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MAIN PARADIGM USED IN THE THEORY OF CALCULATION OF THE HEIGHT AND STRUCTURE OF COMBUSTION AND MELTING ZONES IN COKE-FIRED CUPOLAS

PODSTAWOWY PARADYGMAT DLA TEORII OBLICZANIA WYSOKOŚCI I STRUKTURY STREF SPALANIA ORAZ TOPIENIA W ŻELIWIAKACH KOKSOWYCH

The study proposes a method for evaluation of the accuracy of a new paradigm used in the thermal theory of cupola process. The paradigm has been based on the concept of integral mean volumes and integral mean surfaces of the pieces of metal and coke, used in calculation of the, generated in cupola, height and structure of the melting zone and combustion zone. In particular, the following goals have been set:

– using the differential calculus and integral calculus, derive equations for the calculation of integral mean volumes and integral mean surfaces of the pieces of metal and coke, both of various shapes, melting and burning, respectively, in cupola, and derive formulae for evaluation of the accuracy of a new paradigm by comparing the fundamental dimensions of the pieces (as well as their volumes and surfaces), as calculated from the integral mean volumes and integral mean surfaces;
– derive equations for the calculation of arithmetic mean volumes and arithmetic mean surfaces of the pieces of metal and coke, melting and burning, respectively, in the sequences of pieces formed in the metal melting zone and in the coke combustion zone, and compare the arithmetic mean volumes and arithmetic mean surfaces of the pieces, the values of which depend on their number in a given sequence, with the integral mean volumes and integral mean surfaces, to determine next the minimum number of pieces in a given sequence for which the integral mean quantities (theoretically requiring an infinite number of pieces) hold good.

In the study, two examples of the application of the paradigm in mathematical description of a height of the combustion zone and melting zone were given.

W pracy przedstawia się ocenę dokładności nowego paradygmatu teorii cieplnej procesu żeliwiakowego, dotyczącego zastosowania średnich całkowych objętości i średnich całkowych powierzchni kawałków metalu i koksu, do obliczania generowanych w żeliwiaku wysokości i struktury stref topienia oraz spalania. Praca obejmuje:

– wyprowadzenie wzorów, z wykorzystaniem rachunku różniczkowego i całkowego, do obliczania średnich całkowych objętości i powierzchni topiących się w żeliwiakach kawałków metalu oraz palących się kawałków koksu o różnym kształcie, oraz wyprowadzenie wzorów do oceny dokładności paradygmatu, przez porównanie wielkości podstawowych wymiarów kawałków (a także objętości i powierzchni), obliczonych na podstawie średnich całkowych objętości oraz średnich całkowych powierzchni;
– wyprowadzenie wzorów do obliczania średnich arytmetycznych objętości i średnich arytmetycznych powierzchni topiących się kawałków metalu lub palących się kawałków koksu w ciągach kawałków, utworzonych w strefach topienia oraz spalania koksu; porównanie średnich arytmetycznych objętości i powierzchni kawałków, zależnych od ich liczby w ciągach, ze średnimi całkowymi objętościami i powierzchniami, w celu określenia minimalnej liczby kawałków w poszczególnych ciągach, dla której stosować można wielkości średnie całkowe (które teoretycznie wymagają nieskończonej liczby kawałków).

W pracy przedstawiono dwa przykłady zastosowania paradygmatu, tj. do obliczania wysokości stref spalania i topienia.

Symbols used in the text

a,b,c – the initial thickness, width and length, respectively, of the pieces of coke or metal

$\bar{a}_f$, $\bar{b}_f$, $\bar{c}_f$ – the thickness, width and length, respectively, of the pieces of the surface $\bar{f}$, as indicated by the factor "f",

$\bar{a}_v$, $\bar{b}_v$, $\bar{c}_v$ – the thickness, width and length, respectively, of the volume $\bar{v}$, as indicated by the factor "v",

f_o, $\bar{f}$ – the initial and integral mean surfaces, respectively, of the pieces of coke or metal,

$\bar{f} = 2(\bar{a}_f\bar{b}_f + \bar{a}_f\bar{c}_f + \bar{b}_f\bar{c}_f)$

$\bar{f}_n$ – the arithmetic mean surface of the pieces in a given sequence, depending on the number of pieces,

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$f_o = 2a^2M$
$\bar{f}_v$ – the surface calculated from sides $\bar{a}_v$, $\bar{b}_v$ and $\bar{c}_v$,
$\bar{f}_v = 2(\bar{a}_v\bar{b}_v + \bar{a}_v\bar{c}_v + \bar{b}_v\bar{c}_v)$
$K_{w,4}$ – the content of coke charge in respect of metal, kg coke/100 kg Fe
$M = m_b + m_c + m_b m_c$
$m_b = \frac{b}{a}, m_c = \frac{c}{a}$
n – the number of pieces in a given sequence,
v_o, $\bar{v}$ – the initial and integral mean volumes, respectively, of the pieces of coke or metal
$v_o = a \cdot b \cdot c$
$\bar{v} = \bar{a}_v\bar{b}_v\bar{c}_v$
$\bar{v}_f$ – the volume calculated from sides $\bar{a}_f$, $\bar{b}_f$ and $\bar{c}_f$,
$\bar{v}_f = \bar{a}_f\bar{b}_f\bar{c}_f$
$\bar{v}_n$ – the arithmetic mean volume of the pieces in a given sequence, depending on the number of pieces,
$\varphi_v = \frac{\bar{v}}{v_o}$
$\varphi_f = \frac{\bar{f}}{f_o}$,
$\varphi_{v,n} = \frac{\bar{v}_n}{v_o}$
$\varphi_{v,n}$ – the coefficient expressing the ratio between the arithmetic mean volume of a piece in a given sequence of n pieces and the initial volume of this piece,
$\varphi_{f,n} = \frac{\bar{f}_n}{v_o}$
$\varphi_{f,n}$ – the coefficient expressing the ratio between the arithmetic mean surface of a piece in a given sequence of n pieces and the initial surface of this piece,
$a = 2 \cdot z \cdot n$
2z – the constant difference in linear dimensions of the pieces in a given sequence.

1. Introduction

In the 20th century, searching for practical and theoretical means to describe an optimum cupola process, the following paradigms were formulated: 1. the paradigm of the coke and metal charges arranged in layers running from the upper boundary of preheating zone to the upper boundary of combustion zone, or to the upper boundary of the filling coke column (the author unknown); 2. the paradigm of an optimum blast air volume (J. Buzek, 1908); 3. the paradigm of an equality between the melting time of metallic charge and the burning time of coke charge during optimum running of cupola (the author unknown); 4. the paradigm of an optimum height of the filling coke column during optimum running of cupola (the author unknown); 5. the paradigm of an optimum layout of the melting zone and combustion zone in respect of each other, when the lower boundary of the melting zone is adjacent to the upper boundary of the combustion zone (A. Achenbach et al.).

Some of the above mentioned paradigms comprise contradictory models of the process. For example, paradigm 4. requires the presence of a layer of coke between the upper boundary of the combustion zone and the lower boundary of the melting zone, and hence contradicts paradigm 5. Paradigm 1. is unreal, because in the melting zone the layered arrangement is destroyed very quickly, due to the side feeding of blast air and the column of charge materials rubbing against the cupola lining. Paradigm 3. is inconsistent with J. Buzek's equation, based on the condition of equality between the burning time of coke charge and the melting time of metallic charge, referring to cupolas of the stable performance regime and not only optimum running.

Generally, it should be stated, however, that the above mentioned paradigms did open the way for numerous empirical and theoretical studies, which in the past century contributed a lot to more profound knowledge of the practical aspects of a cupola process, though – in fact – they had no special contribution to the development of an analytical thermal theory of the process, and – more important even, they did not help in development of a very important link in this theory, which is calculation of the height and structure of the combustion zone and melting zone.

The, undertaken in the 20th century, attempts at an analytical description of the combustion zone proposed only a calculation of its stable height, disregarding totally gradual increase of its level from the starting level of the filling coke bed height. The calculations also neglected the structure of the combustion zone, i.e. the size and distribution of the pieces of coke burning in this zone. The situation was even more complicated in the case of the melting zone, which also lacked a relevant theory regarding the distribution of metal pieces melting down in a volume of the coke charge. This was certainly an obstacle to further studies of the problem, and finding a proper solution and means to calculate the melting zone height, the secondary filling coke bed height, and the height of the zone of intensive CO_2 reduction.

In [1] the author formulated a new paradigm using integral mean volumes and integral mean surfaces of the pieces of coke in calculation of the height and structure of the combustion zone and described practical application of this paradigm on an example of the spherically shaped coke pieces. Now, this paradigm is being developed further to serve an empirical and analytical description of the height and structure of the melting zone and combustion zone when the pieces of coke and metal are of different shapes.

Therefore the aim of this study is to describe the characteristic theoretical backgrounds of the paradigm and examine an accuracy of its practical application. The present goal has been obtained solving the following problems:

a) developing theoretical backgrounds for the calculation of integral mean volumes and integral mean surfaces of the pieces of coke or metal (the coefficients φ_v and φ_f),
b) proving the accuracy of the paradigm through theoretical comparative analysis of the fundamental dimensions of the pieces (as well as their volumes and surfaces), as calculated from the integral mean volumes and integral mean surfaces,
d) deriving equations for the calculation of arithmetic mean volumes and arithmetic mean surfaces of the melting pieces of metal or the burning pieces of coke forming a sequence of pieces in the metal melting zone and in the coke combustion zone,
c) conducting theoretical comparative analysis to determine the minimum number of pieces that will enable practical use of the integral mean volume and integral mean surface values (the coefficients $\varphi_{v,n}$ and $\varphi_{f,n}$ as well as φ_v and φ_f);
d) drawing a rough scheme of the application of the developed paradigm in derivation of equations expressing the combustion zone height and the melting zone height.

2. Coefficients φ_v and φ_f resulting from the paradigm

The, determined by the paradigm, integral mean volumes and integral mean surfaces of the pieces of coke in the combustion zone and of the pieces of metal in the melting zone have been computed using the dimensionless coefficients φ_v and φ_f, predicted by the following equations derived for the pieces in the form of rectangular plates and for the particular case of spheres and cubes

$$\varphi_v = \frac{1}{2} - \frac{1}{6m_b} - \frac{1}{6m_c} + \frac{1}{12m_b m_c} \tag{1}$$

$$\varphi_f = \frac{m_b m_c}{m_b + m_c + m_b m_c} = \frac{1}{1 + \frac{1}{m_b} + \frac{1}{m_c}}, \tag{2}$$

where:
$m_b = \frac{b}{a}$; $m_c = \frac{c}{a}$
a, b, c – the initial thickness, width and length, respectively, of the pieces of coke or metal, m.
The physical sense of φ_v results from the following formula

$$\varphi_v = \frac{\bar{v}}{v_o}, \tag{3}$$

where:
v_o, $\bar{v}$ – the initial and integral mean volumes, respectively, of the pieces of coke or metal, m^3.

As follows from (3), the coefficient φ_v is a ratio between the integral mean volume and initial volume of the pieces.
On the other hand, the physical sense of φ_f results from the following formula

$$\varphi_f = \frac{\bar{f}}{f_o}, \tag{4}$$

where:
f_o, $\bar{f}$ – the initial and integral mean surfaces, respectively, of the pieces of coke or metal, m^2.
As follows from (4), the coefficient φ_f is a ratio between the integral mean surface and initial surface of the pieces.
The following formula is also useful in the characterisation of parameter φ_v

$$\varphi_v = \frac{\bar{V}}{V_o} \tag{5}$$

where:
$\bar{V}$ – the integral mean volume of all the burning pieces of coke in the combustion zone, or the integral mean volume of all the melting pieces of metal in the melting zone; for pieces of the same volume and shape, the following equation holds good: $\bar{V} = n \cdot \bar{v}$, m^3
where:
n – the number of pieces in the zone,
V_o – the volume of all the pieces of coke or metal forming volume $\bar{V}$, m^3.
On the other hand, the following formula is also useful in the characterisation of parameter φ_f

$$\varphi_f = \frac{\bar{F}}{F_o}, \tag{6}$$

where:
$\bar{F}$ – the integral mean surface of all the burning pieces of coke in the combustion zone (the combustion zone development surface), or the integral mean surface of all the melting pieces of metal in the melting zone (the melting zone development surface); for pieces of the same volume and shape, the following equation holds good: $\bar{F} = n \cdot \bar{f}$, m^2
F_o – the surface of all the pieces of coke or metal forming surface $\bar{F}$, m^2.
From (3) and (4), the following integral mean modulus of the pieces of coke or metal can be written down:

$$\bar{r} = \frac{\bar{v}}{\bar{f}} = \frac{\varphi_v}{\varphi_f} r_o = \frac{a}{2}\varphi_v, \text{ where} \tag{7}$$

$$r_o = \frac{v_o}{f_o} = \frac{a}{2(1 + \frac{1}{m_b} + \frac{1}{m_c})} = \frac{a}{2}\varphi_f, \tag{8}$$

where:

$\bar{r}$ – the integral mean modulus of the pieces of coke or metal, m

r_o – the initial modulus of the pieces of coke or metal, m.

Formulae (1) and (2) are derived using the differential calculus and integral calculus. Below, the procedure used in derivation of these equations is described.

Formula (1) is derived from the following starting equation, allowing for the volume of a burning piece of coke or melting piece of metal (plate):

$$v_\tau = (a - 2\mu_t\tau)(b - 2\mu_t\tau)(c - 2\mu_t\tau), \quad (9)$$

where:

v_τ – the volume of a burning piece of coke or melting piece of metal after time τ, m^3

μ_t – the linear burning rate of a piece of coke or the linear melting rate of a piece of metal, equal for all the surfaces of a piece, m/s.

Let us extract from brackets the quantities a, b and c

$$v_\tau = v_o(1 - \frac{2\mu_t}{a}\tau)(1 - \frac{2\mu_t}{b}\tau)(1 - \frac{2\mu_t}{c}\tau), \quad (10)$$

where:

$v_o = a \cdot \cdot b \cdot c$.

Applying in (10) the following substitutions: a= 2 μ_t τ_c and X= $\frac{\tau}{\tau_c}$ (where: τ_c – the total burning time of a piece of coke, or the total melting time of a piece of metal, either of the thickness a, s), we obtain

$$v_\tau = v_o(1 - X)(1 - \frac{1}{m_b}X)(1 - \frac{1}{m_c}X). \quad (11)$$

Using (11), the following integral formula for the calculation of φ_v is derived

$$\varphi_v = \frac{\bar{v}}{v_o} = \frac{\int_0^X (1 - X)(1 - \frac{1}{m_b}X)(1 - \frac{1}{m_c}X)dX}{\int_0^X dX}. \quad (12)$$

After integration of (12) and substituting the limits of integration, the following equation is obtained

$$\varphi_v = 1 - \frac{X}{2}(1 + \frac{1}{m_b} + \frac{1}{m_c}) + \frac{X^2}{3}(\frac{1}{m_b} + \frac{1}{m_c} + \frac{1}{m_b m_c}) - \frac{X^3}{4\, m_b\, m_c}). \quad (13)$$

For X = 1 (the plate totally melted or burnt down), equation (13) is simplified to the form of equation (1).

Let us write down the special cases of equation (1) for different shapes:

a) the pieces of coke or metal in the form of cubes or spheres (m_b=m_c=1; a=b=c); the equation is simplified to

$$\varphi_v = \frac{\bar{v}}{v_o} = 0,25. \quad (14)$$

b) the pieces of coke or metal in the form of square based prisms (m_b=1; a=b); the equation is simplified to

$$\varphi_v = \frac{\bar{v}}{v_o} = \frac{1}{3} - \frac{1}{12\, m_c}. \quad (15)$$

c) the pieces of coke or metal in the form of square plates (b=c; m_b=m_c = m); the equation is simplified to

$$\varphi_v = \frac{\bar{v}}{v_o} = \frac{1}{2} - \frac{1}{3m} + \frac{1}{12m^2}, \quad (16)$$

d) the pieces of coke or metal in the form of infinite plates (m_b=m_c=∞); the equation is simplified to

$$\varphi_v = \frac{\bar{v}}{v_o} = 0,5. \quad (17)$$

Let us now write an integral equation for the calculation of φ_f. The surface of the melting plate f_τ, after the melting time τ, can be expressed with the following equation

$$f_\tau = 2(a - 2\mu_t\tau)(b - 2\mu_t\tau) + 2(a - 2\mu_t)(c - 2\mu_t\tau) + \\ +2(b - 2\mu_t\tau)(c - 2\mu_t\tau). \quad (18)$$

After extracting from brackets the main quantities a, b and c and substituting: c=$m_c \cdot$ a, b=m_b·a, a=2·$\mu_t \cdot \tau_c$, equation (18) takes the following form

$$f_\tau = 2a^2[(m_c{-}X)(m_b{-}X)+(m_c{-}X)(1{-}X)+(m_b{-}X)(1{-}X)]. \quad (19)$$

After multiplying out the brackets and extracting from brackets the quantity M=$m_b + m_c + m_b \cdot m_c$, the following equation is obtained

$$f_\tau = f_o[1 - \frac{2(m_b + m_c + 1)X - 3X^2}{m_b m_c + m_b + m_c}], \text{ where} \quad (20)$$

$$f_o = 2a^2(m_b m_c + m_b + m_c) = 2a^2 M. \quad (21)$$

Using (20), let us write down an integral formula for the calculation of φ_f

$$\varphi_f = \frac{\bar{f}}{f_o} = \frac{\int_0^X [1 - \frac{2(m_b+m_c+1)X-3X^2}{m_b m_c+m_b+m_c}]dX}{\int_0^X dX} \quad (22)$$

After integration of (22), substituting the limits of integration and making the necessary simplifications, the following equation is obtained

$$\varphi_f = 1 - \frac{(1 + m_b + m_c)X - X^2}{m_b + m_c + m_b m_c}. \quad (23)$$

For an interesting case of X=1, equation (23) assumes the form of equation (2). Let us write down the special cases of equation (2) for different shapes:

a) the pieces of coke or metal in the form of cubes or spheres (m_b=m_c=1; a=b=c); the equation is simplified to

$$\varphi_f = \frac{\bar{f}}{f_o} = \frac{1}{3}, \quad (24)$$

b) the pieces of coke or metal in the form of square based prisms (m_b=1; a=b); the equation is simplified to

$$\varphi_f = \frac{\bar{f}}{f_o} = \frac{m_c}{1 + 2m_c}, \quad (25)$$

c) the pieces of coke or metal in the form of square plates (m_b=m_c=m; b=c); the equation is simplified to

$$\varphi_f = \frac{\bar{f}}{f_o} = \frac{m}{1 + m}, \quad (26)$$

d) the pieces of coke or metal in the form of infinite plates (m_b=m_c=∞); the equation is simplified to

$$\varphi_f = \frac{\bar{f}}{f_o} = 1. \quad (27)$$

3. Calculation of the fundamental dimensions of the quantities $\bar{v}$ and $\bar{f}$ and accuracy of the paradigm

From equations (1)÷(4), the integral mean quantities $\bar{v}$ and $\bar{f}$, which are functions of m_b and m_c, are calculated. Knowing the values of $\bar{v}$ and $\bar{f}$, their dimensions are calculated.

The starting equation for the calculation of the dimensions of $\bar{v}$ assumes the following form

$$\bar{v} = (a - 2x_v)(b - 2x_v)(c - 2x_v), \quad (28)$$

where: x_v – the thickness of the piece burnt or melted down, equal for all surfaces of this piece and referring to volume $\bar{v}$ m.

Let us substitute b= a·m_b, c=a·m_c to (28) and extract a from brackets

$$\bar{v} = a^3(1 - \frac{2x_v}{a})(m_b - \frac{2x_v}{a})(m_c - \frac{2x_v}{a}). \quad (29)$$

Let us substitute $a^3 = \frac{v_o}{m_b m_c}$ and $\frac{2x_v}{a} = \xi_v$ to (29)

$$\bar{v} = v_o \frac{(1 - \xi_v)(m_b - \xi_v)(m_c - -\xi_v)}{m_b m_c}. \quad (30)$$

From equation (30) follows a definition and a formula for the calculation of φ_v in function of ξ_v

$$\varphi_v = \frac{\bar{v}}{v_o} = \frac{(1 - \xi_v)(m_b - -\xi_v)(m_c - -\xi_v)}{m_b m_c}. \quad (31)$$

After multiplying out the brackets in equation (31) and arranging the obtained quantities, the equation of a third order is obtained:

$$k_3\xi_v^3 - k_2\xi_v^2 + k_1\xi_v - (1 - \varphi_v) = 0 \quad (32)$$

$$k_1 = 1 + \frac{1}{m_b} + \frac{1}{m_c} \quad (33)$$

$$k_2 = \frac{1}{m_b} + \frac{1}{m_c} + \frac{1}{m_b m_c} \quad (34)$$

$$k_3 = \frac{1}{m_b m_c}. \quad (35)$$

Having calculated the quantity ξ_v from equation (32), we calculate next the dimensions of the sides of volume $\bar{v}$, using relation: $2x_v = a\,\xi_v$

$$\bar{a}_v = a - 2x_v = a(1 - \xi_v) \quad (36)$$

$$\bar{b}_v = b - 2x_v = am_b - a\xi_v = a(m_b - \xi_v) \quad (37)$$

$$\bar{c}_v = c - 2x_v = am_c - a\xi_v = a(m_c - \xi_v), \quad (38)$$

where:

$\bar{a}_v$, $\bar{b}_v$, $\bar{c}_v$ – the thickness, width and length, respectively, of the volume $\bar{v}$, as indicated by the factor "v", m.

Using (36), (37) and (38), the volume $\bar{v}$ is calculated

$$\bar{v} = \bar{a}_v\bar{b}_v\bar{c}_v. \quad (39)$$

Like equation (31), equation (39) is also useful in checking the accuracy of calculations. This is done by checking the value of φ_v in accordance with the following formula

$$\varphi_v = \frac{\bar{v}}{v_o} = \frac{\bar{a}_v\bar{b}_v\bar{c}_v}{a^3 m_b m_c}. \quad (40)$$

Now, the formulae for calculation of the dimensions of the integral mean surface $\bar{f}$ are derived.

The starting equation takes the following form

$$\overline{f} = 2[(a - 2x_f)(b - 2x_f) + (a - 2x_f)(c - 2x_f) + (b - 2x_f)(c - 2x_f)], \quad (41)$$

where:
x_f – the thickness of the burnt or melted down piece of coke or metal, respectively, equal for all the surfaces of a piece of coke or metal, and referring to the surface $\overline{f}$,m.
Let us substitute b= a m_b, c=a m_c to equation (41) and extract a^2 from brackets.

$$\overline{f} = 2a^2[(1 - \frac{2x_f}{a})(m_b - \frac{2x_f}{a})(1 - \frac{2x_f}{a})(m_c - \frac{2x_f}{a}) + (m_b - \frac{2x_f}{a})(m_c - \frac{2x_f}{a})]. \quad (42)$$

Let us now substitute $\frac{2x_f}{a} = \xi_f$ to (42) and derive:

$$\overline{f} = 2a^2[(1 - \xi_f)(m_b - \xi_f) + (1 - \xi_f)(m_c - \xi_f) + (m_b - \xi_f)(m_c - \xi_f)] \quad (43)$$

Let us next substitute $a^2 = \frac{f_o}{2M}$ (M= m_b+m_c+$m_b m_c$) to (43) and obtain:

$$\overline{f} = [(1 - \xi_f)(m_b - \xi_f) + (1 - \xi_f)(m_c - \xi_f) + (m_b - \xi_f)(m_c - \xi_f)] \quad (44)$$

From (44) follows a definition and a formula used to calculate φ_f, first, and check the correctness of the calculations, next

$$\varphi_v = \frac{[(1 - \xi_f)(m_b - \xi_f) + (1 - \xi_f)(m_c - \xi_f) + (m_b - \xi_f)(m_c - \xi_f)]}{(m_b + m_c + m_b m_c)}. \quad (45)$$

After multiplying out the brackets and arranging the obtained quantities, equation (45) takes the form of a quadratic equation

$$k_5\xi_f^2 - k_4\xi_f + (1 - \varphi_f) = 0, \quad (46)$$

where:

$$k_4 = \frac{2(1 + m_b + m_c)}{m_b + m_c + m_b m_c} \quad k_5 = \frac{3}{m_b + m_c + m_b m_c}. \quad (47)$$

After solving (46) and substituting (47) and (2), the following formula for the calculation of ξ_f is obtained:

$$\xi_f = \frac{1}{3}[(1 + m_b + m_c) - \sqrt{(1 + m_b + m_c)^2 - 3(m_b + m_c)}]. \quad (48)$$

Having calculated the quantity ξ_f from (48), the dimensions of the sides of the surface $\overline{f}$ are calculated, using relation $2x_f = a\,\xi_f$

$$\overline{a}\xi_f = a - 2x_f = a(1 - \xi_f) \quad (49)$$

$$\overline{b}_f = b - 2x_f = am_b - 2x_f = a(m_b - \xi_f) \quad (50)$$

$$\overline{c}_f = c - 2x_f = am_c - 2x_f = a(m_c - \xi_f) \quad (51)$$

where:
$\overline{a}_f$, $\overline{b}_f$, $\overline{c}_f$ – the thickness, width and length, respectively, of the pieces of the surface $\overline{f}$, as indicated by the factor "f", m.
Using (49), (50) and (51) let us calculate the surface $\overline{f}$

$$\overline{f} = 2(\overline{a}_f\overline{b}_f + \overline{a}_f\overline{c}_f + \overline{b}_f\overline{c}_f). \quad (52)$$

The equations derived for the calculation of ξ_v and ξ_f are considerably simplified when $m_b = m_c = 1$. Equation (32) can be written in two forms:

$$\varphi_v = (1 - \xi_v)^3 \quad (53)$$

$$\xi_v = 1 - \sqrt[3]{\varphi_v}. \quad (53a)$$

Also equation (46) can be written in two forms:

$$\varphi_f = (1 - \xi_f)^2 \quad (54)$$

$$\xi_f = 1 - \sqrt{\varphi_f}. \quad (54a)$$

So far, we have derived formulae for the calculation of $\overline{v}$ by means of sides $\overline{a}_v$, $\overline{b}_v$ and $\overline{c}_v$, and for the calculation of $\overline{f}$ by means of sides $\overline{a}_f$, $\overline{b}_f$ and $\overline{c}_f$. To make the concept of paradigm reliable, the dimensions $\overline{a}_v$, $\overline{b}_v$ and $\overline{c}_v$ as well as $\overline{a}_f$, $\overline{b}_f$ and $\overline{c}_f$ should satisfy the following equalities:

$$\overline{a}_v \cong \overline{a}_f, \overline{b}_v \cong \overline{b}_f, \overline{c}_v \cong \overline{c}_f \tag{55}$$

and

$$\overline{v} \cong \overline{v}_f; \quad \overline{f} \cong \overline{f}_v \tag{56}$$

where:
$\overline{v}_f$ – the volume calculated from sides a $\overline{a}_f$, $\overline{b}_f$ and $\overline{c}_f$.
$\overline{f}_v$ – the surface calculated from sides $\overline{a}_v$, $\overline{b}_v$ and $\overline{c}_v$.
Volume $\overline{v}_f$ can be calculated from a formula analogical to (39):

$$\overline{v}_f = \overline{a}_f\overline{b}_f\overline{c}_f \tag{57}$$

or

$$\overline{v}_f = v_o\frac{(1-\xi_f)(m_b-\xi_f)(m_c-\xi_f)}{m_b m_c} \tag{58}$$

while surface $\overline{f}_v$ can be calculated from a formula analogical to (52)

$$\overline{f}_v = 2(\overline{a}_v\overline{b}_v + \overline{a}_v\overline{c}_v + \overline{b}_v\overline{c}_v) \tag{59}$$

or

$$\overline{f}_v = \frac{f_o}{m_b + m_c + m_b m_c}[(1-\xi_f)(m_b-\xi_v)) + (1-\xi_v)(m_c-\xi_v)) + (m_b-\xi_v)(m_c-\xi_v))] \tag{60}$$

For a quantitative evaluation of the accuracy of paradigm, the following formulae are proposed:

$$\overline{\Delta V} = \frac{\overline{v}-\overline{v}_f}{\overline{v}}100 = \frac{\overline{a}_v\overline{b}_v\overline{c}_v - \overline{a}_f\overline{b}_f\overline{c}_f}{\overline{a}_v\overline{b}_v\overline{c}_v}100 \tag{61}$$

$$\overline{\Delta F} = \frac{\overline{f}-\overline{f}_v}{\overline{f}}100 = \frac{(\overline{a}_v\overline{b}_v + \overline{a}_v\overline{c}_v + \overline{b}_v\overline{c}_v) - (\overline{a}_f\overline{b}_f + \overline{a}_f\overline{c}_f + \overline{b}_f\overline{c}_f)}{(\overline{a}_v\overline{b}_v + \overline{a}_v\overline{c}_v + \overline{b}_v\overline{c}_v)}100. \tag{62}$$

Let us calculate $\overline{\Delta V}$ and $\overline{\Delta F}$ for a wide range of the values of m_b and m_c on which these quantities are dependent.

Calculations 1.

Calculations a):
Given $m_b = 10$, $m_c = 10$, let us calculate $\overline{\Delta V}$ and $\overline{\Delta F}$
$k_1=1+0,1+0,1=1,2$;$k_2=0,1+0,1+0,01=0,21$;$k_3=0,01$;$k_4 = \frac{2(1+10+10)}{10+10+100} = \frac{42}{120}$;$k_5=\frac{3}{120}$; $\varphi_f=0,833$; $\varphi_v=0,4675$; let us substitute the values of factors k_1, k_2 and k_3 to (32) and derive the following equation: $0,01\ \xi_v^3 - 0,21\ \xi_v^2 + 1,2\ \xi_v - (1-0,4675) = 0$; using this equation and the method of successive approximations, we calculate the value of $\xi_v = 0,4838$.
Let us calculate ξ_f. For this purpose, let us substitute the values of factors k_4 and k_5 to (46) and derive equation: $3\ \xi_f^2 - 42\ \xi_f + 120\ (1 - 0,8333\ ) = 0$; from thus derived equation or from equation (48), we calculate the value of $\xi_f = 0,4936$.
From (36), (37) and (38) we calculate : $\overline{a}_v$= a(1-0,4838)=a·0,5162;
$\overline{b}_v = \overline{c}_v$ = a(10-0,4838) = a 9,5162. From (49), (50) and (51) we calculate :
$\overline{a}_f$ = a(1-0,4936)= a·0,5064 ; $\overline{b}_f = \overline{c}_f$ = a·9,5064. From (61) we calculate $\overline{\Delta V} = \frac{0,5162\cdot 9,5162^2 + 0,5064\cdot 9,5064^2}{0,5162\cdot 9,5162^2}100$ = 2,12 %. From (62) we calculate $\overline{\Delta F}$:

$\overline{\Delta F} = \frac{(0,5162\cdot 9,5162\cdot 2+9,5162^2)-(0,5064\cdot 9,5064\cdot 2+9,5064^2)}{(0,5162\cdot 9,5162\cdot 2+9,5162^2)}100$ = 0,39 %.

Calculations b)
Given $m_b = m_c = 5$, let us make the following calculations:
k_1= 1,4 ; k_2= 0,44 ; k_3= 0,04 ; k_4= 0,6286 ; k_5= 0,0857; φ_v= 0,4367; φ_f=0,7143; $0,04\ \xi_v^3 - 0,44\ \xi_v^2 + 1,4\ \xi_v - (1-0,4367)=0$; ξ_v = 0,468 ; : $\overline{a}_v$=a(1-0,468)= a 0,532; $\overline{b}_v = \overline{c}_v$ =a(5-0,468)= a 4,532 ;
$0,0857\ \xi_f^2 - 0,6286\ \xi_f + (1-0,7143)=0$; ξ_f= 0,487; a_f=a(1-0,487)=a 0,513; $\overline{b}_f = \overline{c}_f$ = a(5-0,487)=a 4,513;
$\overline{\Delta V} = \frac{0,532\cdot 4,532\cdot^2 - 0,513\cdot 4,513^2}{0,532\cdot 4,532^2}$ 100 = 4,15 %;
$\overline{\Delta F} = \frac{(0,532\cdot 4,532\cdot 2+4,532^2)-(0,513\cdot 4,513\cdot 2+4,513^2)}{(0,532\cdot 4,532\cdot 2+4,532^2)}$ 100 = 1,36 %

Calculations c)
Given $m_b = m_c = 2$, the following results of the calculations are obtained:
k_1=2 ; k_2= 1,25; k_3= 0,25; k_4= $\frac{10}{8}$; k_5 = $\frac{3}{8}$; φ_v= 0,3542; φ_f = 0,5.
$0,25\ \xi_v^3 - 1,25\ \xi_v^2 + 2\ \xi_v - (1-0,3542) = 0$; ξ_v = 0,4261; : $\overline{a}_v$= a(1-0,4261)= a · 0,5739 ; $\overline{b}_v = \overline{c}_v$ = a(2-0,4261)= a1,5739;
$3\ \xi_f^2 - 10\xi_f + 8(1-0,5) = 0$; ξ_f= 0,4648 ; $\overline{a}_f$=a(1-0,4648)=a 0,5352; $\overline{b}_f = \overline{c}_f$ = a(2-0,4648)= a 1,5352; $\overline{\Delta V} = \frac{0,5739\cdot 1,5739^2 - 0,5352\cdot 1,5352^2}{0,5739\cdot 1,5739^2}100$ = 11,27 %;

$\overline{\Delta F} = \frac{(0{,}5739\cdot 1{,}5739\cdot 2+1{,}5739^2)-(0{,}5352\cdot 1{,}5352\cdot 2+1{,}5352^2)}{(0{,}5739\cdot 1{,}5739\cdot 2+1{,}5739^2)}\,100 = 6{,}62\ \%$.

Calculations d)
Given $m_b=m_c = 1$, the following results of the calculations (formulae (54) and (56)) are obtained:
$\varphi_v= 0{,}25$; $\varphi_f=0{,}3333$; $\xi_v = 1-\sqrt[3]{0{,}25} = 0{,}37$; : $\bar{a}_v=\bar{b}_v = \bar{c}_v$ =a(1-0,37)=a 0,63; $\xi_f = 1-\sqrt{0{,}3333} = 0{,}4226$;
$\bar{a}_f = \bar{b}_f = \bar{c}_f$ =a(1-0,4226)=a 0,5774; $\overline{\Delta V} = \frac{0{,}63^3-0{,}5773^3}{0{,}63^3}$ 100=23 %;
$\overline{\Delta F} = \frac{0{,}63^2-0{,}5774^2}{0{,}63^2}\,100 = 16\ \%$.
The conclusions drawn from the calculations are as follows:

- the difference in the values of $\Delta\xi = \xi_f - \xi_v$ decreases with the increasing values of m_b and m_c ; the maximum difference is equal to 0,0526 for $m_b=1$ and $m_c = 1$,
- for $m_b=10$ and $m_c=10$, the difference $\Delta\xi$ =0,4937-0,4837=0,01; for $m_b=\infty$ and $m_c=\infty$ the difference $\Delta\xi=0$,
- to reduce the values of $\overline{\Delta V}$ and $\overline{\Delta F}$ it is suggested to use in practical calculations an arithmetic mean value of the quantities ξ_v and ξ_f
- the application of an arithmetic mean value of ξ_v and ξ_f in example d) gives the following result: $\overline{\Delta V} = \frac{(1-0{,}37)^3-(1-0{,}3963)^3}{(1-0{,}37)^3}\,100 = 12\ \%$;

$\overline{\Delta F} = \frac{(1-0{,}3963)^2-(1-0{,}4226)^2}{(1-0{,}3963)^2}\,100 = 8{,}52\ \%$; $\frac{0{,}37+0{,}4226}{2} = 0{,}3963$.

4. Calculation of the volume and surface of the sequences of pieces and of the coefficients $\varphi_{v,n}$ and $\varphi_{f,n}$

The, calculated by the method of differential calculus and integral calculus, coefficients φ_v and φ_f strictly relate to the case when the number of the pieces is infinitely large. In practice, however, in the combustion and melting zones of cupola, the number of the pieces of coke and metal is always finite. Hence the question arises: are the calculated coefficients valid in description of the structures present in the above mentioned zones, or what is the minimum number of pieces when these coefficients hold good ? To answer this question, let us define the coefficients $\varphi_{v,n}$ and $\varphi_{f,n}$, remembering that they depend on the number of pieces, and derive respective formulae used for their calculation.
At the beginning of the discussion, some model assumptions regarding the formation of a structure of the combustion zone and melting zone have been adapted. Let us start with the structure of the combustion zone, as simpler because formed of the pieces of coke only.
The structure of the combustion zone is formed in the following way:

- while passing from the melting zone to the combustion zone, the new pieces of coke are forming with the pieces already burning in this zone the sequences of the burning pieces; the number of these sequences equals the number of the coke pieces moving at the same time through an upper level of the combustion zone cross-section,
- in each sequence of pieces, the differences in the fundamental dimensions of the successive pieces are the same and equal to 2z, thus satisfying the equality

$$a = 2zn, \tag{63}$$

where:
2z – the constant difference in the linear dimensions of the pieces in a given sequence, m
n – the number of pieces in a given sequence,

- the filling height in the cupola shaft composed of all the sequences present in the zone makes the height of the combustion zone.

A combustion zone of this specific structure is characterised by the following quantities: arithmetic mean volume and arithmetic mean surface of the pieces in each sequence, where:

- the arithmetic mean volume of the pieces in each sequence is expressed by the following general formula:

$$\bar{v}_n = \frac{\text{total volume of pieces in sequence}}{\text{number of pieces in sequence}}, \tag{64}$$

where:
$\bar{v}_n$ – the arithmetic mean volume of the pieces in a given sequence, depending on the number of pieces, m^3.

- analogically to (29), the arithmetic mean surface of the pieces in each sequence is expressed by the following general formula:

$$\bar{f}_n = \frac{\text{total surface of pieces in sequence}}{\text{number of pieces in sequence}}, \tag{65}$$

where:
$\bar{f}_n$ – the arithmetic mean surface of the pieces in a given sequence, depending on the number of pieces, m^2.
Analogically to the sequences of the pieces of coke burning in the combustion zone, we have the sequences of the pieces of metal melting in the melting zone which, togther with coke, are filling the space in cupola shaft between the lower boundary of the preheating zone and the upper boundary of the combustion zone, thus determining the height of the melting zone. The distribution of the pieces of metal melting in the zone is characterised

by the volumes decreasing in direction towards the lower zone boundary. This makes the coke fraction increase in respect of the metal fraction, in spite of certain losses of coke (strictly speaking of the coal coke), consumed while reducing some fractions of the gaseous CO_2. The mean coke content in the melting zone $K_{w,t}$ can be calculated from the following formula:

$$K_{w,t} = \frac{K_{w,4}}{\varphi_v}, \quad (66)$$

where:
$K_{w,4}$ – the content of coke charge in respect of metal, kg coke/100 kg Fe. Equations (64) and (65) give general prescription for the calculation of an arithmetic mean volume and arithmetic mean surface of the pieces of coke or metal in a given sequence of the pieces; the calculated values depend on the number of pieces in a sequence and on the size of the largest piece in this sequence.
To obtain a formula determining the height of the combustion zone and melting zone, and to describe the geometrical structure of the zones, it is necessary to apply simultaneously the integral mean $\overline{v}$ and $\overline{f}$ and the arithmetic mean $\overline{v}_n$ and $\overline{f}_n$, which is possible only when the values of these parameters satisfy the approximate equalities:

$$\overline{v} \cong \overline{v}_n \text{ and } \overline{f} \cong \overline{f}_n. \quad (67)$$

Let us explain on a specific example the problem of, e.g., $\overline{v} \cong \overline{v}_n$.

Calculations 2.

The burning pieces of coke have a spherical shape and the initial diameter a=10 cm. The initial volume of the spheres is v_o = 523,6 cm^3, while their integral mean volume is equal to $\overline{v}$ = 0,25·523,6 = 130,9 cm^3 [formula (14)]. It is assumed that the sequence of spheres under consideration is composed of 5 spheres of diameters differing by 2 cm [formula (28)]. Using (64), the arithmetic mean volume of the spheres forming the sequence is calculated:
$\overline{v}_5 = \frac{\frac{\pi}{6}(10^3+8^3+6^3+4^3+2^3)}{5}$ =188,5 cm^3. The calculated value considerably exceeds the value of the integral mean volume, which amounts to 130,9 cm^3.
In the calculation of $\overline{v}_5$ it has been assumed that the largest piece in the sequence has an initial volume v_o, which means that it has not decreased in volume while passing from the melting zone to the combustion zone. Let us repeat the calculations assuming now that the largest piece in the sequence has a diameter equal to 8 cm. The calculated arithmetic mean volume of the pieces in the sequence is $\overline{v}_5^* = 83,8 cm^3$, and so it is obviously smaller than the integral mean volume. Let us now, using the calculated mean values, calculate the arithmetic mean value, which is $\frac{188,5+83,8}{2} = 136,15 cm^3$; the mean calculated from the mean values is by only 4 % higher than the integral mean, which can be considered the accuracy sufficient for an equally valid application of the integral mean volume and arithmetic mean volume of the spheres when the number of pieces in a sequence is minimum 5 (with increasing n the accuracy also increases).
Similar conclusions are reached when in an analogical way $\overline{f}_n$ is calculated according to formula (65).
Let us now derive a general formula (plates and any arbitrary values of n) for the calculation of $\overline{v}_n$ and $\overline{f}_n$.
Analogically to the coefficients φ_v and φ_f, we can define the coefficients $\varphi_{v,n}$ and $\varphi_{f,n}$

$$\varphi_{v,n} = \frac{\overline{v}_n}{v_o} \quad (68)$$

$$\varphi_{f,n} = \frac{\overline{f}_n}{\overline{f}_o}, \quad (69)$$

where:
$\varphi_{v,n}$ – the coefficient expressing the ratio between the arithmetic mean volume of a piece in a given sequence of n pieces and the initial volume of this piece,
$\varphi_{f,n}$ – the coefficient expressing the ratio between the arithmetic mean surface of a piece in a given sequence of n pieces and the initial surface of this piece.
The formulae used in calculation of the coefficients $\varphi_{v,n}$ and $\varphi_{f,n}$ are the following:

$$\varphi_{v,n} = \frac{\overline{v}_n}{v_o} = \varphi_v + \frac{2m_b + 2m_c - 1}{12m_b m_c n^2} \quad (70)$$

$$\varphi_{f,n} = \frac{\overline{f}n}{f_o} = \varphi_f + \frac{1 + m_b m_c - m_b - m_c}{2Mn} + \frac{1}{2Mn^2}, \quad (71)$$

where:
$\overline{v}_n$ – the mean volume of the pieces of metal in a sequence of n pieces, m^3 $M = m_b + m_c + m_b m_c$,
$\overline{f}_n$ – the mean surface of the pieces of metal in a sequence of n pieces, m^3.

For n=∞, the formulae (70) and (71) assume the form of formulae (1) and (2)

Let us now describe the theory by which equations (70) and (71) have been obtained.

5. The theory of deriving formula (70)

To obtain formula (70), the following equation expressing the volume of the burning or melting plate will be used (having reduced previously its every dimension by a value 2z·i, where 2z – is the difference in fundamental dimensions of the neighbouring plates in a given sequence of plates; i – is the number of the burnt or melted down plates of the thickness 2z)

$$v_i = (a - 2z \cdot i)(b - 2z \cdot i)(c - 2z \cdot i). \qquad (72)$$

Using (72), the total volume of plates melting in a given sequence will be writen down for the two cases of summation:

$$V'_{c,n} = \sum_{i=1}^{n} (a - 2zi)(b - 2zi)(c - 2zi) \qquad (73)$$

$$V''_{c,n} = \sum_{i=0}^{n} (a - 2zi)(b - 2zi)(c - 2zi), \qquad (74)$$

The difference between the sums obtained in (73) and (74) consists in this that in (73) the first in the sequence is the piece of charge, the fundamental dimensions of which have been reduced by a value 2z(i=1), while in (74) the first in the sequence is the piece of an initial volume abc (2z=0, i=0).

The formulae (73) and (74) are used in calculation of the arithmetic mean volume of the sequences of pieces according to the following formula:

$$V_{c,n} = \frac{V'_{c,n} + V''_{c,n}}{2} \qquad (75)$$

and next of $\bar{v}_n$ ($V_{c,n}/n$) and $\varphi_{v,n}$ ($\bar{v}_n/v_o$).

Let us extract from brackets the quantities a, b and c in formulae (73) and (74) and obtain

$$V'_{c,n} = v_o \sum_{i=1}^{n} (1 - \delta i)(1 - \frac{\delta i}{m_b})(1 - \frac{\delta i}{m_c}) \qquad (76)$$

$$V''_{c,n} = v_o \sum_{i=0}^{n} (1 - \delta i)(1 - \frac{\delta i}{m_b})(1 - \frac{\delta i}{m_c}), \qquad (77)$$

where:

$$\delta = \frac{2z}{a}.$$

Let us denote the sums in equations (76) and (77) with the symbols R_1 and R_2, respectively, and make next the summation of R_1 and R_2.

<u>The summation of R_1</u>

$$R_1 = \sum_{i=1}^{n} (1 - \delta i)(1 - \frac{\delta i}{m_b})(1 - \frac{\delta i}{m_c}) \qquad (78)$$

Let us multiply out the terms in the right side of equation (78)

$$R_1 = 1 - k_1\delta i + k_2\delta^2 i^2 - k_3\delta^3 i^3 \qquad (79)$$

where: k_1, k_2, k_3 – is expresed by formulae (33), (34) and (35).

Let us develop some of the terms in sum (79), i.e. for i=1, 2 ,3 and n:

$$i = 1 \qquad 1 - k_1\delta 1 + k_2\delta^2 1^2 - k_3\delta^3 1^3 \qquad (a)$$

$$i = 2 \qquad 1 - k_1\delta 2 + k_2\delta^2 2^2 - k_3\delta^3 2^3 \qquad (b)$$

$$i = 3 \qquad 1 - k_1\delta 3 + k_2\delta^2 3^2 - k_3\delta^3 3^3 \qquad (c)$$

. . . .

$$i = n \qquad 1 - k_1\delta n + k_2\delta^2 n^2 - k_3\delta^3 n^3 \qquad (d)$$

Let us now sum the columns of expressions (a)÷(d):

$$\text{first column}: \; 1 + 1 + 1 + \ldots.. + 1 = n \qquad (e)$$

$$\text{second column}: \; -k_1\delta(1 + 2 + 3 + \ldots. + n) = -k_1\delta\frac{n(n+1)}{2} \qquad (f)$$

$$\text{third column}: \; k_2\delta^2(1^2 + 2^2 + 3^2 + \ldots. + n^2) = k_2\delta^2\frac{2(2n^2 + 3n + 1)}{6} \qquad (g)$$

$$\text{fourth column}: \; -k_3\delta^3(1^3 + 2^3 + 3^3 + \ldots. + n3) = -k_3\delta^3\frac{n^2(n^2 + 2n + 1)}{4} \qquad (h)$$

The sums (f), (g) and (h) were written down using formulae [3]:

$$1 + 2 + 3 + \cdots + n = \frac{n}{2}(n + 1) \qquad (i)$$

$$1^2 + 2^2 + 3^2 + \ldots n^2 = \frac{n}{6}(2n^2 + 3n + 1) \qquad (j)$$

$$1^3 + 2^3 + 3^3 + \ldots n^3 = \frac{n^2}{4}(n^2 + 2n + 1) \qquad (k)$$

Let us add sums (e)÷(h) :

$$R_1 = n - k_1\delta\frac{n(n + 1)}{2} + k_2\delta^2\frac{n(2n^2 + 3n + 1)}{6} - k_3\delta^3\frac{n^2(n^2 + 2n + 1)}{4} \qquad (80)$$

Let us now simplify (80) by substituting

$$\delta = \frac{1}{n} \qquad (81)$$

The relationship (81) follows from the definition of δ (2z/a) and from the relationship $n = \frac{a}{2z}$.
We obtain the following form of (80)

$$R_1 = (1 - \frac{1}{2}k_1 + \frac{1}{3}k_2 - \frac{1}{4}k_3)n + \frac{1}{2}(k_2 - k_1 - k_3) + \frac{2k_2 - 3k_3}{12n}. \qquad (82)$$

After substituting to (82) the relations k_1, k_2 and k_3 [(33),(34) and (35)], the following equation is obtained

$$R_1 = (\frac{1}{2} - \frac{1}{6m_b} - \frac{1}{6m_c} + \frac{1}{12m_bm_c})n - \frac{1}{2} + \frac{2m_b + 2m_c - 1}{12m_bm_cn}. \qquad (83)$$

The expression in brackets on the right side of equation (83) is used for calculation of the value of coefficient φ_v [equation (1)]; hence formula (83) can be written down as

$$R_1 = \varphi_v n - \frac{1}{2} + \frac{2m_b + 2m_c - 1}{12m_bm_cn}. \qquad (84)$$

Let us substitute (84) to (76) and obtain a final formula for the calculation of $V_(c, n)'$

$$V'_{c,n} = v_o(\varphi_v n - \frac{1}{2} + \frac{2m_b + 2m_c - 1}{12m_bm_cn}). \qquad (85)$$

To produce a formula for the calculation of a mean volume of the piece in a given sequence, we shall divide $V'_{c,n}$ by n and obtain

$$\bar{v}'_{c,n} = \frac{V'_{c,n}}{n} = v_o\varphi'_{v,n}, \qquad (86)$$

where:

$$\varphi'_{v,n} = \varphi_v - \frac{1}{2n} + \frac{2m_b + 2m_c - 1}{12m_bm_cn^2}. \qquad (87)$$

The summation of R_2

$$R_2 = \sum_{i=0}^{n}(1 - \delta i)(1 - \frac{\delta i}{m_b})(1 - \frac{\delta i}{m_c}). \qquad (88)$$

For i=0, the first term in sum (88) is 1, and hence the calculated sum (88) will be larger by 1 than the sum R1, and so it will be

$$R_2 = R_1 + 1 \qquad (89)$$

or after allowing for (84)

$$R_2 = \varphi_v n + \frac{1}{2} + \frac{2m_b + 2m_c - 1}{12m_bm_cn}. \qquad (90)$$

Let us substitute (90) to (77) and obtain

$$V''_{c,n} = v_o(\varphi_v n + \frac{1}{2} + \frac{2m_b + 2m_c - 1}{12m_bm_cn}). \qquad (91)$$

Dividing (91) by n, a formula is obtained to calculate the mean volume of pieces in a given sequence of the pieces

$$\bar{v}''_{c,n} = \frac{V''_{c,n}}{n} = v_o\varphi''_{v,n}, \qquad (92)$$

where:

$$\varphi''_{v,n} = \varphi_v + \frac{1}{2n} + \frac{2m_b + 2m_c - 1}{12m_bm_cn^2}. \qquad (93)$$

Let us calculate the arithmetic means for the derived formulae (85) and (91) as well as (87) and (93) :

$$V_{c,n} = \frac{V'_{c,n} + V''_{c,n}}{2} = v_o(\varphi_v n + \frac{2m_b + 2m_c - 1}{12m_bm_cn}) \qquad (94)$$

$$\varphi_{v,n} = \frac{\varphi'_{v,n} + \varphi''_{v,n}}{2} = \varphi_v + \frac{2m_b + 2m_c - 1}{12 m_b m_c n^2}. \quad (95)$$

For n=∞, equation (95) is simplified to the form of equation (1).

For n< ∞, the difference in the values of the coefficients $\varphi_{v,n}$ and φ_v results from formula (95)

$$\Delta\varphi_{v,n} = \varphi_{v,n} - \varphi_v = \frac{2m_b + 2m_c - 1}{12 m_b m_c n^2}, \quad (96)$$

or (when the difference is expressed in percents)

$$R\varphi_{v,n} = \frac{\Delta\varphi_{v,n} 100}{\varphi_v} = \frac{(m_b + m_c - 1)100}{(6m_b m_c - m_b - m_c + 1)n^2}, \quad (97)$$

where: $R\varphi_{v,n}$ – the difference $\Delta\varphi_{v,n}$ referred to φ_v, %.

Calculations 3.

Using (97), let us calculate the values of $R\varphi_{v,n}$ for the three values of m_b and m_c and for the three values of n.

$m_b=m_c=1$ (spheres, cubes); $R\varphi_{v,n}=\frac{100}{n^2}$; n=2, $R\varphi_{v,2}=25$ %; n=3, $R\varphi_{v,3}=11{,}1$ %; n=5, $R\varphi_{v,5}=4$ %.

$m_b=m_c=5$ (square plates); $R\varphi_{v,n}= \frac{14{,}5}{n^2}$; n=2, $R\varphi_{v,2}=$ 3,625 %; n=3, $R\varphi_{v,3}=1{,}61$ %; n=5, $R\varphi_{v,5} = 0{,}58$ %.

$m_b= 1$, $m_c=3$; $R\varphi_{v,n}=\frac{63{,}64}{n^2}$; n=2, $R\varphi_{v,2}=15{,}91$ %; n=3, $R\varphi_{v,3}=7{,}07$ %; n=5, $R\varphi_{v,5}= 2{,}55$ %.

From the calculations it follows that the values of $R\varphi_{v,n}$ are decreasing with the increasing values of m_b, m_c and n. Let us accept as a maximum the value of $R\varphi_{v,n}=4$ %. For this value we can adopt in calculations $\boldsymbol{\varphi_{v,n} = \varphi_v}$.

6. The theory of deriving formula (71)

To obtain formula (71), the following equation expressing the surface of the burning or melting plate will be used (having reduced previously its every dimension by a value 2z·i, where 2z – is the difference in fundamental dimensions of the neighbouring plates in a given sequence of plates; i – is the number of the burnt or melted down plates of the thickness 2z)

$$f_i = 2(a - 2zi)(b - 2zi) + 2(a - 2zi)(c - 2zi) + 2(b - 2zi)(c - 2zi). \quad (98)$$

After multiplying out the brackets in (98) and arranging we obtain

$$f_i = 2(ab + ac + bc) - 4(a + b + c)2zi + 3(2zi)^2. \quad (99)$$

The first term on the right side of (99) allows for the initial surface of the plate f_o

$$f_o = 2(ab + ac + bc) = 2(m_b + m_c + m_b m_c)a^2 = 2Ma^2, \quad (100)$$

where:

$$M = m_b + m_c + m_b m_c.$$

Let us substitute to (99) the relationship (100), and extract it next from brackets

$$f_i = f_o[1 - \frac{4(a + b + c)2zi}{2Ma^2} + \frac{6(2zi)^2}{2Ma^2}]. \quad (101)$$

By substituting 2zi/a=δ to (101), we obtain

$$f_i = f_o[1 - \frac{2(1 + m_b + m_c)\delta i}{M} + \frac{3(\delta i)^2}{M}] \quad (102)$$

or

$$f_i = f_o(1 - k_4\delta i + k_5\delta^2 i^2), \quad (103)$$

where: k_4 and k_5 are given by formulae (47) and (48)

$$k_4 = \frac{2(1 + m_b + m_c)}{m_b + m_c + m_b m_c}; k_5 = \frac{3}{m_b + m_c + m_b m_c}.$$

Using (103), let us write down the total surface of the plates in a given sequence for the two cases of summation

$$F'_{c,n} = f_o \sum_{i=1}^{n} (1 - k_4\delta i + k_5\delta^2 i^2) \quad (104)$$

$$F''_{c,n} = f_o \sum_{i=0}^{n} (1 - k_4\delta i + k_5\delta^2 i^2). \quad (105)$$

Let us denote the sum in equation (104) with the symbol S_1, and in equation (105) with the symbol S_2

$$F'_{c,n} = f_o S_1 \quad (106)$$

$$F''_{c,n} = f_o S_2, \quad (107)$$

where

$$S_1 = \sum_{i=1}^{n} (1 - k_4\delta i + k_5\delta^2 i^2) \quad (108)$$

$$S_2 = \sum_{i=0}^{n}(1 - k_4\delta i + k_5\delta^2 i^2). \qquad (109)$$

Let us calculate successively the sums S_1 and S_2.

The sum S_1

Let us develop several terms in sum (108), i.e. for i=1, 2 and 3 and n:

$$i = 1 \qquad 1 - k_4\delta 1 + k_5\delta^2 1^2 \qquad (a)$$

$$i = 2 \qquad 1 - k_4\delta 2 + k_5\delta^2 2^2 \qquad (b)$$

$$i = 3 \qquad 1 - k_4\delta 3 + k_5\delta^2 3^2 \qquad (c)$$

. . . .

$$i = n \qquad 1 - k_4\delta 4 + k_5\delta^2 n^2 \qquad (d)$$

Let us add the columns of expressions (a)÷(d):

$$\text{first column}: \ 1 + 1 + 1 + \cdots + 1 = n \qquad (e)$$

$$\text{second column}: \ -k_4\delta(1+2+3+ .. +n) = -k_4\delta\frac{n(n+1)}{2} \qquad (f)$$

$$\text{third column}: \ k_5\delta^2\frac{n(2n^2 + 3n + 1)}{6}. \qquad (g)$$

Let us add the sums (e), (f) and (g):

$$S_1 = n - k_4\delta\frac{n(n + 1)}{2} + k_5\delta^2\frac{n(2n^2 + 3n + 1)}{6}. \qquad (110)$$

By substituting δ=1/n to (110), we obtain

$$S_1 = n - k_4\frac{n + 1}{2} + k_5\frac{2n^2 + 3n + 1}{6n}. \qquad (111)$$

Let us substitute to (111) the expressions for k_4 and k_5

$$S_1 = n - \frac{2(1 + m_b + m_c)}{M}\frac{n + 1}{2} + \frac{3(2n^2 + 3n + 1)}{6nM}. \qquad (112)$$

Let us make a series of transformations (112):

$$S_1 = \frac{1}{M}[nM - (1 + m_b + m_c)n - (1 + m_b + m_c) + n + \frac{3}{2} + \frac{1}{2n}] \qquad (113)$$

$$S_1 = \frac{1}{M}[m_b m_c n + \frac{1 - 2m_b - 2m_c}{2} + \frac{1}{2n}] \qquad (114)$$

$$S_1 = \varphi_f n + \frac{1 - 2m_b - 2m_c}{2M} + \frac{1}{2nM}, \qquad (115)$$

where:
$\varphi_f = \frac{m_b m_c}{M}$ formula (2).
Let us substitute (115) to (106)

$$F'_{c,n} = v_o[\varphi_f n + \frac{1 - 2m_b - 2m_c}{2M} + \frac{1}{2nM}] \qquad (116)$$

Let us now write down a formula for $F''_{c,n}$.

The sum S_2

For i=0, the first term in sum (109) is 1, and hence it is by 1 larger than the sum S_1

$$S_2 = S_1 + 1 = \varphi_f n + \frac{1 - 2m_b - 2m_c}{2M} + \frac{1}{2nM} + 1. \qquad (117)$$

Let us substitute (117) to (107)

$$F''_{c,n} = v_o[\varphi_f n + \frac{1 - 2m_b - 2m_c}{2M} + \frac{1}{2nM} + 1]. \qquad (118)$$

The arithmetic mean can be calculated from equations (116) and (118)

$$F_{c,n} = v_o[\varphi_f n + \frac{1 - 2m_b - 2m_c}{2M} + \frac{1}{2nM} + \frac{1}{2}]. \qquad (119)$$

Equation (119) is the searched equation for calculation of an arithmetic mean surface of the sequence of n pieces. To obtain a mean volume of the piece in a sequence of n pieces, let us divide (119) by n and obtain

$$\overline{v}_{c,n} = \frac{F_{c,n}}{n} = v_o[\varphi_f + frac1 - 2m_b - 2m_c 2Mn + \frac{1}{2n^2M} + \frac{1}{2n}]. \qquad (120)$$

Using (120), let us write down a formula for calculation of the ratio between the arithmetic mean volume of pieces in a sequence of n pieces and the initial volume of pieces

$$\varphi_{f,n} = \frac{\overline{v}_{c,n}}{v_o} = \varphi_f + \frac{1 - 2m_b - 2m_c}{2Mn} + \frac{1}{2n^2M} + \frac{1}{2n} \qquad (121)$$

or after transformation

$$\varphi_{f,n} = \varphi_f + \frac{1 + \frac{1}{m_b m_c} - \frac{1}{m_b} - \frac{1}{m_c}}{2n(1 + \frac{1}{m_b} + \frac{1}{m_c})} + \frac{1}{2n^2(m_b m_c + m_b + m_c)}, \tag{122}$$

Let us write down the special cases of formula (122): For m_b=m_c=1 (spheres, cubes), equation (122) is simplified t

$$\varphi_{f,n} = \varphi_f + \frac{1}{6n^2}. \tag{123}$$

For m_b = m_c= 5, equation (122) is simplified to

$$\varphi_{f,n} = \varphi_f + \frac{2}{9n} + \frac{1}{70n^2}. \tag{124}$$

For m_b = m_c = 10, equation (122) is simplified to

$$\varphi_{f,n} = \varphi_f + \frac{81}{240n} + \frac{1}{240n^2}. \tag{125}$$

For m_b = m_c = ∞, formula (122) is simplified to: $\varphi_{f,n} = 1 + \frac{1}{2n}$, but when n=∞ it takes the form of formula (2), i.e. $\varphi_{f,n}$= φ_f.
Using (122), let us write down a formula for calculation of a difference in the values of coefficients $\varphi_{f,n}$ and φ_f

$$\Delta\varphi_{f,n} = \varphi_{f,n} - \varphi_f = \frac{1 + m_b m_c - m_b - m_c}{2Mn + \frac{1}{2n^2 M}} \tag{126}$$

or (when the value is expressed in percents)

$$R\varphi_{f,n} = \frac{\Delta\varphi_{f,n}}{\varphi_f}100 \tag{127}$$

<u>Calculations 4.</u>

Let us calculate the values of $\Delta\varphi_{f,n}$ and $R\varphi_{f,n}$ for the three values of m_b and m_c and for the three values of n.
m_b=m_c=1 (spheres, cubes), φ_f=1/3, M=3: n=2, $\Delta\varphi_{f,2}$ = 0,0417, $R\varphi_{f,2}$= 12,55 %;
n=3, $\Delta\varphi_{f,3}$=0,0185, $R\varphi_{f,3}$=5,55 %; n=5, $\Delta\varphi_{f,5}$ =6,67·10^{-3}, $R\varphi_{f,5}$=2 %.
m_b=m_c=5, φ_f=0,714, φ_f=0,714, M=35: n=2, $\Delta\varphi_{f,2}$=0,118, $R\varphi_{f,2}$=16,5 %; n=3,
$\Delta\varphi_{f,3}$=0,078, $R\varphi_{f,3}$=11%; n=5, $\Delta\varphi_{f,5}$=0,0642, $R\varphi_{f,5}$=6,5 %.
m_b=m_c=10, φ_f=0,833, M=120: n=2, $\Delta\varphi_{f,2}$=0,17, $R\varphi_{f,2}$=20,4 %; n=3, $\Delta\varphi_{f,3}$=0,113,
$R\varphi_{f,3}$=13,6 %; n=5, $\Delta\varphi_{f,5}$=0,068, $R\varphi_{f,5}$=8 %.
For m_b = m_c = ∞ and n = 5, the value of $\varphi_{f,5}$ = 1,1 , while the value of $\Delta\varphi_{f,5}$=10 %.
From the calculations given above it follows that in practice as soon as we have n=5, formulae (1) and (2) can replace formulae (70) and (71). This solves the problem of calculating the height of combustion zone and melting zone, and of calculating the structure of the zones and other characteristic parameters, e.g. the surface development and output.
Studies [1,2] describe the first attempts at an application of coefficients φ_v and φ_f in calculation of the height of combustion zone and melting zone.

7. Derivation of formulae for calculation of the combustion/melting zone height

Let us apply now the examined paradigm in derivation of the fundamental formulae describing the height of combustion zone and melting zone.

<u>A formula for calculation of the combustion zone height</u>

The following model assumptions have been adapted:
- the combustion zone is located between the level of the tuyères (single-row cupolas are taken into consideration) and lower boundary of the melting zone,
- on passing from the combustion zone to the melting zone, the pieces of coke are characterised by the same weight, the same dimensions, and the same physico-chemical properties (reactivity); the coke weight losses above the combustion zone for CO_2 reduction are neglected;
- the burning pieces of coke form the decreasing sequences of pieces characterised by a random distribution in the zone,
- the bulk density of coke is equal within the whole zone height,
- the mean volume and the mean surface of the burning pieces of coke equals their integral mean,
- along the whole height of the zone, the linear burning rate of the coke is the same,
- the content of carbon monoxide at the upper boundary of the combustion zone is known in advance.

Let us now express with a formula the mass velocity of the coke burning process

$$\dot{m}_k = \mu_k \overline{F}_k \rho_k, \tag{128}$$

where:
$\dot{m}_k$ – the mass velocity of the coke burning process, kg/s
μ_k – the linear velocity of the coke burning process,
$\overline{F}_k$ – the total surface area of the burning pieces of coke (the combustion zone development surface), m^2
ρ_k – the coke pieces density, kg/m^3.

The surface $\overline{F}_k$ can be expressed with the following formula

$$\overline{F}_k = n_k \overline{f}_k, \qquad (129)$$

where:
n_k – the number of the pieces of coke in the zone,
$\overline{f}_k$ – the integral mean surface of the pieces of coke in the zone, m^2.
Now n_k can be expressed with the following formula

$$n_k = \frac{H_s F_{\dot{z}} \rho_{n,k}}{\overline{v}_k \rho_k}, \qquad (130)$$

where:
H_s – the height of the combustion zone, m
$F_{\dot{z}}$ – the internal cupola cross-section in the combustion zone, m^2,
$\rho_{n,k}$ – the coke bulk density in the zone, kg/m^3
$\overline{v}_k$ – the integral mean volume of the pieces of coke in the zone, m^3.
Let us substitute relationships (129) and (130) to (128) and transform the whole to

$$H_s = \frac{\dot{m}_k \overline{r}_k}{F_{\dot{z}} \mu_k \rho_{n,k}}, \qquad (131)$$

where: $\overline{r}_k = \frac{\overline{v}_k}{\overline{f}_k}$ – the integral mean modulus of the pieces of coke in the zone, m.
Let us substitute $\frac{\dot{m}_k}{F_{\dot{z}}} = \frac{P_F}{L_{k,2}}$ to (131) (where: P_F – the blast air volume fed to the combustion zone under normal conditions, $m^3/(m^2 \cdot s)$; $L_{k,2}$ – the blast air volume used to burn down 1 kg coke, calculated from the chemical composition of gases at the upper boundary of combustion zone, m^3/kg coke). Thus, a fundamental form of the equation for calculation of the combustion zone height is obtained

$$H_s = \frac{P_F \overline{r}_k}{\mu_k L_{k,2} \rho_{n,k}} \qquad (132)$$

or

$$H_s = \frac{P_F \tau_{a,k} \varphi_{f,k}}{L_{k,2} \rho_{n,k}}, \qquad (133)$$

where:
$\tau_{a,k}$ – the time necessary for burning down a piece of coke of the thickness a, s
$\varphi_{f,k}$ – the coefficient (2) valid for the pieces of coke.

A formula for calculation of the melting zone height

The following model assumptions have been adapted:

- the melting zone is located in the cupola shaft between the upper boundary of the combustion zone and the lower boundary of the preheating zone; it is a mixture of the melting pieces of metal and the, non-reacting with gases, pieces of coke (the coke weight losses for the CO_2 reaction of decomposition are neglected),
- on passing from the preheating zone to the melting zone, the pieces of metal are characterised by the same shape, the same weight, and the same melting point,
- the mean volume and the mean surface of the melting pieces of metal are equal to their integral mean; the melting pieces of metal form sequences characterised by a random (casual) distribution in the mass of coke,
- the pieces of metal are melting at a mean linear rate equal for the whole zone,
- the height of the melting zone is equal to the sum of the bulk height of the melting pieces of metal and the bulk height of the pieces of coke present in the zone (equal sums of the bulk heights of metal and coke separately and after mixing have been assumed).

Using the last assumption let us write down the sum of the bulk heights of metal and coke in the melting zone

$$H_t F_{\dot{z}} = \frac{\overline{M}_{m,t}}{\rho_{n,m}} + \frac{M_{k,t}}{\rho_{n,k}}, \qquad (134)$$

where:
H_t – the height of the melting zone, m
$F_{\dot{z}}$ – the internal mean cross-section of the melting zone, m^2
$\overline{M}_{m,t}$ – the weight of the melting pieces of metal in the zone (the integral mean), kg
$M_{k,t}$ – the weight of the pieces of coke in the melting zone,kg
$\rho_{n,m}$ – the bulk density of the pieces of metal, kg/m^3.
Let us write down the right side of equation (134)

$$H_t = \frac{\overline{M}_{m,t}}{F_{\dot{z}} \rho_{n,m}} K_{\rho,t}, \qquad (135)$$

where:

$$K_{\rho,t} = 1 + \frac{M_{k,t}}{\overline{M}_{m,t}} \frac{\rho_{n,m}}{\rho_{n,k}} = 1 + \frac{K_{w,4}}{100 \varphi_{v,m}} \frac{\rho_{n,m}}{\rho_{n,k}} \qquad (136)$$

where:
$K_{w,4} = \frac{M_{k,t}}{M_{m,t}} 100 = \frac{m_{n,k}}{m_{n,m}} 100$ - the coke charge consumption rate, kg coke/100 kg Fe
$m_{n,k}$ – the weight of the coke charge burden, kg
$m_{n,m}$ – the weight of the metal burden, kg
$\varphi_{v,m}$ – the coefficient (1) valid for the melting pieces of metal
$\overline{M}_{m,t} = M_{m,t} \varphi_{v,m}$.
By substituting to (135)

$$\frac{\overline{M}_{m,t}}{F_{\dot{z}}} = S_F \tau_{\bar{r}_m} \quad (137)$$

we obtain

$$H_t = \frac{S_F \tau_{\bar{r}_m}}{\rho_{n,m}} K_{\rho,t}, \quad (138)$$

where:

$\tau_{\bar{r}_m}$ – the time necessary for melting the pieces of metal of an integral mean modulus $\bar{r}_m$, s

$$S_F = \frac{\overline{M}_{m,t}}{F_{\dot{z}} \tau_{\bar{r}_m}} \; -- \text{ melting rate, kg Fe/(m}^2 \cdot \text{s)}.$$

Equation (138) can also be written down using relationships resulting from the paradigm

$$H_t = \frac{S_F \tau_{a,m} \varphi_{f,m}}{\rho_{n,m}} K_{\rho,t}, \quad (139)$$

where:

$\varphi_{f,m}$ – the coefficient (2) valid for the pieces of metal,

$\tau_{a,m}$ – the melting time required for melting down a piece of metal of the thickness a , s $\tau_a \varphi_{f,m} = \tau_{\bar{r}_m}$.

Equation (138) is one of the fundamental mutations of the equations used for calculation of the melting zone height. It requires data regarding the melting time $\tau_{a,m}$, which is calculated from the theoretical or empirical formulae, and data on the output S_F, which is calculated from J. Buzek formula.

8. Concluding remarks

According to [4]. "paradigm is a flexible research standard comprising a set of concepts, values and practices that schedule the consecutive research".

Paradigm is formulated for a group of associate problems for which no logical theory and analytical description have been developed, or when the existing theory has been burdened with too many anomalies. Both these situations are relevant in the case of models and analytical descriptions referring to the height and structure of the combustion zone and melting zone in coke-fired cupolas. From the study the following conclusions follow:

- ♦ the equations derived for the calculation of coefficients φ_v and φ_f are functions of the relative dimensions of the pieces of coke and metal m_b and mc;
- ♦ the accuracy of the paradigm, characterised by the main quantities a_f, b_f, c_f and a_v, b_v, c_v, increases with increasing values of m_b and mc;
- ♦ applied in calculation of the cupola zone structure, the paradigm yields sufficiently accurate results when the number of the pieces of metal or coke in the formed sequences is at least 5;
- ♦ the calculation of the structure of combustion zone starts with calculation of its height, followed by calculation of a total volume of the pieces of coke present in this zone, and next – using the concept of integral mean volume, the number of the burning pieces of coke in the zone is calculated; the next operation uses respective formulae and consists in calculating the elements of the zone structure (e.g. distribution of the coke pieces size, zone development surface area); preserving the same order of operations, the number of the pieces of metal melting in the melting zone and the number of the elements forming the zone structure are calculated;
- ♦ for practical application of equation (134) it is necessary to assume the CO content in gas at the upper boundary of combustion zone and to be familiar with the formula used in calculation of the linear coke burning rate [5];
- ♦ the role of equation (139) is to inform us only that it has been derived using a paradigm; the use of the equation in practical calculations requires its further transformations, which consist, among others, in introducing the values of gas temperature at an inlet to and outlet from the melting zone.

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