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CALCULATION OF HEIGHT AND STRUCTURE OF MELTING ZONE OF COKE CUPOLAS

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A set of equations for the calculation of melting zone height and other parameters of coke cupola, which describe its geometrical and kinetic structure (e.g. the distribution of metal mass and the efficiency of melting in dependence on zone height) has been proposed in the study. The set of equations and the enclosed example of calculations were used for conducting a quantitative analysis of metal and coke movement across the melting zone as well as for the identification and description of the process model.

The problem of the zone melting height calculation, although attempted, has not been satisfactorily solved so far, while the calculations of melting zone structure have never been undertaken. Both the problems are organically bounded with each other.

Keywords: coke cupolas, melting zone, height of melting zone, integral modulus, velocity of melting

W pracy wyprowadza się układ równań do obliczania wysokości strefy topienia oraz innych wielkości, charakteryzujących jej geometryczną i kinetyczną strukturę (np. rozkład masy metalu oraz wydajności topienia na wysokości strefy). Układ równań oraz załączony przykład obliczania wysokości strefy topienia i jej struktury, posłużył do przeprowadzenia ilościowej analizy procesu przemieszczania się metalu i koksu przez strefę topienia oraz do identyfikacji i opisu modelu tego procesu.

Problem obliczania wysokości strefy topienia, mimo podejmowanych prób, nie ma w literaturze racjonalnego rozwiązania, natomiast problem obliczania struktury strefy topienia nie był dotąd podejmowany. Oba te problemy są ze sobą organicznie związane.

Symbols used in text

a	–	the smallest out of three dimensions of pieces of metal charge in the form of rectangular slabs, prisms, cubes or spheres, m
C_k	–	coal content in coke, kg_c/kg_k
$c_{g,r}$	–	average specific heat of gases in reduction zone, $J/(m^3 \cdot K)$
$c_{g,t}$	–	average specific heat of gases in melting zone, $J/(m^3 \cdot K)$
F_t	–	development surface of melting zone, m^2
F_c	–	internal surface of cupola cross-section in melting zone perpendicular to the axis of cupola, m^2
H_t	–	height of melting zone, m
$K_{c,4} = K_{w,4}\ C_k$	–	expenditure of coal in charge coke, $kg_c/100\ kg_{Fe}$
$K_{w,4}$	–	expenditure of charge coke, $kg_k/100\ kg_{Fe}$
$K_{c,r}$	–	content of coal in coke in reduction zone, $kg_c/100\ kg_{Fe}$
$K_{\rho,t}$	–	ratio of melting zone volume to bulk volume of metal pieces in the zone, unit fraction

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$L_{c,4}$	–	volume of air blast consumed for burning 1 kg of coal contained in coke, m^3/kg_c
$M_{k,t}$	–	coke mass in melting zone, kg_k
$\bar{M}_{m,t}$	–	mass of melting metal pieces in melting zone, kg
N_c	–	number of series of melting metal pieces in melting zone
n_m	–	number of metal pieces in zone
n_w	–	number of metal and coke layers in the zone
P_F	–	relative amount of blast air, $m^3/(m^2\cdot s)$ or m/s
$P_{\dot{Z}}$	–	volume of blast air delivered to analyzed coke cupola, m^3/s
$\bar{r}_m$	–	mean integral modulus of metal charge pieces, m
r_m	–	initial modulus of metal pieces, m
S_F	–	relative efficiency of melting, $kg/(m^2\cdot s)$
S_c	–	melting efficiency of coke cupola, kg_{Fe}/s
$T_{g,2}$	–	gas temperature at the entry to melting zone, °C
$T_{g,3}$	–	gas temperature at the exit from melting zone, °C
$T_{g,max}$	–	gas temperature at upper boundary of melting zone, °C
$T_{m,3}$	–	temperature of metal charge melting, °C
$T_{m,4}$	–	initial temperature of metal charge, °C
$z = \frac{a}{n_w}$	–	thickness of metal pieces melted down in each zone layer, m
$\alpha_{F,t}$	–	coefficient of heat exchange in the melting zone, $W/(m^2\cdot K)$
$\eta_4, \eta_r, \eta_3;\ \eta_2$	–	degree of gas combustion at the upper boundary of heating, combustion and melting zones, as well as at the lower boundary of melting zone, respectively, in unit fraction.
μ_t	–	average linear velocity of melting, the same for all pieces of zone, m/s
$\rho_{n,m}, \rho_{n,k}$	–	bulk density of melting metal pieces and density of coke pieces, respectively kg/m^3
$\tau_{\bar{r}}$	–	melting time of metal pieces of mean integral modulus, $\bar{r}_m$, s
τ_a	–	time of melting down of thickness a/2, s
$\tau_{\bar{r}}$	–	melting time of metal piece of modulus, $\bar{r}_m$, s
τ_z	–	melting time of metal layer of thickness 0,5 z, s
φ_v	–	dimensionless coefficient for calculation of mean integral volume of metal pieces
φ_f	–	dimensionless coefficient for calculation of mean integral surface of metal pieces

1. Introduction

The melting zone is localized in the coke cupolas between the upper boundary of the main reduction zone and the lower boundary of the heating zone. In spite of more than hundred-year attempts, there has been lack of mathematical approach to the melting zone height based on a rational model of melting metal pieces in the zone together with their displacement with coke as a result of the metal melting as well as burning and the gasification of coke. The difficulties with the mathematical approach of the melting zone height are mainly connected with the description of the melting process of individual metal pieces in the zone, the assessment of their amount and distribution at the zone height as well as other parameters called structure of melting zone in the present study. The problem of the melting zone structure calculation and its organic relation with the melting zone height have not been reported in the literature so far.

The melting zone is an important part of the working height of cupola, hence its knowledge is of essential importance in the designing and exploitation of coke cupolas. Therefore, the solution of the problem of calculation and structure of the melting zone is not only important from the technological and scientific point of view, but also it has become a challenge to more that hundred-year history of struggle with the task, similarly to the issue of calculation of optimum air blast volume [1].

In the present work a theoretical solution of the calculation problem of height and melting zone structure in coke cupolas has been presented based on the paradigm formulated in work [2], formulas resulting from paper [3] as well as new ones derived here.

2. The formulas for calculations of melting zone height

Model assumptions:

- performance of the cupola is stabilized, zone localization with respect to the furnace wall is constant, contact surfaces of the zone with the reduction and preheating ones are two-dimensional and perpendicular to the axis of cupola shaft (Fig. 1),
- pieces of metal charge drifting from the heating zone to the melting one have the identical shape, mass, density and melting temperature; the pieces, distributed within the coke, form, at the zone height, an assembly of partially melted pieces of various degree of melting, whose mass decreases in the direction of lower part of the melting zone; mean mass and mean surface of pieces in the zone are equal to their mean integrals defined in work [2],
- total mass of metal pieces in the zone is equal to

$$\bar{M}(m,t) = S_c \tau_{\bar{r}} \qquad (1)$$

where:
$\bar{M}(m,t)$ – mass of the melting metal pieces, kg
S_c – efficiency of melting, kg_{Fe}/s
$\tau_{\bar{r}}$ – melting time of metal pieces of mean integral module $\bar{r}_m$ [2], s

- temperature of gases at the entry and the exit of the melting zone are constant; the value of temperature at the entry is equal to the temperature of gases at the higher boundary of the combustion zone reduced by the decrease of gas temperature in the main reduction zone, while gas temperature at the exit from the zone is equal to the gas temperature at the entry reduced by the decrease of temperature brought about by the process of metal melting,
- mean difference of gas temperatures and melting metal pieces equals to their mean logarithmic described with the following formula of theory of heat exchanger [4, 5]

$$\overline{\Delta T}_{g,t} = \frac{\Delta T_{g,t}}{\ln \frac{T_{g,2}-T_{m,3}}{T_{g,2}-\Delta T_{g,t}-T_{m,3}}} \qquad (2)$$

where:
$\overline{\Delta T}_{g,t}$ – mean temperature difference of the gases and the surface of the melting metal pieces in the melting zone, K,
$\Delta T_{g,t} = (T_{g,2} - T_{g,3})$ decrease of gas temperature at the height of melting zone, K
$T_{g,2}$ – temperature of gases at the entry to the melting zone, °C
$T_{g,3}$ – temperature of gases at the exit of the melting zone, °C
$T_{g,3} = T_{g,2} - \Delta T_{g,t}$, °C
T_{m3} – melting temperature of metal charge (constant), °C

- temperature of gases at the upper boundary of combustion zone is given ($T_{g,max}$); the temperature depends on the amount of heat fed to the combustion zone, heat loss on the expense of the furnace wall in the combustion and melting zones, expenditure of heat for the superheating of the liquid metal in the combustion zone and other processes taking place in the combustion zone (in the presented study, due to its limited volume, the method of calculation of individual values of heat losses and gains is not reported).

Balance of the metal and coke bulk volumes:

It is assumed, that the volume occupied by the melting zone is equal to the sum of the bulk volume of metal pieces and bulk volume of coke pieces, which can be described in formula:

$$H_t F_c = \frac{\bar{M}_{m,t}}{\rho_{n,m}} + \frac{M_{k,t}}{\rho_{n,k}} = \frac{\bar{M}_{m,t}}{\rho_{n,m}} K_{\rho,t} \qquad (3)$$

in which:

$$K_{\rho,t} = 1 + \frac{M_{k,t}}{\bar{M}_{m,t}} \frac{\rho_{n,m}}{\rho_{n,k}} \qquad (4)$$

where

H_t – height of melting zone, m

F_c – internal area of cupola cross-section in the melting zone, perpendicular to the cupola axis, m^2

$M_{k,t}$ – coke mass in melting zone, kg_k

ρ_{nm}, $\rho_{n,k}$ – bulk density of melting metal pieces and coke pieces, respectively, kg/m^3

The physical sense of $K_{\rho t}$ results from equation (3); it is the ratio of melting zone volume ($H_t \cdot F_c$) to bulk volume of metal lumps ($\frac{\bar{M}_{m,t}}{\rho_{n,m}}$).

Introducing (1) into (3)

$$\boldsymbol{H_t = \frac{S_F \tau_{\bar{r}}}{\rho_{n,m}} K_{\rho,t}} \qquad (5)$$

in which:

$$S_F = \frac{S_c}{F_c} \qquad (6)$$

where:

S_F – relative efficiency of melting, kg/(m^2·s).

Equation (5) is the first variant of the formula of calculation H_m in the present work.

The use of (5) requires the calculation of values S_F, $\tau_{\bar{r}}$ and $K_{\rho,t}$, which are depenldent on initial parameters of the process.

Calculation of cupola efficiency, S_F :

Relative efficiency of coke cupolas S_F can be calculated based on the following formula of J. Buzek [6]

$$S_F = 100\frac{P_F}{K_{c,4}L_{c,4}} \qquad (7)$$

where:

$$L_{c,4} = 4,45(1+\eta_{v,4}) \qquad (8)$$

$$\eta_{v,4} = \frac{(CO_2)_{v,4}}{(CO_2)_{v,4}+(CO)_{v,4}} \qquad (9)$$

where:

P_F – relative air blast volume delivered to the cupola, recalculated to normal conditions, m^3/(m^2·s) or m/s

$K_{c,4}$ – contribution of coal in the coke charge, kg$_c$/100 kg$_{Fe}$

$L_{c,4}$ – air blast volume consumed for burning 1 kg of coal contained in coke, air with normal content of oxygen of pressure 0.1 MPa m^3/kg$_c$

$(CO_2)_{v,4}$, $(CO)_{v,4}$ – CO_2 and CO content, respectively, in gases at the upper boundary of heating zone, vol.%.

Note: Equation (9) can be used for arbitrary cross-sections of all zones of the cupola [3].

After the incorporation of (7) into (5)

$$\mathbf{H_t = \frac{100 P_F \tau_{\bar{r}}}{K_{c,4}L_{c,4}\rho_{n,m}} K\rho, t} \qquad (10)$$

Equation (10) is a second variant of the formula of H_t calculation in the present work.

Calculation of $K_{\rho,t}$ quantity:

Quantity $\overline{M}_{m,t}$ in equation (4) is a sum of masses of melting metal lumps in the zone; lumps have different masses, which decrease in the direction of lower zone boundary. $\overline{M}_{m,t}$ is calculated using dimensionless coefficient φ_v [2], defined with the following ratio:

$$\varphi v = \frac{\bar{M}_{m,t}}{M_{m,t}} \qquad (11)$$

where:

$M_{m,t}$ – primary mass of metal of which mass $\overline{M}_{m,t}$ was formed, kg.

The following relationship results from (11)

$$\bar{M}_{m,t} = \varphi_v M_{m,t} \qquad (12)$$

In which φ_v is calculated from formula [2]

$$\varphi_v = \frac{1}{2} - \frac{1}{6m_b} - \frac{1}{6m_c} + \frac{1}{12m_b m_c} \qquad (13)$$

where:

$m_b = \frac{b}{a}$; $m_c = \frac{c}{a}$

a, b, c – thickness width and length of metal charge pieces, respectively, in the form of rectangular slabs, prisms, cubes or balls (a=b=c=d), m.

Relation (12) substituted into (4) gives

$$K_{\rho,t} = 1 + \frac{M_{k,t}}{\varphi_v M_{m,t}} \frac{\rho_{n,m}}{\rho_{n,k}} \qquad (14)$$

Expenditure of charge coke, $K_{w,4}$ is introduced into equation (14)

$$K_{\rho,t} = 1 + \frac{K_{w,4}}{\varphi_v 100} \frac{\rho_{n,m}}{\rho_{n,k}} \qquad (15)$$

where:

$K_{w,4} = \frac{M_{k,t}}{M_{m,t}}$ 100 – expenditure of charge coke, kg$_k$/100 kg$_{Fe}$

Calculation of $\tau_{\bar{r}}$, $\bar{r}_m$ and μ_t:

According to assumption (1) time $\tau_{\bar{r}}$, present in formula (1) can be defined as

$$\tau_{\bar{r}} = \frac{\bar{r}_m}{\mu_t} \qquad (16)$$

where:

$\bar{r}_m$ – average integral modulus of metal charge pieces in the melting zone, m

μ_t – average linear rate of melting, the same for all pieces of zone, m/s.

Introducing (16) into (10)

$$\mathbf{H_t = \frac{100 P_F \bar{r}_m}{K_{c,4}L_{c,4}\mu_t\rho_{n,m}} K_{\rho,t}} \qquad (17)$$

Equation (17) is a third variant of the calculation formula of H_t

Value $\bar{r}_m$ is calculated based on paper [2]

$$\bar{r}_m = r_m \frac{\rho_v}{\rho_f} = \frac{\bar{v}_m}{\bar{f}_m} \qquad (18)$$

in which

$$r_m = \frac{v_m}{f_m} = \frac{a}{2(1+\frac{1}{m_b}+\frac{1}{m_c})} \qquad (19)$$

$$\varphi_f = \frac{m_b m_c}{m_b + m_c + m_b m_c} = \frac{1}{1+\frac{1}{m_b}+\frac{1}{m_c}} \qquad (20)$$

where:

v_m, $\bar{v}_m$ – starting and mean integral volume of metal pieces, respectively $\bar{v}_m = \varphi_v v_m$ m^3

f_m, $\overline{f}_m$ – starting and mean integral surface of metal pieces, respectively $\overline{f}_m = \varphi_f\, f_m$, m^2
r_m – initial modulus of metal pieces, m.

The following thermal balance of the melting metal piece is a basis to mathematical definition of value μ_t and to obtain an equation to calculate it

$$q_g f_x d\tau = -\rho_m L_{f,p} dv_x \quad (21)$$

where:
q_g – thermal flux density, transferred by gases in elementary time $d\tau$ to the surface of the melting piece of metal charge of current surface f_x, resulting in melting down of the elementary volume dv_x W/m^2

$L_{f,p}$ –heat of melting of 1 kg metal melted in the zone increased by preliminary superheating of the liquid metal, which brings about the trickling down of metal drops from the melting pieces, J/kg$_{Fe}$

ρ_m – density of melting metal mass, kg/m^3

Negative sign in (22) effects from the negative value of dv_x

Equation (22) can be separated into two ones

$$\mu_t = -\frac{dv_x}{f_x d\tau} \quad (22)$$

$$\mu_t = \frac{q_g}{L_{f,p}\rho_m} \quad (23)$$

Equation (22) will be used to record the mathematical definition of linear melting velocity, while equation (23) to obtain a formula of linear melting velocity.

Mathematical definition of linear melting velocity:

Equation (22) may be written down for the melting metal pieces in the form of balls, cylinders and slabs.

Volume and surface of a melting ball is described with formulas

$$v_x = 4/3\pi x^3 \quad (24)$$

$$f_x = 4\pi x^2 \quad (25)$$

where:
x – ball radius, m.

Let us differentiate (24)

$$dv_x = 4\pi x^2 dx \quad (26)$$

After substituting (25) and (26) into (22) and after reduction the following formula is obtained

$$\boldsymbol{\mu_t = -\frac{dx}{d\tau}} \quad (27)$$

where:
dx – elementary melted thickness (negative increase) at surface of arbitrary radius of curvature, m
$d\tau$ – elementary time of melting, s.

Formula (27) is the mathematical definition of linear velocity of melting.

The process of melting cylinder from the side surface (the cylinder is theoretically infinitely long). Its volume and surface are written down with formulas

$$v_x = \pi x^2 c \quad (28)$$

$$f_x = 2\pi x c \quad (29)$$

Equation (28) after differentiation

$$dv_x = 2\pi x c \quad (30)$$

After incorporating (29) and (3) into (22) and after the reduction formula (27) is obtained

As a third shape, infinite plate, e.x. not melting from its side surfaces will be discussed. Its volume and melting surface can be written down with the following formulas

$$v_x = f \cdot x \quad (31)$$

$$f_x = f \quad (32)$$

where:
f – total surface of the melting plate (both main plate surfaces are constant during melting), m^2
x – half thickness of the melting plate, m.

After differentiation of (31)

$$dv_x = f dx \quad (33)$$

Substituting (32) and (33) into (22) and after the simplification formula (27) is obtained

Based on the considerations carried out, it can be concluded, that formula (27) represents the mathematical definition of linear melting velocity for plates of arbitrary curvature radii.

Formula of linear velocity of melting :

The formula to calculate the linear velocity of melting can be derived using (23)

Density of thermal flux q_g can be written down in the form of Newton formula

$$q_g = \alpha_{F,t}\overline{\Delta T}_{g,t} \quad (34)$$

where:

$\alpha_{F,t}$ – coefficient of heat exchange between gases and the total surface of the melting metal charge pieces (surface of development) in the melting zone, average value for the whole zone, W/(m^2·K)

Formula μ_t is obtained after incorporating (34) and (2) into (23)

$$\boldsymbol{\mu_t = \frac{\alpha_{F,t}\Delta T_{g,t}}{L_{f,p}\rho_m \ln\frac{T_{g,2}-T_{m3}}{T_{g,2}-\Delta T_{g,t}-Tm,}}} \quad (35)$$

and (35) into (16)

$$\tau_{\bar{r}} = \frac{\bar{r}_m L_{f,p} \rho_m \ln \frac{T_{g,2} - T_{m3}}{T_{g,2} - \Delta T_{g,t} - T_{m,3}}}{\alpha_{F,t} \Delta T_{g,t}} \quad (36)$$

Formula (36) serves to calculate average melting times of metal pieces of mean integral modulus $\bar{r}_m$.

Developed formula to calculate the height of melting zone:

Let us insert (35) into (17)

$$H_t = \frac{100 P_F \bar{r}_m L_{f,p} \rho_m K_{\rho,t}}{K_{c,4} L_{c,4} \alpha_{F,t} \Delta T_{g,t} \rho_{n,m}} \ln \frac{T_{g,2} - T_{m,3}}{T_{g,2} - \Delta T_{g,t} - T_{m,3}} \quad (37)$$

Expression (37) is the fourth variant of formula for the H_t calculation. Its use requires the calculation of value $\Delta T_{g,t}$ and temperature $T_{g,2}$

Calculation of $\Delta T_{g,t}$:

Using the basis of theory of heat exchangers which binds the decrease of gas temperature with the process of metal melting the following equation of balance can be written

$$W_{g,t} \Delta T_{g,t} = L_{f,p} S_c \quad (38)$$

in which:

$$L_{f,p} = L_f + c_{m,f} \Delta T_{p,t} \quad (39)$$

where:

$W_{g,t}$ – temperature power of gases in the melting zone (name W.L.), called previously aqueous number, W/K

L_f – heat of metal melting, J/kg_{Fe}

$c_{m,f}$ – specific heat of liquid metal, J/(kg_{Fe}·K)

$\Delta T_{p,t}$ – degree of superheating of metal liquid drops, which trickle off the melting metal pieces, K

Value $W_{g,t}$ for coke cupolas can be written down with the following formula

$$W_{g,t} = V_{g,t} c_{g,t} m_{c,t} \quad (40)$$

where:

$V_{g,t}$ – mean volume of gases in the melting zone, normal conditions, m^3/kg_C

$c_{g,t}$ – mean specific heat of gases in the melting zone, J/(m^3·K)

$m_{c,t}$ – mass velocity of burning coal corresponding to volume of gases $V_{g,t}$ and their specific heat $c_{g,t}$, kg_c/s

$V_{g,t}$ $m_{c,t}$ – gas volume expense in the melting zone, normal conditions, m^3/s

After incorporating (40) into (38) and after transformation, the following formula for the calculation of decrease of gas temperature in the melting zone is derived

$$\Delta T_{g,t} = \frac{100 L_{f,p}}{V_{g,t} c_{g,t} K_{c,4}} \quad (41)$$

where:

$$K_{c,4} = \frac{S}{m_{c,t}} 100 \left[\frac{kg_{Fe}}{kg_c} 100 \right].$$

Expression (41) neglects the coal losses due to the CO_2 reduction (assumption).

Values $V_{g,t}$ can be calculated as a mean arithmetic of gas volume values at the zone boundaries based on formulas (blast with the normal content of oxygen)

$$V_{g,2} = 5.39(1 + 0.65\eta_{v,2}) \quad (42)$$

$$V_{g,3} = 5.39(1 + 0.65\eta_{v,3}) \quad (43)$$

where:

$V_{g,2}$, $V_{g,3}$ – volume of gases at lower and upper boundary of zone, respectively, normal conditions, m^3/kg_c

$\eta_{v,2}$, $\eta_{v,3}$ – degree of gas burning at the lower and upper boundary of melting zone, respectively, unit fraction.

Finally, quantity $c_{g,t}$ has remained uncalculated. The course of its calculation according to the method elaborated by the author for given temperatures of gases at the higher and lower boundaries of melting zone (or the remaining zones) is as follows:

– it is assumed that temperatures of gases at the higher and lower boundary of melting zone are $T_{g,2}$ and $T_{g,3}$ as multiplies of 100°C; i.e. 1600 and 1400°C

– specific heat of gases $c_{g,2}$ is calculated for an assumed $T_{g,2}$ temperature out of the formula

$$c_{g,2} = A + B \frac{\eta_{v,2}}{1 + 0.65 \cdot \eta_{v,2}} \quad (44)$$

where:

A, B – coefficients taken from Table 1 for assumed temperature $T_{g,2}$

– specific heat $c_{g,3}$ for the assumed $T_{g,3}$ temperature is calculated in the same way.

It should be emphasized, that calculated specific heat $c_{g,2}$ is an average specific heat of gases for the temperature range from 0°C up to temperature $T_{g,2}$, while $c_{g,3}$ is an average specific heat of gases for temperatures from 0°C up to $T_{g,3}$.

After the calculation of $c_{g,2}$ and $c_{g,3}$, mean value of gas specific heat is calculated in the area between $T_{g,2}$ a $T_{g,3}$ from the formula

$$c_{g,t} = \frac{T_{g,2} c_{g,2} - T_{g,3} c_{g,3}}{T_{g,2} - T_{g,3}} \quad (45)$$

TABLE 1

Auxiliary table for the calculation of mean specific heat of cupola gases from 0°C up to a required temperature of gases T_g (e.g. $T_{g,2}$ and $T_{g,3}$ as well as $\eta_v = \eta_2$ and $\eta_{v,3}$) for the oxygen content in the blast air 21 vol.%

Calculation formula $C_g = A + B\frac{\eta_v}{1+0.65\eta_v}$					
T_g°C	A	B	T_g°C	A	B
0	1282.0	103.1	**1300**	1417.6	285.0
100	1283.7	135.9	**1400**	1427.6	289.9
200	1288.0	163.8	**1500**	1436.6	293.7
300	1296.1	185.7	**1600**	1445.6	297.5
400	1306.8	203.9	**1700**	1453.6	301.3
500	1318.5	219.4	**1800**	1461.2	305.4
600	1331.2	231.5	**1900**	1467.9	308.7
700	1345.5	242.4	**2000**	1474.9	311.2
800	1358.9	251.8	**2100**	1480.9	314.0
900	1372.2	259.9	**2200**	1486.9	316.0
1000	1384.6	267.0	**2300**	1491.9	318.5
1100	1396.2	274.2	**2400**	1497.9	320.2
1200	1406.6	279.8	**2500**	1502.9	321.9

Comment: The presented method of calculation $c_{g,t}$ refers to the situation, when temperature data $T_{g,2}$ and $T_{g,3}$ are the multiplies of 100°C. However, if they are not, the calculation may proceed in two ways:

– the application of approximation methods known from the thermal engineering, or

– correction of temperatures $T_{g,2}$ and $T_{g,3}$ (e.g. calculated from the zone heat balances) up or down to full multiplies of 100°C, which may cause small increase or a drop of calculated values $c_{g,2}$ and $c_{g,3}$

In this way a simple method of calculation of gas specific heat is derived. It should be underlined that, generally accepted in literature values of specific heat of gases for the coke cupola process are usually underrated. For example, calculated values $c_{g,3}$ and $c_{g,4}$ for temperatures $T_{g,3}$= 1700°C and $T_{g,3}$ =1400°C ($\eta_{v,4}$ =0.525) amount up to 1571 and 1539 J/(m^3·K), while the value of $c_{g,t}$ =1722 J/(m^3·K). The calculated $c_{g,t}$ value is higher than the $c_{g,2}$ one, which is a rule, if mean heats $c_{g,2}$ and $c_{g,3}$ increase together with temperature.

Calculation of temperature $T_{g,2}$:

The sequence of $T_{g,2}$ calculation is as follows:

– a decrease of gas temperature in the main reduction zone is calculated from the formula analogous to (41)

$$\Delta T_{g,r}=\frac{100q_r}{V_{g,r}c_{g,r}K_{c,r}} \quad (46)$$

where:

$\Delta T_{g,r}$ – decrease of gas temperature in the main reduction zone due to the decomposition of CO_2 with the coal contained in the coke charge, K

q_r – physical heat of gases, used up for the reduction of CO_2 in 1 kg of the iron charge, J/kg$_{Fe}$

$V_{g,r}$ – average volume of gases in the reduction zone, normal conditions, m^3/kg$_c$

$c_{g,r}$ – average specific heat in the reduction zone of pressure 0.1 MPa, J/(m^3·K)

$K_{c,r}$ – fraction of coke coal in the reduction zone, kg$_c$/100 kg$_{Fe}$

The formulas for calculation of q_r; $V_{g,r}$; $c_{g,r}$ and $K_{c,r}$ are given successively

Calculation of q_r :

Using the data of work [3] the following formula for calculation of expenditure of thermal energy for the CO_2 reduction between the upper boundary of the combustion zone and the lower one of the melting zone can be introduced

$$\Delta Q_r= 13.54 \cdot 10^6 \left(\frac{\eta_{v,r}}{1+\eta_{v,r}}-\frac{\eta_{v,2}}{1+\eta_{v,2}}\right)(1+\eta_{v,4})K_{c,r} \quad (47)$$

where:

ΔQ_r – expenditure of thermal physical energy of gases for reaction: $CO_2+C\rightarrow 2\ CO$, J/100 kg$_{Fe}$

$13.54\cdot10^6=(2\cdot120.72-403.93)/12$ (negative sign is neglected), J/kg$_c$

$\eta_{v,r}$ – degree of gas burning at the upper boundary of combustion, unit fraction

In order to calculate the $\eta_{v,r}$ value, the contribution of CO in the gases at the upper boundary of combustion $(CO)_{v,r}$, is assumed based on literature data (e.g. [7]) and next, the value of $(CO_2)_{v,r}$ is calculated from formula

$$(CO_2)_{v,r}=\frac{34.7-(CO)_{v,r}}{1.65} \quad (48)$$

Knowing $(CO)_{v,r}$ and $(CO_2)_{v,r}$ from (9) $\eta_{v,r}(_{v,4=v,r})$ can be calculated

After the calculation of Q_r from (47) q_r is calculated

$$q_r = \frac{\Delta Q_r}{100} \quad (49)$$

Calculation of $V_{g,r}$:

Value $V_{g,r}$ can be derived from the formula analogous to (43)

$$V_{g,r} = 0.5 \cdot 5.39[(1 + 0.65\eta_{v,r}) + (1 + 0.65\eta_{v,2})] \quad (50)$$

Calculation of $c_{g,r}$:

Mean value of specific heat of gases in the zone of reduction $c_{g,r}$ can be obtained in the identical way like the mean value of specific heat in the melting zone, $c_{g,t}$

Calculation of $K_{c,r}$:

In the main zone of reduction the mass of coal contained in the coke significantly decreases. Based on work [3] the following formula for obtaining the expenditure of coke at an arbitrary zone height, x can be written

$$K_{c,x} = \frac{1+\eta_{v,4}}{1+\eta_{v,x}} K_{c,4} \quad (51)$$

where:

$\eta_{v,x}$ – degree of gas burning at a given level of the main reduction zone.

Using (51) the formula to calculate an average expenditure of coke in the zone as an arithmetic mean of boundary expenditures will be written down as follows

$$K_{c,r} = 0.5\left(1+\frac{1+\eta_{v,4}}{1+\eta_{v,r}}\right)K_{c,4} \quad (52)$$

After calculating the value of $\Delta T_{g,r}$ from (46), the temperature of gases $T_{g,2}$ can be derived from formula

$$T_{g,2} = T_{g,max} - \Delta T_{g,r} \quad (53)$$

where:

$T_{g,max}$ – temperature of cupola gases at the contact boundary of combustion zone and the main reduction zone, °C.

According to common opinion of researchers, temperature $T_{g,max}$ is the maximum temperature of gases in a given process. Its calculation is not derived in the present work.

So far, the formulas to calculate H_t as a geometrical characteristics of the melting zone as well as the calculation formula for μ_t as a kinetic characteristic factor of zone have been derived. Now, formulas to calculate further characteristic geometric and kinetic parameters will be consecutively derived.

3. Calculation of remaining geometric parameters of zone

The following formulas will be written down successively: surfaces of development; number of melting metal charge pieces; amount of series of melting metal charge pieces; calculation mass of metal and coke in the zone.

Surface of development of melting metal charge pieces:

The formula for calculation of total surface of the melting metal charge lumps, i.e. the development surface of the melting zone can be derived starting from the following equation

$$F_t = n_m \bar{f}_m = \frac{\bar{M}_{m,t}}{\rho_m \bar{r}_m} \quad (54)$$

in which

$$n_m = \frac{\bar{M}_{m,t}}{\rho_m \bar{v}_m} \quad (55)$$

where:

F_t – development surface of the melting zone, m^2

n_m – number of melting metal charge lumps.

$\bar{M}_{m,t}$ is calculated from equation (3) and substituted into (54) obtaining the formula for the F_t calculation

$$\mathbf{F_t = \frac{F_c H_t \rho_{n,m}}{K_{\rho,t} \rho_m \bar{r}_m}} \quad (56)$$

Expression H_t can be derived from (37).

Number of melting metal charge lumps:

The number of melting metal charge lumps is contained in formula (55), into which mass $\overline{M}_{m,t}$, calculated from (3), is incorporated

$$\mathbf{n_m = \frac{F_c H_t \rho_{n,m}}{\rho_m \bar{v}_m K_{\rho,t}}} \quad (57)$$

H_t can be obtained from (37).

Number of sequences of melting metal charge lumps:

The melting metal charge lumps in amount n_m form identical (assumption) sequences of decreasing pieces at the height of the melting zone. It is assumed that the number of sequences is equal to the number of metal charge of initial average size, $\overline{X}$, moving together with the coke to the zone through its upper boundary. The following formula containing the number of initial metal pieces moving from the zone of heating to the melting one is proposed to calculate the number of series:

$$\mathbf{N_c = \frac{F_c}{\bar{X}^2 K\rho, p}} \quad (58)$$

in which

$$\bar{X} = \frac{a+b+c}{3} \quad (59)$$

where:

$K_{\rho,p}$ – criterion for the heating zone, calculated from expression (15) at φ_v=1,

N_c – number of sequences formed of the melting metal pieces

Calculation of metal and coke masses in the zone:

The mass of metal melting in the zone can be calculated from (3)

$$\bar{\mathbf{M}}_{\mathbf{m,t}} = \frac{\mathbf{F_c H_t \rho_{n,m}}}{\mathbf{K_{\rho,t}}} \quad (60)$$

The mass of coke in the zone $M_{k,t}$ can be calculated from expression

$$M_{k,t} = \frac{K_{w,4}}{\varphi_v 100} \bar{M}_{m,t} \quad (61)$$

4. Calculation of kinetic mean parameters of zone

The formula for the calculation of average linear velocity of melting metal pieces of arbitrary radius of curvature (35) has been derived so far as well as the formula to calculate melting time of metal pieces of mean integral modulus (36). Now the formulas to calculate other average kinetic parameters of the melting zone will be derived: melting time of arbitrary mass of metal; linear velocity of melting based on efficiency and surface of development, melting time of a single lump in dependence on its characteristic linear size.

Linear velocity of melting calculated based on efficiency:

The linear velocity of melting can be calculated using efficiency of melting and surface of development F_t. The efficiency of melting can be expressed with the formula

$$S_c = \mu_t F_t \rho_m \quad (62)$$

μ_t can be obtained from (62)

$$\mu_t = \frac{S_c}{F_t \rho_m} \quad (63)$$

F_t can be calculated from (56), while S_c from (7) (S_c=S_F F_c)

Calculation of zone melting time of arbitrary mass of metal:

The efficiency of zone melting can be written down with formula

$$S_c = \frac{\bar{M}_{m,t}}{\tau_{\bar{M}}} \quad (64)$$

where:

$\tau_{\bar{M}}$ – mass melting time of mass $\bar{M}_{m,t}$, s.

$\tau_{\bar{M}}$ can be derived from (64)

$$\tau_{\bar{M}} = \frac{\bar{M}_{m,t}}{S_c} \quad (65)$$

Melting time of metal mass equal to the mass of a cartridge of metal charge $m_{n,m}$ (kg_{Fe}) can be calculated from (65) after substituting

$\bar{M}_{m,t}$=$m_{n,m}$

$$\tau_{n,m} = \frac{m_{n,m}}{S_c} \quad (66)$$

where:

$\tau_{n,m}$ – melting time of metal mass equal to the mass of a cartridge of metal charge, s.

Melting time of individual pieces of the metal charge

Melting time of individual pieces of the metal charge depends on their defined characteristic size and on the linear velocity of melting. Typical sizes of metal pieces in the melting zone can be described as:

– the smallest initial size of lump, a/2
– initial modulus of lumps r_m
– mean integral modulus of lumps for the whole zone $\bar{r}_m$

Since the linear melting rate is constant for the stabilized melting zone [formula (35)] and it does not depend on the curvature of melting metal piece surface, the formulas for its calculation can be put down in the form of the following sequence valid for the defined characteristic values

$$\mu_t = \frac{\frac{a}{2}}{\tau_a} = \frac{r_m}{\tau_r} = \frac{\bar{r}_m}{\tau_{\bar{r}}} \quad (67)$$

where:

τ_a – real melting time of metal charge pieces of simple shape (described in a general case by thickness, width and length) as well as of melted thickness a/2, s

τ_r – melting time of metal charge pieces of modulus r_m, s.

The following formulas to calculate the defined melting times result from (67)

$$\tau_a = \frac{\frac{a}{2}}{\mu_t} \quad (68)$$

$$\tau_r = \frac{r_m}{\mu_t} \quad (69)$$

$$\tau_{\bar{r}} = \frac{\bar{r}_m}{\mu_t} \quad (70)$$

The following relations between the defined melting times also come from (67):

– comparison of term one and two

$\frac{\frac{a}{2}}{\tau_a}=\frac{r_m}{\tau_r}$ lub $\tau_r=\frac{r_m}{\frac{a}{2}}\tau_a$ or after taking into consideration (20)

$$\tau_r=\tau_a\varphi_f \tag{71}$$

– comparison of term two and three

$\frac{r_m}{\tau_r}=\frac{\bar{r}_m}{\tau_{\bar{r}}}$ lub $\tau_{\bar{r}}=\frac{\bar{r}_m}{r_m}\tau_r$ and after consideration of (13) and (20)

$$\tau_{\bar{r}}=\frac{\varphi_v}{\varphi_f}\tau_r \tag{72}$$

And taking into account (71) as well

$$\tau_{\bar{r}}=\tau_a\varphi_v \tag{73}$$

5. Layers of melting zone and their geometric and kinetic parameters

Calculation of features characteristic for the structure of particular levels of the melting zone.

Number of layers in the melting zone:

The height of melting zone can be separated into horizontal layers, which consist of a mixture of melting metal pieces and heated up coke assuming that the number of layers equals to the number of metal charge lumps in the series of pieces. Hence, the number of zone layers can be calculated from the formula

$$n_w=\frac{n_m}{N_c} \tag{74}$$

where:

n_w – number of layers in the melting zone

The volumes of particular layers are the sum of bulk volumes of metal and coke lumps; the bulk metal volumes decrease towards the lower boundary of melting zone, while the bulk coke volumes remain the same.

Volumes and masses of metal charge pieces in individual layers of zone:

In order to calculate the volumes and masses of metal charge pieces, which melt in particular zone layers it is assumed [2], that the differences of elementary dimensions of metal pieces in adjoined layers are equal to z (m). It follows from such an assumption, that e.g. the first upper zone layer can contain metal lumps of primary volume (pieces, which passed out of the heating zone unchanged) or pieces, whose basic dimensions were diminished by value z (they underwent partial melting during their way from the heating zone to the first layer of the melting zone). It may be proved [2], that the first volume out of mentioned ones, will be too big, while another will be too small hence in the calculations it is recommended to take arithmetic means of volumes of metal charge pieces in individual layers, calculated for the two characterized cases, which is equivalent to a linear decrease of lump volumes when passing from one layer to another.

In further considerations, it is assumed, that the melting metal charge lumps have a form of slabs. The volumes of slabs in particular layers, according to the accepted rule of arithmetic means, is contained in formula

$$v_{m,i}=\frac{[a-(i+1)z]\cdot[b-(i-1)z]\cdot[c-(i-1)z+(a-iz)\cdot(b-iz)\cdot(c-iz)}{2} \tag{75}$$

in which

$$z=\frac{a}{n_w} \tag{76}$$

where:

$v_{m,i}$ – volumes of melting metal pieces in particular zone layers (m^3), where i = 1, 2, 3,.. n_w subsequent numbers of zone layers, starting from the highest layer.

The volume and mass of all metal pieces in an i-layer can be calculated from the following formulas

$$V_{m,i}=N_c v_{m,i} \tag{77}$$

$$M_{m,i}=\rho_m V_{m,i} \tag{78}$$

where:

$V_{m,i}$ – volume of metal pieces in i-layer, m^3

$M_{m,i}$ – mass of metal lumps in i-layer, kg.

The bulk volume of metal in an i-layer and its height is obtained from the following formulas

$$V_{m,n,i}=\frac{M_{m,i}}{\rho_{n,m}}=\frac{\rho_m}{\rho_{n,m}}V_{m,i} \tag{79}$$

$$h_{m,n,i}=\frac{V_{m,n,i}}{F} \tag{80}$$

where:

$V_{m,n,i}$ – bulk metal volume in i-layer, m^3

$h_{m,n,i}$ – bulk height of volume $V_{m,n,i}$, m.

Volume and mass fractions of coke with respect to metal:

The bulk mass and volume of coke in each layer are equal and can be calculated from the relation (the assumption neglects the reduction process and other losses of coke)

$$M_{k,w}=\frac{M_{k,t}}{n_w} \tag{81}$$

$$V_{k,w}=\frac{M_{k,w}}{\rho_{n,k}} \tag{82}$$

where:

$M_{k,w}$; $V_{k,w}$ – mass and volume of coke in each layer, respectively, kg and m^3.

The bulk height of volume $V_{k,w}$ can be derived from relation

$$h_{k,w}=\frac{V_{k,w}}{F_c} \tag{83}$$

The volume and mass ratios of coke to metal in particular layers of zone can be found from expression

$$K_{v,i}=\frac{V_{k,w}}{V_{m,n,i}} \tag{84}$$

$$K_{w,i}=\frac{M_{k,w}}{M_{m,i}}100 \tag{85}$$

where:

$K_{v,i}$ – ratio of coke volume to metal in i-layer, m^3coke/m^3metal

$K_{w,i}$ – mass fraction of coke with respect to metal in i-layer kg_k/100 kg_{Fe}

The contribution of metal mass in individual layers with respect to the metal mass in the zone is interesting for the characteristics of the melting zone, and can be obtained from relationship

$$U_{m,i}=\frac{M_{m,i}}{\bar{M}_{m,t}}100 \tag{86}$$

where:

$U_{m,i}$ – mass of metal in i-layer with respect to the metal mass in the zone, in mass %

Development surface of metal charge pieces in individual layers and efficiency of layer melting:

Accepting an analogous assumption as for formula (75), the development surface of melting metal pieces in individual layers can be expressed as follows

$$f_{m,i}=\frac{f'_{m,i}+f''_{m,i}}{2} \tag{87}$$

in which:

$$f'_{f,i}=2[a-(i-1)z][b-(i-1)z]+2[a-(i-1)z]$$
$$[c-(i-1)z+2[b-(i-1)z][c-(i-1)z] \tag{88}$$

$$f''_{f,i}=2(a-i\cdot z)(b-i\cdot z)+2(a-i\cdot z)$$
$$(c-i\cdot z)+2(b-i\cdot z)(c-i\cdot z) \tag{89}$$

where:

$f_{m,i}$ – surface of melting metal charge pieces in i-layer, m^2

The surface of all the melting pieces, $F_{n,i}$ in i-layer is derived from expression

$$F_{m,i}=N_c f_{m,i} \tag{90}$$

Knowing $F_{m,i}$, the melting efficiency of individual layers in the zone is calculated

$$s_i=\mu_t F_{m,i}\rho_m \tag{91}$$

where:

s_i – melting efficiency of i-layer, kg/s.

Time spent by metal charge pieces in i-layer

In each layer the metal piece melts down thickness z during passing to the next layer; melting time of layer of z thickness will be

$$\tau_z=\frac{0.5a}{\mu_t n_w} \tag{92}$$

where:

τ_z – stay of metal pieces in particular layers and simultaneously melting time of layers with thickness 0.5 z, s.

The rate of lowering of individual layers:

The lowering rate of zone layers due to metal melting results from the following equation of metal volume balance

$$W_{t,i}\rho_{n,m}F_c=s_i \tag{93}$$

where:

$W_{t,i}$ – lowering rate of the i-layer during metal melting, m/s

s_i – melting efficiency in i-layer, kg/s.

The formula to calculate $W_{t,I}$ is obtained from (93),

$$W_{t,i}=\frac{s_i}{F_c\rho_{n,m}} \tag{94}$$

The same way refers to obtaining the formula for the whole melting zone

$$W_{t,m}=\frac{S_F}{\rho_{n,m}} \tag{95}$$

where:

$W_{t,m}$ – lowering rate of metal height in the melting zone, m/s.

6. Example of calculation of melting zone height according to (37)

Starting data:

P_F=1.6 m/s; F_c= 0.502 m^2 (internal diameter of the cupola d_c=0.8 m); $K_{w,4}$=12 kg_k/100 kg_{Fe}; C_k=0.86 kg_c/kg_k; a=0.05 m; b=0.2 m; c=0.3 m; $\alpha_{F,t}$ = 200 W/(m^2·K)(estimated value); L_f=268000 J/kg_{Fe}; $\Delta T_{p,t}$=50 K; $c_{m,f}$=837 K/(kg_{Fe}·K); ρ_m=7000 kg/m^3; $\rho_{n,m}$=2500 kg/m^3; $\rho_{n,k}$=500kg/m^3; $m_{n,m}$=400kg;$(CO_2)_{v,4}$=13, 6 vol%; $(CO)_{v,4}$=12,3 vol.%; $\eta_{v,2} = \eta_{v,3} = \eta_{v,4}$; $(CO)_{v,2}$=4 vol.%; $T_{m,f}$=1150°C; $T_{g,max}$=1750°C (estimated value)

Calculation of formula elements (37)($\bar{r}_m$; $L_{f,p}$; $K_{\rho,t}$; $\Delta T_{g,t}$; $\Delta T_{g,r}$ and H_t)

$m_b=\frac{0.2}{0.05}=4; m_c=\frac{0.3}{0.05}=6; v_m=0.2\cdot 0.3\cdot 0.05=3\cdot 10^{-3} m^3$;

$f_m=2(0.2{\cdot}0.3+0.2{\cdot}0.05+0.3{\cdot}0.05)=0.17 m^2$;

$r_m=\frac{3\cdot 10^{-3}}{0.17}=\frac{3\cdot 10^{-3}}{0.017}=0.0176$ m according to (19);

$\varphi_v=\frac{1}{2}-\frac{1}{6\cdot 4}-\frac{1}{6\cdot 6}+\frac{1}{12\cdot 4\cdot 6}=0.434$ acc. to (13);

$\varphi_f=\frac{6\cdot 4}{6+4+6\cdot 4}=0.706$acc.(20)$\bar{r}_m=0.0176\frac{0.434}{0.706}=0.011$ m acc. (18);

$K_{c,4}=12\cdot 0.86=10.32 kg_c/100kg_{Fe}$; $\eta_{v,4}=\frac{13.6}{13.6+12.3}=0.525$ acc. to (9);

$L_{c,4}=4.45(1+0.525)=6.79 m^3/kg_c$ acc. to (8);

$L_{f,p}=268000+50{\cdot}837=309850 J/kg_{Fe}$ acc. to (39);

$K_{\rho,t}=1+\frac{12}{0.434\cdot 100}\frac{2500}{500}=2.38$ acc. to (14);

$c_{g,t}=\frac{1600\cdot 1572-1400\cdot 1541}{200}=1709 J/(m^3\cdot K)$ acc. to (45)

where: 1600 and 1400 – assumed limits of temperatures of the melting zone in °C; 1572 and 1541 – calculated specific heats of gases for the boundary temperatures acc. to the method described in the presented work in J/(m^3·K);

$V_{g,t}=5.39(1+0.65{\cdot}0.525)=7.23 m^3/kg_c$ acc. to (43)

$\Delta T_{g,t}=\frac{100\cdot 309850}{7.23\cdot 1709\cdot 10.32}=243$Kacc.to(41);

$(CO_2)_{v,r}=\frac{34.7-4}{1.65}=18.6$vol.%acc.to(48);

$\eta_{v,r}=\frac{18.6}{18.6+4}=0.823$ acc. to (9) for 4 = r;

$V_{g,r}=5.39\cdot 0.5[(1+0.65\cdot 0.823)+(1+0.65\cdot 0.525)]=7.75 m^3/kg_c$ acc. to (50);

$c_{g,r}=\frac{1750\cdot 1620-1600\cdot 1572}{1750-1600}=2132 J/(m^3\cdot K)$ acc. to (45) for boundary temperatures of main reduction zone, where: 1750 and 1600 – limits of temperatures in °C; 1620 and 1572 – specific heat of gases for boundary temperatures, calculated according to the method described in this work, in J/(m^3 · K5 $\left(1+\frac{1+0.525}{1+0.823}\right)$10.32 = 9.48$kg_c/100kg_{Fe}$w);

$K_{c,r}$= 0, acc. to (52); $\Delta Q_{c,r}=13.54\cdot 10^6\left(\frac{0.823}{1+0.823}-\frac{0.525}{1+0.525}\right)$ $(1+0.525)9.48=22.84\cdot 10^6 J/100kg_{Fe}$ acc. to (47);

$q_r=\frac{22.84\cdot 10^6}{100}=228400 J/kg_{Fe}$ acc. to (49);

$\Delta T_{g,r}=\frac{100\cdot 228400}{7.75\cdot 2132\cdot 9.48}=146$Kacc.to(46);

$T_{g,2}=1750-146=1604$°C acc. to (53)

$$\mathbf{H_t=\frac{100\cdot 1.6\cdot 0.011\cdot 309850\cdot 7000\cdot 2.38}{10.32\cdot 6.79\cdot 200\cdot 243\cdot 2500}\ln\frac{1604-1150}{1604-243-1150}=0.8\ m\ acc.\ to\ (37).}$$

Calculations of S_F, F_t, n_m, N_c, $\overline{M}_{m,t}$; $M_{k,t}$; μ_t

$S_F=100\frac{1.6}{10.32\cdot 6.79}=2.283 kg_{Fe}/(m^2\cdot s)$ acc. to (7), (8), (9);

$F_t=\frac{0.503\cdot 0.8\cdot 2500}{2.38\cdot 7000\cdot 0.011}=5.5 m^2$ acc. to (56);

$n_m=\frac{0.503\cdot 0.8\cdot 2500}{7000\cdot 3\cdot 10^{-3}\cdot 0.434\cdot 2.38}=46.4$ acc. to (57) (verification $F_t=46.4\cdot 0.17\cdot 0.706=5.57 m^2$);

$\bar{X}=\frac{0.05+0.2+0.3}{3}=0.183$ m acc. (59);

$N_c=\frac{0.503}{0.183^2\cdot 1.6}=9.39$ acc. (58);

$\bar{M}_{m,t}=\frac{0.503\cdot 0.8\cdot 2500}{2.38}=423 kg_{Fe}$ acc. to (60);

$M_{k,t}=\frac{12\cdot 423}{0.434\cdot 100}=117 kg_k$ acc. to (61).

$\mu_t=\frac{200\cdot 243}{309850\cdot 7000\cdot \ln\frac{1610-1150}{1610-243-1150}}=2.98\cdot 10^{-5}$ m/s acc. to (35);

Calculation of times $\tau_{\bar{r}}$;$\tau_{\bar{M}}$;$\tau_{n,m}$;τ_a;τ_r;

$\tau_{\bar{r}}=\frac{0.011}{2.98\cdot 10^{-5}}=369$ s acc. to (36);

$H_t=\frac{100\cdot 1.6\cdot swg(65369\cdot 238)}{10.32\cdot 6.786\cdot 2500}=0.8$ m acc. to (10) – verifying calculation;

$\mu_t=\frac{1.15}{5.5\cdot 7000}=2.99\cdot 10^{-5}$ m/s acc. to (63);

$\tau_{\bar{M}}=\frac{423}{1.15}=368(=\tau_{\bar{r}})$;$\tau_{n,m}=\frac{400}{1.15}=348$ s (5.8 min) acc. to (66);

$\tau_a=\frac{0.5\cdot 0.05}{2.98\cdot 10^{-5}}=839$ s (14 min) acc. to (68);

$\tau_r=\frac{0.0176}{2.98\cdot 10^{-5}}=591$ s (9.8 min) acc. to (69);

$\tau_r=839\cdot 0.706=592$ s acc. to (71); $\tau_{\bar{r}}=839\cdot 0.434=365$ acc. to (73).

Calculations of geometric and kinetic parameters of zone layers

$n_w=\frac{46.4}{9.39}=4.94$ acc. to (74); assumed n_w = 5 and corrected N_c:

$N_c=\frac{46.4}{5}=9.28; z=\frac{0.05}{5}=0.01$ m acc. to (76);

calculation of $v_{m,i}$, $V_{m,i}$ and $M_{m,i}$ acc. to (75), (77) and (78); i=1: $v_{m,1}=2.602 \cdot 10^{-3}$ m^3; $V_{m,i}=9.28 \cdot 2.602 \cdot 10^{-3}$ 0.02415 m^3;

$M_{m,1}=7000 \cdot 0.02415=169$ kg.

The calculated masses of metal in subsequent layers:

$M_{m,2}=120.6$ kg; $M_{m,3}=79.4$ kg; $M_{m,4}=42.6$ kg; $M_{m,5}=13.2$ kg; sum of metal masses in layers is 424.8 kg (sum $\bar{M}_{m,t}=423$ kg)

Calculation of volume and bulk height of metal in layers acc. to (79) and (80): i=1: $V_{m,n,1}=\frac{169}{2500}=0.0676$ m^3; $h_{m,n,1}=\frac{0.0676}{0.503}=0.134$ m

The calculated volumes and bulk metal heights in subsequent layers are: $V_{m,n,2}=0.0482$; $V_{m,n,3}=0.0318$; $V_{m,n,4}=0.017$; $V_{m,n,5}=5.28 \cdot 10^{-3}$m^3; $h_{m,n,2}=0.096$ m; $h_{m,n,3}=0.063$ m; $h_{m,n,4}=0.0339$ m; $h_{m,n,5}=0.0105$ m. The total height of metal layers equals to $h_{m,n}=$ 0.337 m $\left(\frac{\bar{M}_{m,t}}{F_{\dot{z},t}\rho_{n,m}}=\frac{423}{0.503 \cdot 2500}=0{,}336 \text{ m}\right)$.

The calculated volume and mass fractions of coke with regard to metal:

$M_{k,w}=\frac{117}{5}=23.4$ kg acc. to (81);

$V_{k,w}=\frac{23.4}{500}=0.0468$ m^3;

$h_{k,w}=\frac{0.0468}{0.503}=0.093$ m acc. to (83);

$K_{v,1}=\frac{0.0468}{0.0676}=0.692$ acc. to (84);

$K_{v,2}=\frac{0.0468}{0.0482}=0.97$; $K_{v,3}=\frac{0.0468}{0.0318}=1.47$; $K_{v,4}=\frac{0.0468}{0.017}=2.75$;

$K_{v,5}=\frac{0.0468}{5.28 \cdot 10^{-3}}=8.86$; $K_{w,1}=\frac{23.4}{169}$ 100=13.8 kg$_k$/100 kg$_{Fe}$ acc. to (85);

$K_{w,2}=\frac{23.4}{120.6}$ 100=194; $K_{w,3}=\frac{23.4}{79.4}$ 100=295; $K_{w,4}=\frac{23.4}{42.6}$ 100=55;

$K_{w,5}=\frac{23.4}{13.2}$ 100=177 kg$_k$/100 kg$_{Fe}$

$U_{m,1}=\frac{169}{423}$ 100=40%; $U_{m,2}=\frac{120.6}{423}=28.5\%$; $U_{m,3}=\frac{79.4}{423}$ 100=18.8%

$U_{m,4}=\frac{42.6}{423}$ 100=10.1%; $U_{m,5}=\frac{13.2}{423}$ 100=3.12%.

Commentary: The bulk volumes of coke related to the bulk volumes of metal increase in the direction of lower boundary of the zone from value 0.692 up to value 8.86 (13 times); mass fractions of coke related to mass fractions of metal also grow from 13.8 up to 177 kg$_k$/100 kg$_{Fe}$ (13 times also). However, metal masses in individual layers decrease with respect to summary metal mass in the zone from 40% do 3.12 mass %

The calculated development surfaces of metal charge pieces in individual layers as well as melting efficiencies of individual layers: surfaces $f_{m,i}$ acc. to (87) in m^2 and $F_{m,i}$ acc. to (90) in m^2:

$f_{m,1}=\frac{0.18+0.15}{2}=0.165$; $f_{m,2}=0.145$; $f_{m,3}=0.13$; $f_{m,4}=0.11$;
$f_{m,5}=0.045$; $F_{m,1}=9.28 \cdot 0.165=1.53$; $F_{m,2}=1.346$; $F_{m,3}=1.206$;

$F_{m,4}=1.020$; $F_{m,5}=0.418$; calculated efficiencies of layers acc. to (91): $s_1=2.98 \cdot 10^{-5} \cdot 1.53 \cdot 7000=0.319$ kg/s; $s_2=0.281$; $s_3=0.252$; $s_4=0.213$;
$s_5=0.087$; sum of efficiencies: $S_c=1.152$ kg$_{Fe}$/s; $S_F=\frac{1.152}{0.503}=2.29$ kg/(m^2·s); calculation of τ_z acc. to (92): $\tau_z=\frac{0.5 \cdot 0.05}{2.98 \cdot 10^{-5} \cdot 5}=167.8$ s

The passage of metal layer boundaries acc. to (94)
$W_{t,1}=\frac{0.319}{0.503 \cdot 2500}=2.54 \cdot 10^{-4}$ m/s; $W_{t,2}=2.235 \cdot 10^{-4}$; $W_{t,3}=2.004 \cdot 10^{-4}$; $W_{t,4}=1.694 \cdot 10^{-4}$; $W_{t,5}=6.92 \cdot 10^{-5}$; sum: $9.165 \cdot 10^{-4}$m/s;
$W_{t,m}=\frac{2.283}{2500}=9.132 \cdot 10^{-4}$m/s acc. to (95); $h_{m,n}=9.132 \cdot 10^{-4} \cdot 365=0.333$ m (verification).

Commentary: Metal pieces in each zone melt down by thickness layer z =0.01 m during $\tau_z=168$ s; time spent by the pieces in the zone is equal to: $\tau_a=5 \cdot 168=840$ s. In each zone layer the coke charge mass is $M_{k,w}=\frac{117}{5}=23.4$ kg; burning time of the coke mass equals: $\tau_k=\frac{M_{k,w}}{P_F F_{\dot{z},t}} L_{k,4}=\frac{23.4}{1.6 \cdot 0.503}$ 5.84=169 s,
where: $L_{k,4}=4.45(1+0.525)0.86=5.84$; height of coke mass $M_{k,w}$ is: $h_{k,n}=\frac{M_{k,w}}{F_{\dot{z},t}\rho_{n,k}}=\frac{23.4}{0.503 \cdot 500}=0.096$ m; the height lowering velocity of combustion zone $W_s=\frac{P_F}{L_{k,4}\rho_{n,k}}=\frac{1.6}{5.84 \cdot 500}=5.48 \cdot 10^{-4}$ m/s; in time τ_z, height of combustion zone lowers by value $h_{k,s}=W_s \cdot \tau_z=5.48 \cdot 10^{-4} \cdot 169=0.093=h_{k,n}$.

Equality of times τ_z and τ_k as well as equality of heights $h_{k,s}$ and $h_{k,n}$ means, that the procces of coke passage from the melting zone to the combustion one is a continuous process; during loosing thickness "z" due to melting of metal pieces; mass of coke equal to the height of coke burnt in the combustion zone. The calculated continuity of the process increases together with the decrease of values "z" and the growth of values n_w

The following model of stabilization of melting zone height results also from the presented calculations: metal mass in zone $\bar{M}_{m,t}=423$ kg; coke mass in zone $M_{k,t}=117$ kg; $\tau_{\bar{r}}=365$ s; $\tau_a=839$

s; mass of metal melted in time τ_a: $M_{m,a} = \tau_a S_{\dot{z}}$=839·1.148=963.2 kg; difference: $\Delta M_{m,a}=M_{m,a}-\bar{M}_{m,t}$=963.2-423=540.2 kg; time of mass melting $\Delta M_{m,a}$: $\frac{540.2}{1.148}$ =470 s; coke mass used up for the mass melting $\Delta M_{m,a}$: $\Delta M_{k,a}=\frac{540.2 \cdot 12}{100}$=64.8 kg of coke; burning time of coke mass $\Delta M_{k,a}= \frac{64.8}{1.6 \cdot 0.503}$5.84=470 s; as well as balance of passes of metal and coke: in time τ_a, 963,2 kg_{Fe} passes to the zone and melts down, out of which 423 kg is comsumed for a total renewal of the zone and 540.2 kg for the additional mass increase of the melted down metal. The metal passing from the heating zone supplies 117 kg of coke, which is precisely as much coke is present in the zone; coke in the zone melts down the zone metal as well as the metal excess ΔM_{ma}.

7. Conclusions

The set of equations for the calculation of melting height zone as well as the following parameters describing its structure: development surface of the melting metal pieces; linear velocity of melting; number of melting metal lumps and number of their series: number of metal and coke layers in the zone; masses, volumes and melting metal heights in individual zone layers; melting efficiency of the whole zone and others have been presented in the work.

The following model of the melting process and movements of metal and coke throughout the zone results from the derived set of equations:

- a constant number of melting metal lumps n_m of total mass $\bar{M}_{m,t}$ are in the zone; mass $\bar{M}_{m,t}$ formed from a greater mass $M_{m,t}$, which passed from the heating zone undergoing partial melting, at which $\bar{M}_{m,t}=M_{m,t}\varphi_v$,
- mass of charge coke is $M_{k,t}$ and it is equal to the coke mass, which passed from the heating zone together with the metal,
- coke to metal ratio in the melting zone is higher than that in the heating zone, which results from inequality:$\bar{M}_{m,t} < M_{m,t}$
- mean linear velocity of melting metal charge pieces μ_t is equal and constant for the whole zone and its layers,
- melting efficiency of individual layers decreases in the direction of its lower boundary, because their surface of development decreases; total efficiency of all zone layers is equal to the zone efficiency, calculated according to the derived formulas or following the Buzek's formula (compatibility of formulas),
- in time τ_a (true melting time of an individual metal lump), the melting zone melts down $M_{m,t}$ kg of metal, which passes from the heating zone bringing about the exchange of metal mass $\bar{M}_{m,t}$ in the zone as well as additional melting of the metal mass $\Delta M_t= (M_{m,t} - \bar{M}_{m,t}) = M_{m,t}(1-\varphi_v)$; mass $M_{m,t}$ can be calculated from formula: $M_{m,t} = \tau_a\ S_{\dot{z}}$,
- masses of metal in individual layers of zone decrease fast in the direction of the lower zone boundary, which effects in the growth of coke to metal ratio at unchanged coke mass; in the lower part of the melting zone a layer of coke appears containing fine pieces of metals, which are about to end the melting process. There are favorable conditions for the proceeding of CO_2 reduction reaction in this layer, which can be considered as a part of the main reduction zone.

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