A R C H I V E S O F M E T A L L U R G Y A N D M A T E R I A L S
Volume 54 2009 Issue 4

A. GRADOWSKI*

THE NUMERICAL SIMULATION OF THE HEATING SUB-AREAS OF A TRADITIONAL CUPOLA AND HEAT LOSSES TO THE ENVIRONMENT

NUMERYCZNA SYMULACJA PROCESU NAGRZEWANIA PODOBSZARÓW ŻELIWIAKA TRADYCYJNEGO ORAZ STRAT CIEPŁA DO OTOCZENIA

The present paper focuses on the idea of the numerical analysis of the heating processes of the sub-areas of a cupola. The calculation algorithm is based on the radial, single-dimensional model. The developed numerical model includes the conjugation of thermal conduction, free convection and radiation processes in the examined system.
This enables the analysis of the variation of the broad range of parameters that describe the course of thermal exchange in the examined system.

Definitions of new heat parameters were introduced to facilitate the multi-variant heat and economic analysis of the cupola process. The developed numerical model enabled the construction of a program for a computer simulation of the process under investigation. The results of numerical experiments were compared to the data available in literature [2].

Keywords: numerical simulation, cupola, heat exchange, convection, radiation

Praca zawiera koncepcję numerycznej analizy procesu nagrzewania podobszarów żeliwiaka tradycyjnego. Algorytm obliczeniowy oparto na modelu radialnym, jednowymiarowym. Skonstruowany model numeryczny ujmuje sprzężenie zachodzących w badanym układzie procesów przewodzenia ciepła, konwekcji swobodnej i promieniowania. Umożliwia analizę zmienności podstawowych parametrów opisujących przebieg wymiany ciepła w badanym układzie.

Wprowadzono definicje nowych parametrów cieplnych, ułatwiających wielowariantową analizę cieplną i ekonomiczną przebiegu procesu żeliwiakowego. Opracowany model numeryczny umożliwił skonstruowanie programu komputerowej symulacji badanego procesu. Wyniki eksperymentów numerycznych porównano z danymi uzyskanymi przez autorów pracy [2].

1. Introduction

The paper is devoted to elaboration of the numerical model describing heat losses during the cupola process.

The economic optimization of the work parameters of a cupola requires knowledge of the elements of the heat balance of the operation of a cupola. One of the important elements is the value of heat losses to the walls of the cupola and the losses from giving off heat to the environment. The concept of the simplified model stated in the title of the paper refers to the method of understanding the complex heat exchange in the furnace shaft by means of the established type 1 boundary condition. The present paper pertains to a traditional cupola where heat exchange on the outer surface is completed by means of free convection and radiation. The number of scientific papers concerning heat losses connected with the heating of walls of the cupola (1, 2, 3, 4) is relatively low, which is probably due to the theoretical difficulties of the quantitative view of the course of such a process.

The solution of the analyzed problem requires the simultaneous treatment of the heat conduction process in a multi-layered cupola wall and the process of radiation and free convection at the changing type 3 boundary conditions. The construction of a model of the numerical simulation of the process should be preceded by the identification of the so-called conditions of uniqueness, where the most important ones are geometrical, physical and boundary conditions. For that reason, it is necessary to establish the heat parameters of the following main structural elements of the cupola wall:

a) the layer of refractory lining, usually chamotte,
b) the expansion gap filled with loose insulation material,

* FACULTY OF FOUNDRY ENGINEERING, DEPARTMENT OF FOUNDRY PROCESSES ENGINEERING, AGH-UNIVERSCITY OF SCIENCE AND TECHNOLOGY, 30-059 KRAKÓW, 23 REYMONTA STR., POLAND

c) the outside jacket of the cupola.

In the present paper, the concept of heat losses is defined as the sum of heat accumulated in the 3-layer cupola wall and the heat lost at the cupola's outer surface by means of radiation and convection. The analysis of the types of heat losses is presented in Section 4 of the present paper.

The base to geometrical and physical analysis of cupola sub-areas was work [5], containing simplify solutions for heat transfer processes following from assumptions different to numerical model used in this article.

2. Assumptions of the numerical model

For a comprehensive view of the various heat processes in the system under investigation, the numerical simulation method was chosen. The calculation algorithm was based on a single-dimension numerical model using radial differential digitization of the system, with the predicted presentation of calculations in the visual and graphic form thanks to saving the calculations in the form of a file that allowed for their numerical and graphical processing using different applications (e.g. Excel).

Difference equations necessary for the numerical simulation of a single-dimensional heating process of a multi-layered cylindrical wall come from the so-called method of elementary balance and can be found in the literature (6, 7, 8, 9).

The basic assumptions of a mathematical model of the process are following:

a) The examined system includes cylindrical, 3-layered walls of the cupola heated from the inside according to the established 1 type boundary condition,
b) Heat exchange with the environment is completed by means of free convection with changing, un-specified type 3 boundary conditions with a simultaneous participation of the process of radiation,
c) The elements of the cupola walls do not undergo any dimensional changes during the heating, i.e. the effect of chemical erosion is eliminated,
d) In the area of the usable height of the cupola the temperature of the internal surface is fixed,
e) In the porous area of the expansion gap only the process of heat conduction occurs and the potential influence of radiation is low and thus can be omitted.

For the mathematical presentation of the variation of the basic thermo-physical parameters that de-scribe the free convection process, an analysis of the data from the literature was performed (10, 11, 12, 13, 14). The analysis resulted in the development of the following functions describing the changing of parameters for dry air (temperatures t_C given in °C, temperature range 0 to 1000°C):

a) thermal conductivity of air

$$\lambda_{air} = (241 + 0.706 \cdot t_c - 0.00016 \cdot t_c^2) \cdot 10^{-4} \quad [W/(mK)]$$

b) kinematic viscosity of air

$$\nu_{air} = (13.3 + 0.098 \cdot t_c + 0.000081 \cdot t_c^{1.97}) \cdot 10^{-6} \quad [m^2/s]$$

c) specific heat at a fixed pressure:

$$C_{air} = 1005 + 0.18 \cdot t_C \quad [J/(kgK)]$$

d) density in the function of temperature (°C)

$$\rho_{air} = \frac{352}{t_c + 273} \quad [kg/m^3]$$

e) coefficient of thermal expansion

$$\beta_{air} = \frac{1}{t_c + 273} \quad [1/K]$$

f) Prandtl number

$$Pr_{air} = \frac{3538 + 0.6336 \cdot t_c}{t_c + 273} \cdot \frac{13.3 + 0.098 \cdot t_c + 0.000081 \cdot t_c^{1.97}}{241 + 0.706 \cdot t_c - 0.00016 \cdot t_c^2}.$$

The relations obtained were used in simulation software for a free, vertical flow of air along the out-side wall of the cupola which physically matches the process of heat exchange of natural convection (15, 16, 17). For the preliminary determination of the Grashof number, a value of the characteristic dimension was used comparable with the usable height of a typical cupola, i.e. D_{ch} = 4 m.

The product Gr.Pr [13, 15, 16, 17] that determines the algorithm for calculating the heat exchange by way of convection exceeds the boundary value (Gr Pr = 2^*10^7) as early as the stowed temperature at the outside surface of the cupola higher than 0.004 K (T_{os} > 20.004°C). The mathematical consequence of that is the lack of influence of the characteristic dimension on the parameters of the convection process of heat [15, 17]. The table below shows the variation of:

a) Value of Gr * Pr numbers,
b) Coefficients of heat exchange by way of convection (α_{con}), radiation (α_{rad}) and the value of the coefficient of effectiveness α_{ef} (summary).

T_{os}, °C	20.004	21	25	30	50	100	300	500	1000
$GrPr * 10^{-7}$	2.6	645	3123	6002	15400	–	–	–	–
α_{con}	0.26	1.62	2.76	3.45	4.87	6.40	8.19	8.59	8.51
α_{rad}	4.56	4.59	4.68	4.80	5.31	6.80	16.3	33.0	121.2
α_{ef}	4.82	6.21	7.44	8.26	10.2	13.2	24.5	41.6	129.7

3. Data for numerical calculations

For the preliminary calculations a base (model) cupola was chosen with a shaft diameter (internal diameter) D_i = 800 mm. The data from the literature make it possible to use the assumed diameter of the shaft for the arbitrary determination of essential base dimensions of a model cupola, which are given in Table 1.

The approximate value of a melting rate of the studied cupola S_h was defined for the assumed the amount of air V_b = 6000 m^3/ (m^2 s) and for the melting ratio at 12%, which matches the specific melting rate S_{sh} = 8570 kg/ (m^2 h).

TABLE 1

Selected geometric parameters of the examined model cupola

Internal diameter	Thickness of refractory	Thickness of expansion gap	Thickness of the steel jacket	Usable height of the cupola	Approximate effectiveness S_h
D_i, mm	X_r, mm	X_g, mm	X_s, mm	H_u, mm	kg/ h
500	156	18.8	9.4	2375	1680
800	245	20	10	3800	4300
1200	368	20	10	5700	9700
1600	450	20	10	7600	17200

For calculating the parameters of the process of free convection, the assumed value of the characteristic dimension of the system was equal to the usable height of the cupola H_u, proportional to the internal diameter of the cupola, i.e. H_u/ D_i = 4.75.

For determining the thickness of refractory on the cupola wall, the geometric similarity rule was used, without any changes to the thickness of the expansion gap and the steel jacket (with a minimum deviation of 500 mm for the diameter).

The parameters used to describe the physical and initial conditions are described by the following data:

a) The used temperature of the internal surface of the cupola was 1600°C, at the one of ambient temperature was 20° C,
b) The thermo-physical properties of the refractory of the cupola wall [9]: λ_r= 1.45 W/(m K); c_r = 1260 J/(kg K), ρ_r = 1900 kg/ m^3
c) The parameters of the expansion gap are due to the use of quartz sand: λ_g = 0.4 W/(m K); c_g = 980 J/(kg K), ρ_g = 1300 kg/ m^3
d) Thermo-physical properties of the steel jacket of the cupola: λ_s = 44 W/(m K); c_s = 480 J/(kg K), ρ_s = 7900 kg/ m^3

To obtain the effective coefficient of heat exchange with the environment, taking into account the processes of convection and radiation that take place, the value of the coefficient of radiation emission matching the area of the oxidized steel sheet was taken, i.e. ε = 0.8. The structure of the software takes into account the possibility of expansion for the mathematical view of the variation of the thermo-physical parameters of the refractory and the material of the expansion gap owing to the temperature.

4. Results of the numeric simulation of the process

The values of geometrical parameters used for the numerical calculations are shown in Table 1. The calculation program makes it possible to obtain the following results of the numerical simulation of heat losses in the analyzed system:

a) Temperature variation of heat exchange for free convection and radiation,
b) The accumulated heat and the heat wasted to the environment referenced to the individual height of the cupola [MJ/m],
c) Total accumulated heat and the heat wasted to the environment referenced to the area of usable height [MJ],
d) The influence of the thickness of the refractory lining

and the expansion gap on the accumulated heat and the heat wasted to the environment,

e) The influence of the internal diameter of the cupola on the total and individual accumulated heat and the heat wasted to the environment,

f) Undetermined temperature area in the sub-areas of a 3-layer cupola wall,

g) Temperature variation of the external surface of the steel jacket.

The results of the calculation software can be stored in a file in a tabular form or after processing in the form of simple or web charts. Table 2 presents the data illustrating the course of the process of heating the model cupola with the shaft diameter D_i = 800 mm by means of the temporal variation of heat parameters:

1. the unitary heat accumulated in the 3-layer cupola wall referenced to the unit of usable height Q_{1m}, [MJ/ m],
2. the unitary heat losses to the environment through the external surface of cupola self-cooling by means of radiation and free convection at the changing type 3 boundary conditions (referenced to the unit of usable height) Q_{2m}, [MJ/ m],
3. the unitary heat losses for the heating of a cupola (heat losses for the heating of the cupola and losses to the environment $Q_{3m} = Q_{1m} + Q_{2m}$), Q_{3m}, [MJ/ m],
4. the heat losses for the heating of a cupola, referenced to a unit of internal shaft area, Q_{3a}, [MJ/ m^2],
5. the amount of heat losses for the heating of the cupola, referenced to value of usable height of a cupola Q_c, [MJ],
6. the temperature of the outside surface of the steel jacket of a cupola T_{os}, [°C],

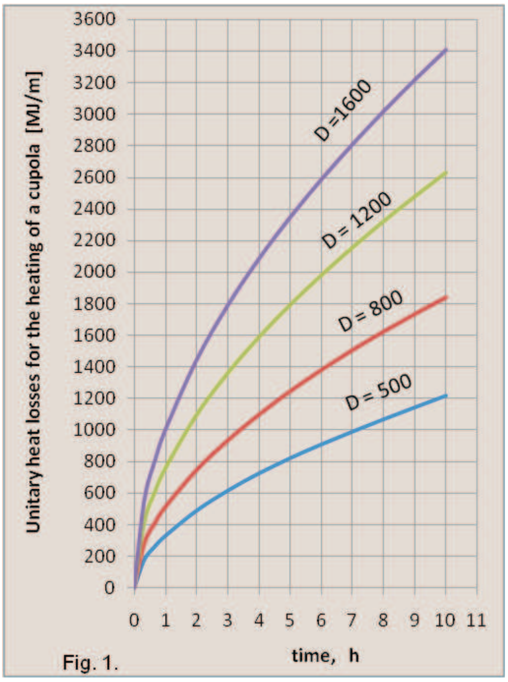
The effective coefficient of heat exchange at the outside surface of the cupola taking into account the simultaneous processes of free convection and radiation α_{ef} [W/ (m^2 K)].

TABLE 2

The results of the numerical simulation of the heat exchange process for the system: cupola – environment

Time	Q_{1m}	Q_{2m}	Q_{3m}	Q_{3a}	Q_c	T_{os}	α_{ef}
h	MJ/ m	MJ/ m	MJ/ m	MJ/ m^2	MJ	°C	W/ (m^2 K)
1	513	0.00	513	204	1953	20.02	5.02
2	747	0.58	747	297	2841	23.3	7.04
3	930	4.14	934	372	3551	39.1	9.27
4	1084	12.9	1097	437	4171	66.6	11.30
5	1216	28.2	1244	495	4731	98.0	13.09
10	1637	202	1839	732	6992	208	18.86
15	1820	487	2307	918	8766	250	21.26
20	1896	822	2719	1082	10333	266	22.24

A graphical presentation of the results of the numerical simulation of the unitary heat losses for the heating of a cupola and amount of heat losses for the heating of different internal diameters is shown in Fig. 1, 2.



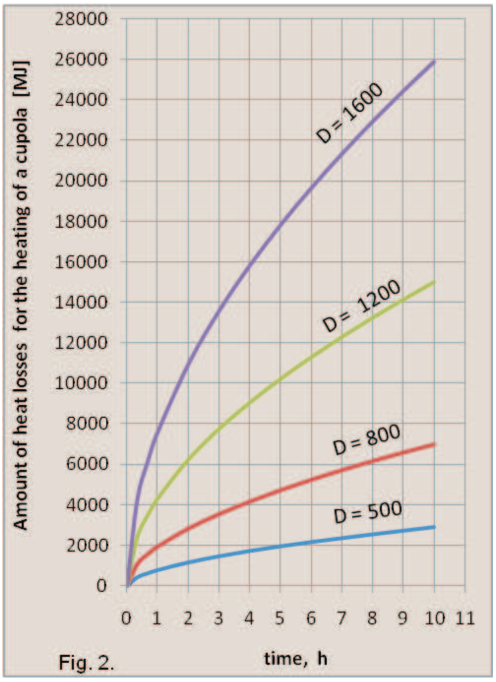


Fig. 1. Unitary heat losses for the heating of a cupola (Q_{3m}) for the changing its internal diameters at temperature of the internal surface of 1600°C ($Q_{3m} = Q_3 / H_u$).

Fig. 2. Fig. 2. Amount of heat losses for the heating of a cupola (Q_3) for the changing its internal diameters at temperature of the internal surface of 1600°C

According to the initial assumption, in the present paper the idea of heat losses is understood as the sum of the heat used for the heating of the cupola and the heat wasted to the environment. The complete analysis of the course of the studied heat processes requires the formulation of some additional concepts. All the concepts (parameters) defined below pertain to the heat losses referenced to the "working area" of the cupola, i.e. the subarea limited by the value of its usable height.

The concept of *temporary rate of heat losses* [W] (power of heat losses) is defined as the relation of the increase of the lost heat to the increase of time, i.e. the derivative of a theoretical curve of the lost heat Q_c in a function of time. This parameter describes the kinetics of heat losses for a specific moment in time. According to the accepted definition, the momentary heat losses can be recorded using the following expression:

$$U_\tau = \frac{dQ_c}{dm_t} \quad [\mathrm{W, MJ/h}]. \tag{1}$$

The momentary heat losses are probably impossible to indicate by means of experimental research but there are no difficulties for analyzing their course using the numerical simulation method.

It is worth noting that there is a possibility to interpret this concept differently in relation to the increase in the mass of melted cast iron which results from the general melting rate (efficiency) of a cupola S. Taking into account the time interval necessary to obtain the first portion of liquid cast iron, the mass of melted metal could be determined with some approximation using the following relationships:

$$m_t = S_h(\tau - \tau_0), \qquad [\mathrm{kg}] \tag{2}$$

where:

τ_0 – initial time when the first portion of liquid metal is obtained.

For all the following considerations, the value of this time will be assumed as 1 hour.

Temporary mass heat losses is defined as the relation of the increase of the lost heat Q_c in reference to the increase of the mass of the molten metal, i.e.

$$U_m = \frac{dQ_c}{dm_t}, \qquad [\mathrm{J/kg}] \tag{3}$$

The next concept used to describe the analyzed processes is the *average mass heat loss*, that is the relation of summary waste heat to the total weight of the molten metal:

$$Q_t = \frac{Q_c}{m_t}, \qquad [\mathrm{J/kg, MJ/100kg}] \tag{4}$$

where:

m_t – total mass of the molten cast iron.

In the literature on the analyzed problem, the concepts used for the determination (estimation) of heat losses for the heating of a cupola are described by equation (3) or (4), with a "charge" of the mass 100 kg used in place of the unit of weight.

TABLE 3

The values of different types of heat lost due to: the unitary heat accumulated in the walls of a cupola (Q_{1m}), unitary heat losses to the environment (Q_{2m}), summary unitary heat loss for the heating of a cupola (Q_{3m}) and average mass heat loss in cupola process (Q_t) in a function of thickness of refractory lining and the time of work of a cupola (in case of a model cupola additionally temporary rate of heat losses U_τ)

X_r, mm	Heat parameters	Unit	Duration of the process [h]							t_e
			1h	2h	4h	6h	8h	10h	20h	
60	Q_{1m}	MJ/ m	489	603	637	638	638	638	638	4 h
	Q_{2m}	MJ/ m	31.4	125	373	632	891	1150	2442	
	Q_{3m}	MJ/ m	520	728	1010	1270	1529	1788	3080	
	Q_t	MJ/ 100 kg	–	64.2	29.7	22.4	19.3	17.5	14.3	
	T_{os}	°C	217	396	445	447.2	447.2	447.2	447.2	
	α_{ef}	W/ (m^2 K)	19.4	31.8	36.1	36.3	36.3	36.3	36.3	
90	Q_{1m}	MJ/ m	510	699	827	850	855	855	855	8 h
	Q_{2m}	MJ/ m	10.6	61.0	243	461	687	914	2050	
	Q_{3m}	MJ/ m	521	760	1070	1312	1542	1770	2905	
	Q_t	MJ/ 100 kg	–	67.1	31.5	23.2	19.4	17.3	13.5	
	T_{os}	°C	94.0	264	383	401	404	405	405	
	α_{ef}	W/ (m^2 K)	12.3	22.1	30.6	32.1	32.4	32.4	32.4	
120	Q_{1m}	MJ/ m	513	732	953	1030	1056	1064	1068	10 h
	Q_{2m}	MJ/ m	3.14	28.1	147	321	515	716	1732	
	Q_{3m}	MJ/ m	516	760	1101	1352	1571	1780	2800	
	Q_t	MJ/ 100 kg	–	67.1	32.4	23.9	19.8	17.5	13.0	
	T_{os}	°C	42.5	154	306	351	364	368	370	
	α_{ef}	W/ (m^2 K)	9.05	16.1	24.9	28.1	29.1	29.5	29.6	
180	Q_{1m}	MJ/ m	513	745	1058	1245	1352	1413	1486	15 h
	Q_{2m}	MJ/ m	0.17	5.05	48.7	139	246	408	1235	
	Q_{3m}	MJ/ m	514	750	1107	1384	1616	1821	2721	
	Q_t	MJ/ 100 kg	–	66.2	32.5	24.4	20.4	17.9	12.6	
	T_{os}	°C	21.2	48.0	159	235	274	294	317	
	α_{ef}	W/ (m^2 K)	6.3	10.0	16.2	20.4	22.8	24.1	25.6	
245	Q^{1m}	MJ/ m	513	746	1084	1330	1508	1636	1891	>20h
	Q_{2m}	MJ/ m	0	0.58	12.8	49.9	115	203	828	
	Q_{3m}	MJ/ m	513	747	1097	1380	1623	1839	2719	
	Q_c	MJ	1953	2841	4171	5245	6170	6992	10330	
	U_τ	MJ/ h	1050	773	588	486	432	396	–	
	m_t	kg	0	4310	12920	21540	30160	38770	81850	
	Q_t	MJ/ 100 kg	–	65.9	32.3	24.4	20.5	18.0	12.6	
	T_{os}	°C	20.02	23.3	66.6	128	176	208	266	
	α_{ef}	W/ (m^2 K)	5.02	7.04	11.3	14.6	17.1	18.9	22.2	
368	Q_{1m}	MJ/ m	513	747	1090	1362	1592	1788	2405	>20 h
	Q_{2m}	MJ/ m	0	0	0.65	5.43	18.6	42.7	329	
	Q_{3m}	MJ/ m	513	747	1091	1368	1611	1831	2734	
	Q_t	MJ/ 100 kg	–	65.9	32.1	24.1	20.3	17.9	12.7	
	T_{es}	°C	20.0	20.01	22.7	36.3	59.0	83.9	169	
	α_{ef}	W/ (m^2 K)	0	4.96	6.89	9.00	10.8	12.3	16.7	
490	Q_{1m}	MJ/ m	513	747	1090	1365	1603	1815	2610	>20 h
	Q_{2m}	MJ/ m	0	0	0.02	0.43	2.43	7.61	118	
	Q_{3m}	MJ/ m	513	747	1090	1365	1605	1823	2728	
	Q_t	MJ/ 100 kg	–	65.9	32.1	24.1	20.2	17.9	12.7	
	T_{os}	°C	20	20	20.08	21.5	26.4	35.1	94.8	
	α_{ef}	W/ (m^2 K)	0	0	5.26	6.44	7.70	8.87	12.9	

The concepts introduced are an important element of the numerical simulation software of the ana-lyzed heat processes and could be useful for analyzing the variation of the cost of a melt of a single unit of mass of metal for the function of the duration of the heat. The concept of mean heat loss contains an important component that includes the heat lost until the first drops of liquid metal are obtained. This justifies the usefulness of this component for the analysis of the variation of the cost of heat for a single unit of mass of metal in a function of the time of duration of heat.

The elaborated program of numerical simulation of the course of heat processes in a cupola enables the analysis of heat losses while utilizing all of the concepts defined above. The next series of calculations included the numerical simulation of the course of heat processes for a wide scope of variation in the thickness of lining with the unchanged expansion gap. The results of these calculations are presented in Table 3.

The conclusions from the analysis of the results of the numerical calculations shown in Table 3 are the following:

a) There exists a specific boundary value of a dimensionless thickness of the refractory lining ($X_r /D_i > 0.15$) above in which the variation of the heat used (losses) for the heating process of the cupola (Q_{3m}) is similar. This is due to the different course of both of its components (Q_{1m}, Q_{2m}),

b) Over the course of time the value of accumulated heat (Q_{1m}) stabilizing and the speed of heat losses to the environment rises at the same time; in the case of small thicknesses of the lining the value of accumulated heat is stabilized after the shorter duration of process compared to larger values of thickness, e.g. in the case of the thickness of lining of 60 mm (ca. 8% of shaft diameter) this occurs after ca. 4 hours, and for the thickness of 90 mm (ca. 12% of shaft diameter) this occurs after ca. 6 hours,

c) The summary value of the lost heat runs to a specific boundary that stems from the conditions re-quired to achieve the state fixed heat flow ; the approximate values of times necessary to obtain such a state (t_e) vary from 4 hours for the 60 mm thick lining to over 20 hours for the thickness over 240 mm,

d) The speed of the rise of the temperature of the outside surface of the cupola steel jacket depends on the thickness of the lining and rises when the thickness is lower,

e) For linings over 180 mm thick (20% of the shaft diameter), the courses of variation of summary heat (Q_{3m}) and average mass heat losses (Q_t) referenced to the mass of melt metal are very similar.

The second series of the numerical modeling of the process concerned influence of the refractory materials thermo-physical properties (heat accumulation and thermal conductivity) . The results of this series are collected in Table 4.

TABLE 4

he influence of the thermal and physical parameters of the insulating lining of the cupola

b_r	λ_r	Parameter	Unit	Time, h						
				1	2	4	6	8	10	20
1313	0.72	Q_{3m}	MJ/ m	360	519	753	939	1102	1248	1844
		Q_t	MJ/ 100kg	–	45.8	22.1	16.6	13.9	12.2	8.6
1695	1.2	Q_{3m}	MJ/ m	466	677	990	1243	1462	1658	2451
		Q_t	MJ/ 100kg	–	59.7	29.1	21.9	18.4	16.3	11.4
1863	1.45	Q_{3m}	MJ/ m	513	747	1097	1380	1623	1839	2719
		Q_t	MJ/ 100kg	–	65.9	32.3	24.4	20.5	18.0	12.6

As assumed, the increase of the coefficient of the heat conduction of the insulation lining (as a con-sequence of the coefficient of accumulation) causes the increase in the intensity of heat exchange in the system with the simultaneous increase of heat losses. The double decrease of the coefficient of the lining's heat conduction causes a drop of the average mass heat losses (Q_t) by about 30%, and this regularity is valid for the whole time interval of the study.

The numerical analysis of the broad scope of the results of the simulation of the analyzed process concerning the total heat lost to the walls and the environment of the cupola enabled the generalization of the results for the cupolas with the lining thicker than 20% of the shaft diameter. Passing over the quantitative view of the influence of the temperature of the internal surface of cupola, the amount of heat losses for the heating of the cupola can be approximately described using the following function of four variables:

$$Q_c = 0,345 \cdot b_r \cdot m_c \cdot D_i^{1,92} \cdot (\tau_h^{0,58} + 0,1) \text{.....[MJ]} \quad (5)$$

b_r – coefficient of heat accumulation of cupola insulation (refractory material),
m_c – slenderness of the internal, usable shaft area (H_u/D_i),
D_i – internal diameter [m],
τ_h – time [h].

This function after differentiation in relation to time enables the recording of the derivative describing the temporary rate of heat losses in the analyzed system:

$$U_\tau = \frac{dQ_c}{d\tau} = 0.181 \cdot b_r \cdot m_c \cdot D_i^{1,92} \cdot \tau_h^{-0,42} ....[\text{MJ/h}] \quad (6)$$

or for b_r = 1863 W $s^{1/2}$ / (m^2 K)

$$U_\tau = 337 \cdot m_c \cdot D_i^{1,92} \cdot \tau_h^{-0,42} ....[\text{MJ/h}]. \quad (7)$$

For the purpose of verifying the obtained results of numerical calculations, certain inconveniences should be noted connected with their confrontation with experimental data published in [1, 2], among other things owing to the lack of thermal and physical parameters of the sub-areas of the cupola. Other difficulties stem from the fact of the omission of the initial heating of the cupola stage in the measurement research, e.g. [1, 2]. For instance, the position [2] from the literature contains the presentation of the values of temporary heat losses as moderate for the range from 2 to 5 hours of operation of the cupola, and in the book [1] on the chart containing a time scope of 2 to 10 hours but without the basic theoretical assumptions necessary for its verification. This allows us to make the supposition that in the cited research the idea of heat losses does not include the component of losses for the initial stage of heating, which is difficult to determine.

In order to verify the obtained numerical results in spite of the difficulties we shall assume that the physical sense of the results of Patterson's research justifies the use of the idea of the temporary mass heat losses U_m [MJ/ kg]. The results of the measurement research [2, p.46] shown in the web chart concern a cupola with a diameter of 750 mm and a usable height of H_u = 4930 mm, with a mean specific melting rate at S_{sh} = 140 kg/ (m^2 min), which corresponds to general melting rate S_h = 3700 kg/ h. Based on that chart for the assumed time interval of 2 to 5 hours, the summary heat losses for the accumulation and radiation are obtained, which fall in the following limits:
19.5 - 31.4 MJ/ 100 kg_{Fe} (according to the original scale of the chart: 4650 - 7500 kcal/ 100 kg_{Fe}).

Using equation (6) for calculating the theoretical range of heat losses including the melting rate of the cupola and the linear rise of the mass of the melt metal in a function of time enables the calculation of the value of heat losses in relation to the mass of the melt metal.

When using a numerical-analytical method for the time interval assumed earlier, the following heat losses are obtained:

a) for the time of 2 hours: U_{m1} = 25.8 MJ/ 100 kg_{Fe} = 6160 kcal/ 100 kg_{Fe},
b) for the time of 5 hours: U_{m2} = 17.5 MJ/ 100 kg_{Fe} = 4190 kcal/ 100 kg_{Fe}.

The comparison between the numerical and experimental results ** shows the common range of the values of heat losses (heat lost for accumulation, radiation and convection) included in the boundaries 19.5 - 25.8 MJ/ 100 kg_{Fe} (4650 - 6160 kcal/ 100 kg_{Fe}).
This fact proves that the model assumptions used for creating the calculation algorithm were correct.

5. Summarize and final conclusions

1. The created algorithm and numerical model include the conjugation of thermal conduction, free con-vection and radiation processes in the studied system. The developed numerical program enables the analysis of the variation of a broad range of parameters that decide about the course of heat losses in the cupola process.
2. The results of the numerical simulation of heat flow in the system make it possible to present them in the form of tables, charts and monograms, which enables a multi-variant optimization of a cupola design to minimize the general heat losses and the parameters that relate these losses to the mass of the metal in the melt.
3. The developed calculation program enables the simulation of the examined process including the in-fluence of the following factors:
   a) internal diameter of the cupola (shaft),
   b) mean temperature of the internal surface of the cupola in the area of usable height,
   c) thermo-physical parameters (heat resistance) of the refractory lining and expansion gap,
   d) parameters of the process of convection and radiation at the external boundary of the examined system.
4. For a comprehensive view of the problem of heat losses in a cupola process, the definitions of some new parameters were introduced in the present paper that facilitate the multi-variant analysis of both heat and economic analysis. They also enable the com-

parison of the results obtained by different authors who used different research ideas.
5. The comparison of the results of numerical modeling of the process with the selected experimental data from the literature show a common range of heat losses, which proves that the model assumptions used were correct.

The algorithms elaborated during this work will be developed during the next investigations, taking into account more complex model of the process, results of which will be published in a close future.

Final results of numerical analysis of the investigated processes is a computer Cupola-T.exe, of which J. S. Suchy is a co-author.

Important symbols used in the text

A_s – internal, convention shaft area referenced to usable height, [m^2],
b_r – coefficient of heat accumulation of cupola insulation (refractory material), [W s$^{1/2}$/ (m^2 K],

$$b_r = \sqrt{\lambda_r\, c_r\, \rho_r}$$

c_{air} – specific heat of air, [J/ (kg K)],
c_r – specific heat of refractory the cupola wall, [J/ (kg K)],
D_i – internal diameter (shaft diameter), [m],
D_{ch} – characteristic dimension (height) of a cupola, [m],
G_r – Grashof number,
H_u – usable height of the cupola,
K_r – melting ratio (coke ratio), [%],
m_c – slenderness of the usable area of the cupola (H_u/ D_i),
m_t – total mass of the molten metal, [kg],
Pr_{air} – Prandtl number,
Q_{1m} – the unitary heat accumulated in the 3-layer cupola wall referenced to the unit of height (1 m) of the cupola, [MJ/ m],
Q_{2m} – the unitary heat losses to the environment through the external surface of cupola self-cooling by means of radiation and free convection at the changing type 3 boundary conditions (referenced to the unit of usable height), [MJ/ m],
Q_{3m} – the unitary heat losses for the heating of a cupola (heat losses for the heating of the cupola and losses to the environment $Q_{3m} = Q_{1m} + Q_{2m}$), Q_{3m}, [MJ/ m],
Q_{3a} – heat losses for the heating of a cupola, referenced to a unit of internal shaft area, [MJ/ m^2],
Q_c – amount of heat losses for the heating of the cupola, referenced to value of usable height of a cupola, [MJ],
Q_t – average mass heat losses in cupola process, [J/ kg, J/ 100 kg],
S_h – melting rate of a cupola, [kg/ h],
S_{sh} – specific melting rate ;of a cupola, [kg/ (m^2 h)],
t_e – approximate values of time to achieve the state fixed heat flow, [h],
T_{is} – temperature at the internal surface of the cupola, °C,
T_{os} – temperature at the outside surface of the cupola, °C,
U_m – temporary mass heat losses, [J/ kg], [dQ_c /d τ],
U_τ – temporary rate of heat losses [W],
V_b – amount of air blast (blast volume), [m^3/ (m^2 s)],
X_r – thickness of refractory lining (chamotte), [m, mm],
X_g – thickness of expansion gap, [m, mm],
X_s – thickness of steel jacket (shell), [m, mm],
α_{con}, α_{rad}, α_{ef} – coefficients of heat exchange by way of convection, radiation and the value of effectiveness α_{ef} (summary), [W/ (m^2 K)],
β_{air} – coefficient of thermal expansion of air, [K^{-1}],
ε – coefficient of radiation emission,
λ_{air} – thermal conductivity of air, [W/ (m K)],
λ_r – thermal conductivity of refractory material, [W/ (m K)],
ν_{air} – kinematic viscosity of air, [m^2/ s],
ρ_{air} – density of air, [kg/ m^3],
ρ_r – density of refractory lining, [kg/ m^3],
τ – time [s],
τ_h – time in hour [h],
τ_0 – initial time when the first portion of liquid metal is obtained.

Acknowledgements

This paper was supported by Polish Ministry of Science and Higer Education, own research project. No. **N N508 469234** (AGH University project no. **18.18.170.327**).

REFERENCES

[1] M. C z y ż e w s k i, O oszczędności koksu w żeliwiaku. Przegląd Odlewn. nr 2/1952.

[2] W. P a t t e r s o n, H. S i e p m a n n, H. P a c y n a, Die Stoffbilanz und Wärmebilanz des Kaltwind-Kupolofens. MKO, Wiedeń 1961, s. 29-52.

[3] M. C z y ż e w s k i, Szybkość ogrzewania i topienia metalu w zależności od warunków pracy żeliwiaka. Hutnik nr 1/ 1948.

[4] W. L o n g a, Analiza i korekta wykresu M. Czyżewskiego do określania strat cieplnych na ogrzanie żeliwiaków koksowych. Przegląd Odlewnictwa 5/6 (2009).

[5] Cz. P o d r z u c k i, J. S z o p a, Piece i urządzenia metalurgiczne stosowane w odlewnictwie. Wyd. Śląsk. Katowice 1983.

[6] B. M o c h n a c k i, J. Sz. S u c h y, Modelowanie i symulacja krzepnięcia odlewów. PWN. W-wa 1993.

[7] J. S z a r g u t, Metody numeryczne w obliczeniach cieplnych pieców przemysłowych, Wyd. Śląsk. Katowice 1977.

[8] A. G r a d o w s k i, Numerical simulation of porosity field formation in cast steel casting. 2nd International Conference. Theory and Practice in Foundry. Acta Metallurgica Slovaca **3** (1/2), 363-368 (2001).

[9] A. G r a d o w s k i, Modelowanie numeryczne skuteczności zasilania odlewu przy zastosowaniu nadlewów krzepnących w otulinach izolacyjnych.III Międzynarodowa Konferencja „Nauka dla przemysłu odlewniczego". Kraków 2001, s. 49-54.

[10] Y. S. T o u l o u k i a n, Thermophysical properties of matter **3**. Thermal Conductivity-Nonmetallic Liquids and Gases. IFI. New York-Washington 1970.

[11] Y. S. T o u l o u k i a n, Thermophysical properties of matter **6**. Specific Heat-nonmetallic Liquids and Gases. IFI. New York-Washington 1970.

[12] Y. S. T o u l o u k i a n, Thermophysical properties of matter **11**. Viscosity. IFI. New York-Washington 1970.

[13] Sprawocznik konstruktora piecziej prokatnogo proizwodstwa. Praca zbiorowa pod red. W. M. Tymczaka. Wyd. Metallurgija. Moskwa 1970.

[14] K. R a z n j e v i ć, Handbook of Termodynamics Tables and Charts. McGraw-Hill Company Washington London.

[15] W. L o n g a, Krzepnięcie odlewów w kokilach. Wyd. Śląsk. Katowice 1978.

[16] M. A. M i c h i e j e w, Osnowy tiepłopieredaczy. II izd. Goseniergoizdat, Moskwa-Leningrad 1956.

[17] B. S t a n i s z e w s k i, Wymiana ciepła, Podstawy teoretyczne. PWN, W-wa 1979.

Received: 7 March 2009.