sqrt{\pi}}{2} \frac{\Gamma\left(\frac{r+2}{2}\right)}{\Gamma\left(\frac{r+3}{2}\right)} \left\langle D^{r+1} \right\rangle \langle D \rangle^{-1} (r = -1, 0, 1, \ldots), \quad (3)$$

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where Γ is the Euler gamma function [4,6]. The PDFs $f_3(D)$ and $f_2(d)$ satisfy the Wicksell equation [4–7]:

$$f_2(d) = \langle D \rangle^{-1} d \int_d^{\infty} \frac{f_3(D)}{\sqrt{D^2 - d^2}} dD. \qquad (4)$$

3. LN stereology of spheres

For random diameters of spheres the LN PDF is the following:

$$f_3(D) = \frac{1}{D\alpha\sqrt{2\pi}} \exp\left(-\frac{(\ln D - \beta)^2}{2\alpha^2}\right). \qquad (5)$$

where $\alpha \geqslant 0$ and β are parameters.

The substitution of Eq. (5) into Eq. (1) results in, [5],

$$\langle D^r \rangle = \exp\left(r\left(\beta + \frac{r}{2}\alpha^2\right)\right), (r = 0, 1, 2, \ldots). \qquad (6)$$

Eq. (6) may be written in the form

$$\langle D^r \rangle = \exp\left(\frac{r(r-1)}{2}\alpha^2\right)\langle D \rangle^r. \qquad (7)$$

The substitution of Eq. (7) into Eq. (3) gives

$$\langle d^r \rangle = \frac{\sqrt{\pi}}{2}\frac{\Gamma\left(\frac{r+2}{2}\right)}{\Gamma\left(\frac{r+3}{2}\right)} \exp\left(\frac{r(r+1)}{2}\alpha^2\right)\langle D \rangle^r \quad (r = -1, 0, 1, \ldots). \qquad (8)$$

From Eq. (8), for $r = 1$ and $r = 2$, the result is:

$$\frac{\langle d^2 \rangle}{\langle d \rangle^2} = \frac{32}{3\pi^2} e^{\alpha^2}. \qquad (9)$$

From Eqs. (8) and (6) (for $r = 1$, the result is:

$$\langle d \rangle = \frac{\pi}{4} e^{\beta + \frac{3}{2}\alpha^2}. \qquad (10)$$

Eqs. (9) and (10) connect the α, β parameters of the PDF $f_3(D)$ given by Eq. (5) with particle section diameter moments $\langle d \rangle$ and $\langle d^2 \rangle$. They are fundamental stereological equations in the LN stereology of spheres, and when the moments $\langle d \rangle$ and $\langle d^2 \rangle$ are expressed as values that are determined from simple counting measurements made on sections, they are equivalent to DeHoff's equations, [5].

4. Quantitative metallography

The Eqs. (9) and (10) form the basis for a stereological measurement method of LN particle size distributions in quantitative metallography. One may distinguish two cases. In the first case (i), the particle size distribution follows the LN one, but the α, β parameters are unknown. In the second case (ii), the particle size distribution is unknown.

In the case (i), the α, β parameters will be estimated by Eqs. (9) and (10) usually given in DeHoff's form [5]. Then, the $\langle D^r \rangle$ moments will be determined by Eq. (6).

In the case (ii), a testing of logarithmico-normality should be made. Here, the $f_2(d)$ should be measured – the result is given in a discrete form, i.e, $f_2(d_i), i = 1, \ldots, k$. A possible testing procedure is as follows [4]. An unknown particle size distribution follows the LN distribution when the $f_2(d_i)$, may be fitted by a function that is calculated by the Wicksell Eq. (4) in which $f_3(D)$ is given by Eq. (5) with empirical parameters α, β. The α, β parameters will be determnied by Eqs. (9) and (10) from $f_2(d_i)$.

5. Experimental

The aim of the experimental studies was to describe an unknown size distribution of Fe_3C particles in steels by the LN distribution according to case (ii), presented above in section 4.

For these investigations, existing $f_2(d_i)$ functions for coarse Fe_3C-particles in two carbon steels (A, B) are used, i.e., A: Fe-0.26%C (700°C/1200 h), [8]; B: Fe-0.40%C (650°C/450 h), [9]; (in brackets, there are given heat treatment conditions, i.e., temperature and time for the coarsening process).

a)

b)

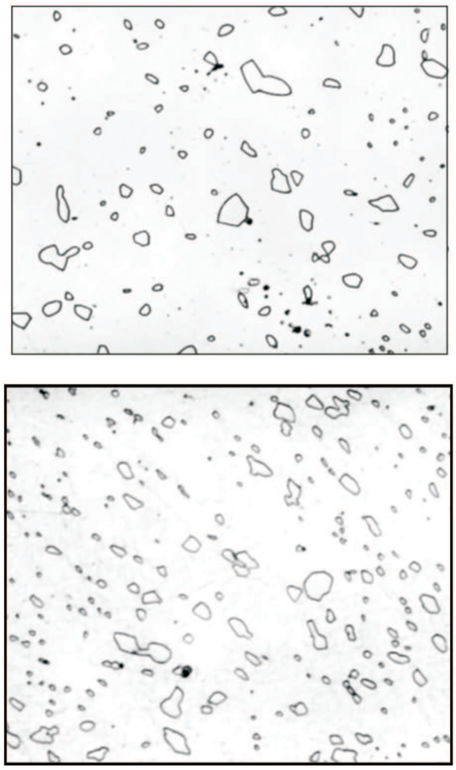

Fig. 1. Steel microstructures: a) steel A (Fe-0.27%C, 700°C/1200h), b) steel B (Fe-0.40%C, 650°C/450h)

Fig. 1 and Fig. 2 show the microstructure of steels and the empirical $f_2(d_i)$ functions, respectively, [8, 9]. The $f_2(d_i)$ functions are quite regular and unimodal with positive asymetry.

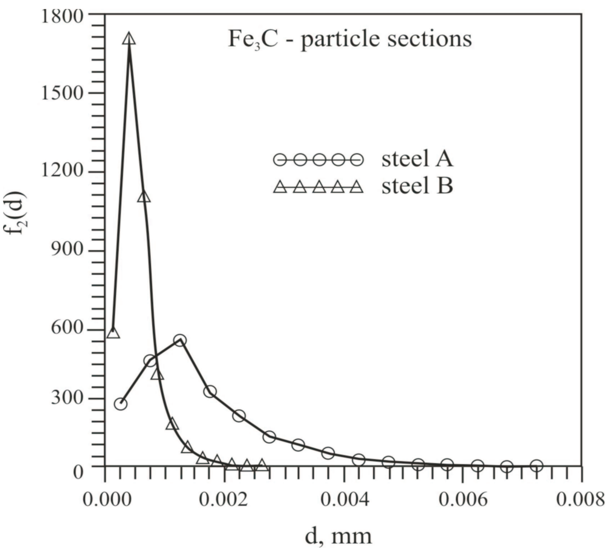

Fig. 2. Fe_3C-particles sections $f_2(d_i)$ PDFs

For the given $f_2(d_i)$, the $\langle d\rangle$ and $\langle d^2\rangle$ moments were determined. The Fe_3C particle characteristics: volume fraction V_V, mean diameter $\langle d\rangle$ and $\frac{\langle d^2\rangle}{\langle d\rangle^2}$ ratio are presented in Table 1. Table 1 and Fig. 2 demonstrate, that for the investigated steels, the V_V, $\langle d\rangle$ and $f_2(d_i)$ differ significantly when the $\frac{\langle d^2\rangle}{\langle d\rangle^2}$ ratio difference is rather small.

Using Eqs. (9), (10), and the empirical $\langle d\rangle$ and $\frac{\langle d^2\rangle}{\langle d\rangle^2}$ values, the α, β parameters of the PDF $f_3(D)$ given by Eq. (5) were calculated for both steels; the results are presented in Table 1.

TABLE 1
Quantitative characteristics of the Fe_3C-particles and their planar sections for the investigated steels

steel	V_V %	$\langle d\rangle \times 10^3$ mm	$\frac{\langle d^2\rangle}{\langle d\rangle^2}$	α	β	$\langle D\rangle \times 10^3$ mm
A	4.0	1.545	1.47	0.55	–6.69	1.446
B	6.2	0.528	1.36	0.48	–7.65	0.534

For testing of the logarithmico-normality, the empirical $f_2(d_i)$ PDFs were compared to $f_2(d)$ functions, which are calculated using Eq. (4) (composed of the PDF $f_3(D)$, given by Eq. (5) with the empirical parameters α, β) in an approximated form of the so-called discrete Saltykow algorithm [4].

Fig. 3 shows that the calculated $f_2(d)$ functions fit to the the empirical $f_2(d_i)$ functions. This suggests that the analysed Fe_3C-particle size distributions follow, in principle, the LN distribution; the estimated $f_3(D)$ PDFs are also presented in Fig. 3.

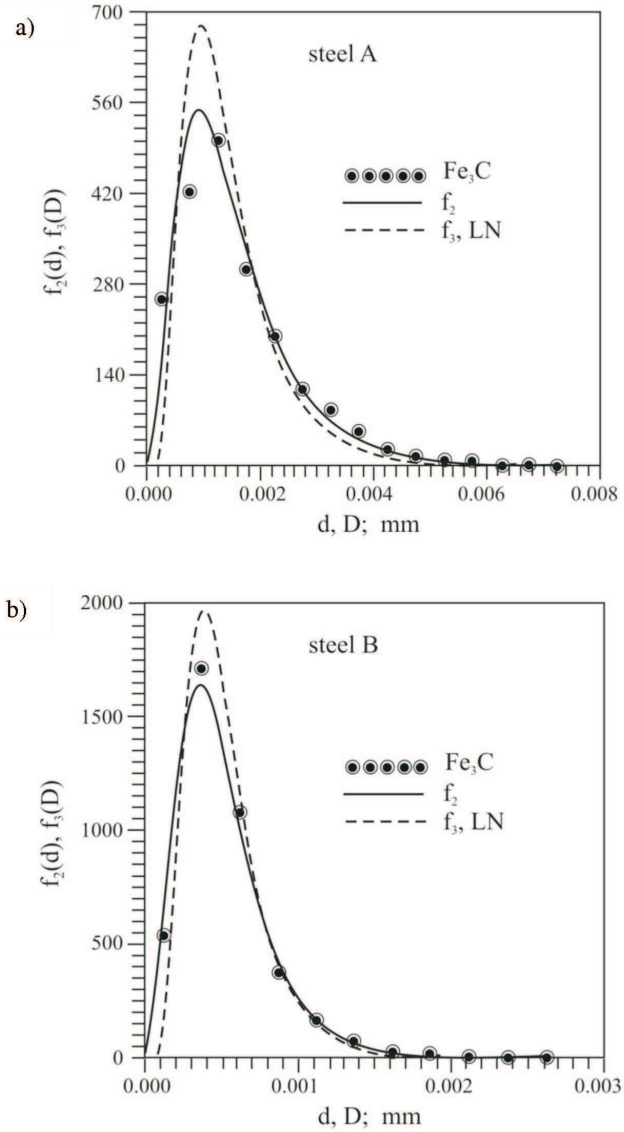

Fig. 3. Fe_3C-particle sections $f_2(d_i)$ PDFs, its approximating $f_2(d)$ funcions and the logarithimiconormal $f_3(D)$ PDFs: a) steel A, b) steel B

Finally, for the given empirical α, β parameters, the mean sphere diameters $\langle D\rangle$ were estimated by Eq. (6). The results are presented in Table 1. It should be noted that the relations between mean diameters, $\langle D\rangle$ and $\langle d\rangle$, agree with the one given in [4].

6. Conclusions

1. If a particle size distribution corresponds to the LN distribution then the sterological estimation of its PDF $f_3(D)$ may be reduced to estimation of the α, β parameters by means of Eqs. (9) and (10), using the planar section moments $\langle d\rangle$ and $\langle d^2\rangle$.
2. A particle size distribution corresponds to the LN distribution if the $f_2(d)$ function, given by

Eq. (4) composed of a LN PDF $f_3(D)$, fits to the empirical $f_2(d_i)$ function.
3. The analysed Fe_3C-particle size distributions follow the LN distribution.

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