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NATURAL VIBRATIONS OF LONG VIBRATORY CONVEYERS

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Analysis of natural vibrations of vibratory conveyers troughs supported by a leaf spring system on a vibroinsulated frame, utilised in metallurgical industry, is presented in the paper. The analysis of vibrations was done by decomposing the system and separating the solution of the problem in terms of two separate analyses. The first analysis for the system EI_b and $EI_r \rightarrow \infty$, causes discretisation of the system and allows to determine its first four natural frequencies. The second one is the analysis of the continuous system: body – frame – frame supporting system, which allows to find the remaining natural frequencies.

The methodology of an analytical estimation of natural frequencies of conveyers supported on vibroinsulating frames is given in the paper. In order to verify the correctness of analytical equations the vibration frequencies of the conveyer with typical parameters were determined and then compared with the results obtained in computer simulation by the Finite Element Method. It has been proved that the mathematical model provides the correct results for transverse vibration of the system at frequencies up to 100[Hz]. Above this frequency the influence of axial deflections of leaf springs is clearly visible.

W pracy przedstawiono analizę drgań własnych rynien przenośników wibracyjnych stosowanych w przemyśle hutniczym podpartych przy pomocy układu listew resorujących na wibroizolowanej ramie. Analizę drgań dokonano przez dekompozycję układu i rozdzielanie zagadnienia na analizę układu dla EI_b i $EI_r \rightarrow \infty$, co prowadzi do dyskretyzacji układu i pozwala na wyznaczenie czterech pierwszych częstości własnych układu i na analizę układu ciągłego: korpus – rama – układ podparcia rami, co prowadzi do znalezienia pozostałych częstości własnych.

Praca przedstawia metodologię analitycznego wyznaczenia częstości drgań własnych przenośników podpartych na wibroizolowanej ramie. W celu sprawdzenia poprawności równań analitycznych wyznaczono częstości drgań przenośnika o typowych parametrach i porównano je z wynikami uzyskanymi na drodze symulacji komputerowej metodą elementów skończonych. Wykazano że model matematyczny daje poprawne wyniki dla częstotliwości drgań giętych układu do 100[Hz], powyżej tej częstotliwości zaznacza się wyraźnie wpływ ugięcia poosiowego listew resorujących.

1. Introduction

Vibratory conveyers are utilised in metallurgical industry for continuous transport – usually at short distances up to 20m – of hot materials (furnace slag, small steel elements etc.), caustic substances or substances emitting gases hazardous for an environment. In addition, vibratory conveyers enable cooling of feeds, recovery of heat from the transported materials (used later e.g. for warming furnace blowers), drying, humidifying etc. Furthermore they allow transporting in closed conduits.

Free vibrations of troughs of the vibratory conveyers, revealing themselves as double-sided bending of those troughs in the plane of symmetry of the conveyer, disturb the transporting process and therefore constitute

one of the main constructional problems of the conveyers of several meter lengths, at an average operational frequency being in the range of 13 to 50 Hz [1]. Knowledge of the basic frequencies of natural vibrations allows to reconstruct the system in such a way that the forced frequency of the vibrator will not occur in the vicinity of any natural frequency.

This problem, analysed in detail [2] for short and medium length conveyers supported on elements with omni-directional compliance (coil springs, rubber vibroinsulators), was not – until recently – discussed sufficiently in relation to long conveyers supported usually on a system of parallel steel leaf springs.

Connection of troughs with the stiff foundation by means of a system of leaf springs stiffens troughs but

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does not completely protect against transverse vibrations [3]. Moreover, cause transferring of large dynamic loads on the foundation – in the spring axial direction [4]. Due to this feature, long conveyers are often supported on heavy frames vibroinsulated from the foundation. In such case the trough co-operates in bending with the frame creating jointly the system presented in Fig. 1.

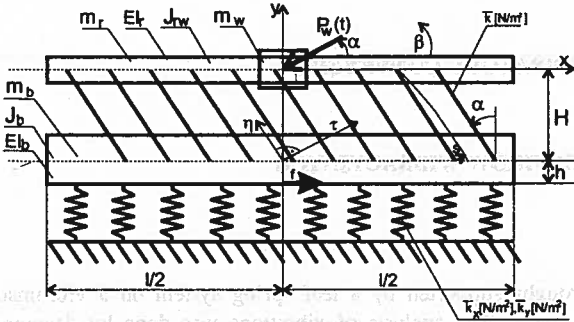


Fig. 1. Model of the conveyor with the vibroinsulating frame

As it will be shown in the further part of the paper, the analysis of vibrations for this type of construction – for the typical parameter values – can be done by decomposing the system and dividing the problem into:

- Analysis of the system for EI_b and $EI_r \rightarrow \infty$, which leads to discretising of the system and allows determining its first four natural vibrations.
- Analysis of the continuous system: trough – frame – system of frame support, which allows finding the remaining natural vibrations.

2. Determination of the first four natural frequencies for the system: trough-vibroinsulating frame. Analysis of the discrete system.

Let's consider the system presented in Fig. 1 as a system of solids: trough – frame, connected mutually and with the foundation by elastic elements with constants k , k_x , k_y .

The Lagrangian Function of the system takes a following form:

$$E_k - E_p = \frac{1}{2}(m_r + m_w) \cdot (\dot{x}^2 + \dot{y}^2) + \frac{1}{2}J_{rw}\dot{\beta}^2 + \frac{1}{2}m_b [(\dot{x} + \dot{s} \cos \alpha + H\dot{\beta})^2 + (\dot{y} + \dot{s} \sin \alpha)^2] + \frac{1}{2}J_b\dot{\beta}^2 - \int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{1}{2}\bar{k}_y(y + s \cdot \sin \alpha + \beta \cdot f)^2 df - \frac{1}{2}\bar{k}_x l [x + s \cdot \cos \alpha + (H + h)\beta]^2 - \frac{1}{2}\bar{k}_l s^2, \tag{1}$$

where:

- x, y – Co-ordinates of the mass centre of the trough together with the vibrator,
- β – Angle of rotation of the trough,
- s – Spring deflection, measured as in Fig. 1,
- f – Auxiliary co-ordinate, as in Fig. 1,
- m_r – Mass of the trough,
- m_w – Total mass of the vibrator,
- J_{rw} – Total mass moment of inertia of the trough with the vibrator versus the mass centre of $m_r + m_w$.
- m_b – Mass of the vibroinsulating frame,
- J_b – Mass moment of inertia of the vibroinsulating frame versus the mass centre of m_b ,
- \bar{k} – Stiffness coefficient of leaf springs in direction s for the unit of length of the trough,
- \bar{k}_x – Stiffness coefficient of coil springs in direction x for the unit of length of the trough,
- \bar{k}_y – Stiffness coefficient of coil springs in direction y for the unit of length of the trough.

Remaining parameters, as in Fig. 1.

The equations of motion for this system are as follows:

$$\begin{bmatrix} m_r + m_w + m_b & 0 & m_b \cdot H & m_b \cdot \cos \alpha \\ 0 & m_r + m_w + m_b & 0 & m_b \cdot \sin \alpha \\ m_b \cdot H & 0 & J_{rw} + J_b + m_b \cdot H^2 & m_b \cdot H \cdot \cos \alpha \\ m_b \cdot \cos \alpha & m_b \cdot \sin \alpha & m_b \cdot H \cdot \cos \alpha & m_b \end{bmatrix} \cdot \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\beta} \\ \ddot{s} \end{bmatrix} + \begin{bmatrix} \bar{k}_x l & 0 & \bar{k}_x l (H + h) & \bar{k}_x l \cos \alpha \\ 0 & \bar{k}_y l & 0 & \bar{k}_y l \sin \alpha \\ \bar{k}_x l (H + h) & 0 & l^3 \bar{k}_y / 12 + \bar{k}_x l (H^2 + h^2 + 2hH) & \bar{k}_x l \cos \alpha (H + h) \\ \bar{k}_x l \cos \alpha & \bar{k}_y l \sin \alpha & \bar{k}_x l \cos \alpha (H + h) & l(\bar{k}_y \sin^2 \alpha + \bar{k}_x \cos^2 \alpha + \bar{k}_s) \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ \beta \\ s \end{bmatrix} = [0]. \tag{2}$$

Expecting the solution as:

$$\begin{aligned} x &= a \cos(\omega t - \psi) \\ y &= b \cos(\omega t - \psi) \\ \beta &= c \cos(\omega t - \psi) \\ s &= d \cos(\omega t - \psi) \quad \psi = \text{const} \end{aligned} \quad (3)$$

and substituting functions (3) and their derivatives into equations of motion (2) the set of homogeneous equa-

tions is obtained with amplitudes a , b , c and d as unknown values. Due to the physical meaning of amplitudes this system should have a non-zero solution. The condition necessary for obtaining non-zero solutions of a homogeneous set of equations is that the determinant – formed of coefficients at unknowns – equals zero. In consequence, we obtain equations (4) for the natural frequencies from ω_1 to ω_4 .

$$\begin{vmatrix} -\omega^2(m_r + m_w + m_b) + \bar{k}_x l & 0 & -\omega^2(m_b H) + \bar{k}_x l (H + h) & -\omega^2(m_b \cos \alpha) + \bar{k}_x l \cos \alpha \\ 0 & -\omega^2(m_r + m_w + m_b) + \bar{k}_y l & 0 & -\omega^2(m_b \sin \alpha) + \bar{k}_y l \sin \alpha \\ -\omega^2(m_b \cdot H) + \bar{k}_x l (H + h) & 0 & -\omega^2(J_{rw} + J_b + m_b \cdot H^2) + l^3 \bar{k}_y / 12 + k_x l (H^2 + h^2 + 2hH) & -\omega^2(m_b \cdot H \cdot \cos \alpha) + \bar{k}_x l \cos \alpha (H + h) \\ -\omega^2(m_b \cdot \cos \alpha) + \bar{k}_x l \cos \alpha & -\omega^2(m_b \cdot \sin \alpha) + \bar{k}_y l \sin \alpha & -\omega^2(m_b \cdot H \cdot \cos \alpha) + \bar{k}_x l \cos \alpha (H + h) & -\omega^2 m_b + l(\bar{k}_y \sin^2 \alpha + \bar{k}_x \cos^2 \alpha + k_s) \end{vmatrix} = 0. \quad (4)$$

In order to verify the correctness of the given above calculations and to compare the obtained results with the simulation results, the first four natural frequencies

for typical parameters of the vibratory conveyer were determined for machine parameters:

$$\begin{aligned} m_b &= 9596 [kg] \\ m_r &= 2256 [kg] \\ m_w &= 850 [kg] \\ H &= 0,3784 [m] \\ h &= 0 [m] \\ l &= 16 [m] \\ J_{rw} &:= 48128 [kg \cdot m^2] \\ J_b &= 204714 [kg \cdot m^2] \\ \alpha &= \frac{\pi}{6} \\ \bar{k}_x &= 112490 \left[\frac{N}{m^2} \right] \\ \bar{k}_y &= 112490 \left[\frac{N}{m^2} \right] \\ \bar{k} &= 95734 \left[\frac{N}{m^2} \right] \end{aligned} \quad (5)$$

Substituting the above parameters into the expanded determinant (4) we obtain the frequency equation as:

$$0,95754 \cdot \omega^8 - 0,10782 \cdot 10^4 \omega^6 + 0,34202 \cdot 10^6 \omega^4 - 0,428 \cdot 10^8 \omega^2 + 0,18804 \cdot 10^{10} = 0. \quad (6)$$

The graphical interpretation of equation (6) is presented in Fig. 2:

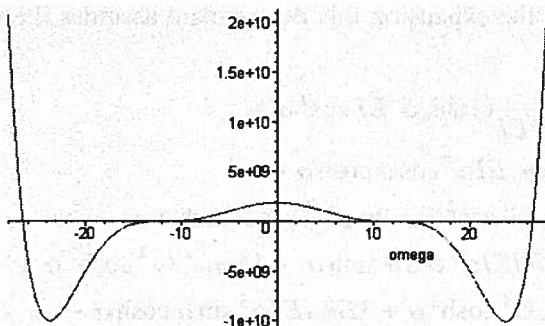


Fig. 2. Graphical interpretation of equation (6)

Equation (6) has four positive roots, out of which three are within the range from $\omega = 10$ to $\omega = 14$.

The natural frequencies $f_i = \frac{\omega_i}{2\pi}$, obtained from solving equation (6) are as follows:

$$\begin{aligned} f_1 &= 1.81 [\text{Hz}] \\ f_2 &= 1.89 [\text{Hz}] \\ f_3 &= 1.97 [\text{Hz}] \\ f_4 &= 4.17 [\text{Hz}] \end{aligned}$$

3. Determination of the natural frequencies for the system: trough-vibroinsulating frame. Analysis of the continuous system.

Utilising the stability of the deformed form of the system (Fig. 1) for $s \cong 0$ and $0 \leq \alpha < \pi/2$, the substitute scheme for the symmetric vibrations – as shown in Fig. 3 – was assumed. In this scheme the system: trough – frame constitutes a beam with mass m , where

$$\begin{aligned} \bar{m} &= \frac{m}{l} \\ m &= m_b + m_r \end{aligned} \quad (7)$$

$$EI = EI_b + EI_r \tag{8}$$

$$P(t) = P_w(t) \sin(\alpha) \tag{8}$$

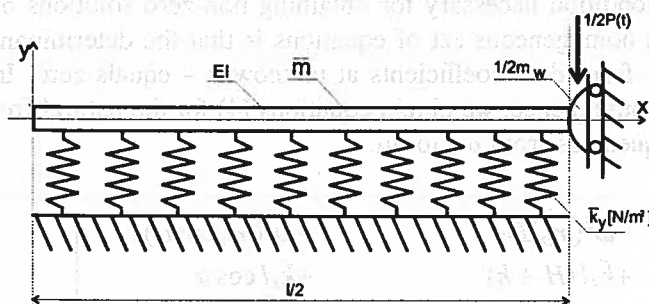


Fig. 3. Substitute scheme for symmetric vibrations

The presented above scheme remains the proper one as long as the axial compliance of springs interconnecting the trough with the frame can be omitted. This assumption is the right one for the first few forms of vibrations – as the analysis performed by the Geometric Element Method showed.

The equation of natural vibrations of the system presented in Fig. 3 is given below:

$$EI \frac{\partial^4 y}{\partial x^4} + \bar{m} \frac{\partial^2 y}{\partial t^2} + \bar{k}_y y = 0. \tag{9}$$

This equation should be supplemented with the boundary conditions for the symmetric forms of vibrations:

$$a) \frac{\partial^3 y}{\partial x^3}(0, t) = 0, \quad b) \frac{\partial^2 y}{\partial x^2}(0, t) = 0$$

$$c) \frac{\partial y}{\partial x} \left(\frac{l}{2}, t \right) = 0, \quad d) \frac{1}{2} m_w \frac{\partial^2 y}{\partial x^2} \left(\frac{l}{2}, t \right) = EI \frac{\partial^3 y}{\partial x^3} \left(\frac{l}{2}, t \right).$$

Looking for solution of natural frequencies formulated as:

$$y(x, t) = f_x(x) \cdot f_t(t), \tag{11}$$

we obtain the set of equations:

$$f_x^{(4)} - \lambda^4 f_x = 0 \tag{12}$$

$$f_t^{(2)} + \left(\lambda^4 + \frac{\bar{k}_y}{EI} \right) \frac{EI}{\bar{m}} f_t = 0 \tag{13}$$

Denoting

$$\omega_n = \sqrt{\left(\lambda^2 + \frac{\bar{k}_y}{EI} \right) \frac{EI}{\bar{m}}}. \tag{14}$$

Solutions of those equations can be written as:

$$f_x = C_1 \sin(\lambda x) + C_2 \cos(\lambda x) + C_3 \sinh(\lambda x) + C_4 \cosh(\lambda x) \tag{15}$$

$$f_t = D_1 \sin(\omega t) + D_2 \cos(\omega t) \tag{16}$$

$$\alpha = \lambda \cdot \frac{l}{2}. \tag{17}$$

After substituting those functions into the boundary conditions (a) and (b) we obtain for $m_w \neq 0$:

$$C_1 = C_3, \quad C_2 = C_4. \tag{18}$$

While the remaining conditions conduce to the matrix equation:

$$\begin{bmatrix} a_1(\alpha) & a_2(\alpha) \\ a_3(\alpha) & a_4(\alpha) \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = 0, \tag{19}$$

where:

$$a_1 = \cos(\alpha) + \cosh(\alpha)$$

$$a_2 = \sinh(\alpha) - \sin(\alpha)$$

$$a_3 = \frac{1}{2} \frac{m_w}{\bar{m}} \left(\alpha^4 \frac{16}{l^4} + \frac{\bar{k}_y}{EI} \right) (\sin(\alpha) + \sinh(\alpha)) + \alpha^3 \frac{8}{l^3} (-\cos(\alpha) + \cosh(\alpha))$$

$$a_4 = \frac{1}{2} \frac{m_w}{\bar{m}} \left(\alpha^4 \frac{16}{l^4} + \frac{\bar{k}_y}{EI} \right) (\cos(\alpha) + \cosh(\alpha)) + \alpha^3 \frac{8}{l^3} (\sin(\alpha) + \sinh(\alpha))$$

$$\alpha = \lambda \frac{l}{2}.$$

The condition for obtaining the non-zero solution of equation (19) is:

$$\det \begin{bmatrix} a_1(\alpha) & a_2(\alpha) \\ a_3(\alpha) & a_4(\alpha) \end{bmatrix} = 0. \tag{21}$$

After the expansion this determinant assumes the form

$$\frac{1}{2\bar{m}_c l^4 EI} (16m_w \alpha^4 EI \cos^2 \alpha + 32m_w EI \alpha^4 \cos \alpha \cosh \alpha + m_w k l^4 \cos^2 \alpha + 2m_w k l^4 \cos \alpha \cosh \alpha + 32\bar{m} l EI \alpha^3 \cos \alpha \sinh \alpha + 16m_w EI \alpha^4 \cosh^2 \alpha + m_w k l^4 \cosh^2 \alpha + 32\bar{m} l EI \alpha^3 \sin \alpha \cosh \alpha - 16m_w EI \alpha^4 \sinh^2 \alpha - m_w k l^4 \sinh^2 \alpha + 16m_w \alpha^4 EI \sin^2 \alpha + m_w k l^4 \sin^2 \alpha) = 0. \tag{22}$$

which leads to the transcendental equation in relation to α . Values $\omega_1, \omega_2 \dots \omega_n$ and natural frequencies $f_1, f_2 \dots f_n$, given by equation (14) correspond to successive roots of the transcendental equation $\alpha_1, \alpha_1 \dots \alpha_n$. Due to the reasons discussed in the introduction, the high accuracy can be obtained usually for the first four natural frequencies of the continuous system: frame – trough. Forms of natural vibrations corresponding to the successive frequencies can be determined by e.g. calculating the ratio C_1/C_2 from equation (19). Taking into account that $C_1 = C_3$ and $C_2 = C_4$ we obtain:

$$\begin{aligned} C_2 &= C_1 \frac{\cos(\alpha_n) + \cosh(\alpha_n)}{\sin(\alpha_n) - \sinh(\alpha_n)} \\ C_4 &= C_1 \frac{\cos(\alpha_n) + \cosh(\alpha_n)}{\sin(\alpha_n) - \sinh(\alpha_n)}. \end{aligned} \quad (23)$$

Those dependencies, after including that:

$$\lambda_n = \frac{2\alpha_n}{l}, \quad (24)$$

determine the natural vibration of the transverse form with the accuracy to the constant.

In order to compare the results of the introduced above considerations with the results obtained by the computer simulation applying the Geometric Elements Method the first few natural frequencies of the system were determined – for the typical parameters of the vibratory conveyer:

$$\begin{aligned} \bar{m} &= \frac{m}{l} = \frac{9596 + 2856}{16} = 778.25 \left[\frac{kg}{m} \right] \\ EI &= EI_b + EI_r = 841705 \cdot 10^{10} [Nm^2] \\ \bar{k}_y &= 112490 \left[\frac{N}{m^2} \right]. \end{aligned}$$

The remaining parameters are given in equation (5). Substituting those values to equation (22) we obtain:

$$\begin{aligned} 0.0266 \cdot \alpha^4 \cos \alpha \cosh \alpha + 0.1459 \cdot \cos \alpha \cosh \alpha + \\ + 0.3906 \cdot \alpha^3 \cos \alpha \sinh \alpha + 0.3906 \cdot \alpha^3 \sin \alpha \cosh \alpha + \\ + 0.0266\alpha^4 + 0.1459 = 0 \end{aligned} \quad (25)$$

Successive roots of this equation correspond to the respective natural frequencies (on the basis of equation (14)). Thus, the successive natural frequencies of the continuous system, where:

$f_i = \frac{\omega_i}{2\pi}$, are as follows:

$$\begin{aligned} f_1 &= 4.74 \text{ [Hz]} \\ f_2 &= 23.5 \text{ [Hz]} \\ f_3 &= 57.5 \text{ [Hz]} \\ f_4 &= 108.2 \text{ [Hz]} \\ f_5 &= 174.5 \text{ [Hz]}. \end{aligned}$$

Those frequencies correspond to the transverse symmetric vibration forms. Since vibrations of anti-symmetric form will not be excited in typical systems: trough-frame, with the drive in the middle of the trough, their determination seems not necessary.

4. Determination of the frequency and form of natural vibrations by the computer simulation utilising the Geometric Element Method.

To verify the correctness of analytical calculation the natural frequencies of the system: trough-vibroinsulating frame (presented in Fig. 1.), were determined by the Finite Element Method (Geometric) with an application of the software package Pro/Mechanica.

A sector of the simulated model is presented in Fig. 4. Leaf springs are modelled by two coil springs, one of which is of a small stiffness in the vibration direction while the second is of much higher stiffness in the axial direction of the spring. Such modelling decreases the time of numerical calculations increasing simultaneously their accuracy. This happens due to assigning one equation to each spring in the model.

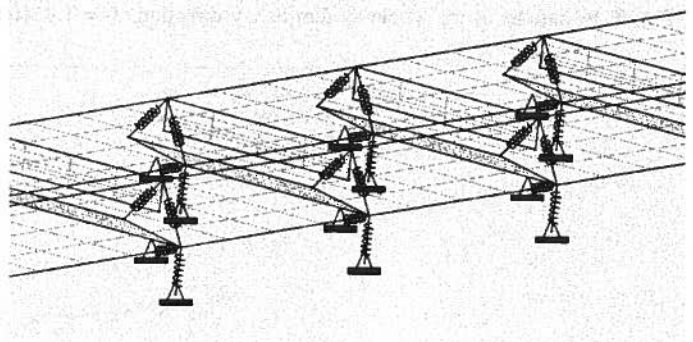


Fig. 4. Simulated system

Graphs in Figures 5 to 15 present successive forms of vibrations of the system: trough-vibroinsulating frame as well as the corresponding natural frequencies. The solid line shows the input state of the system, while the broken line illustrates the deformation of the system corresponding to the successive forms of natural vibrations.

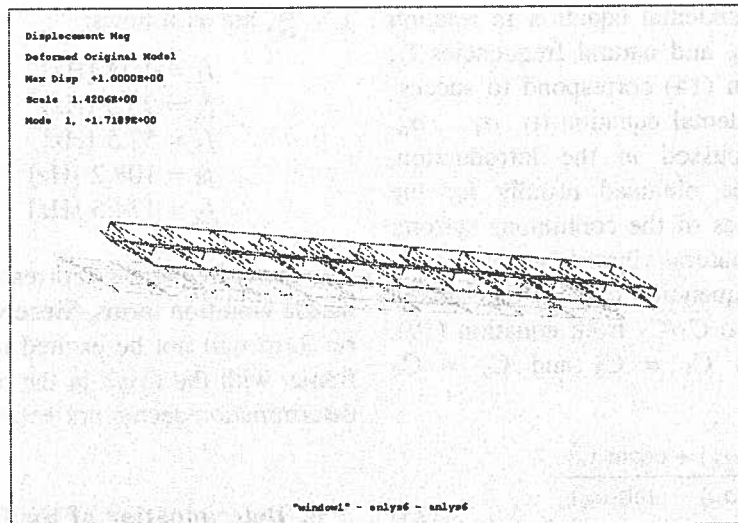


Fig. 5. Vibrations of the whole system in x-x direction. $f_1 = 1.718[\text{Hz}]$

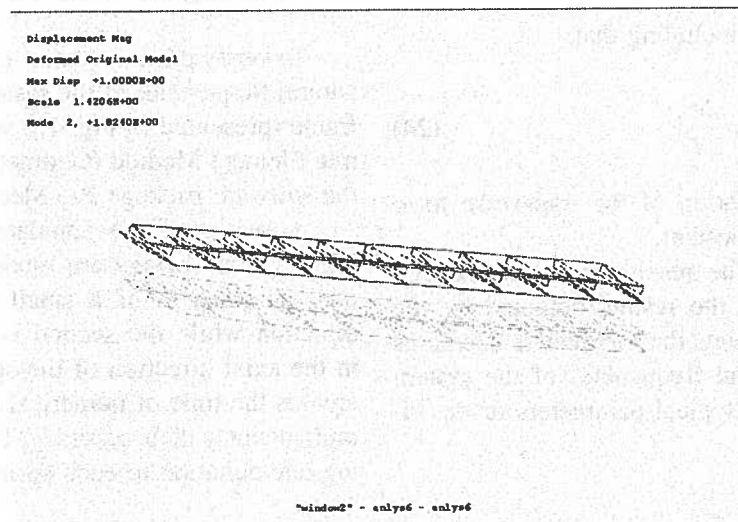


Fig. 6. Vibrations of the whole system in y-y direction. $f_2 = 1.824[\text{Hz}]$

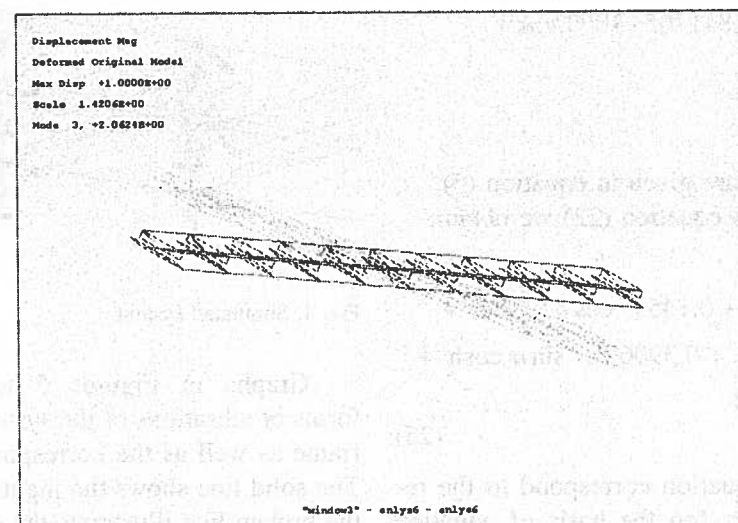


Fig. 7. Rotational vibrations β of the whole system, $f_3 = 2.062[\text{Hz}]$

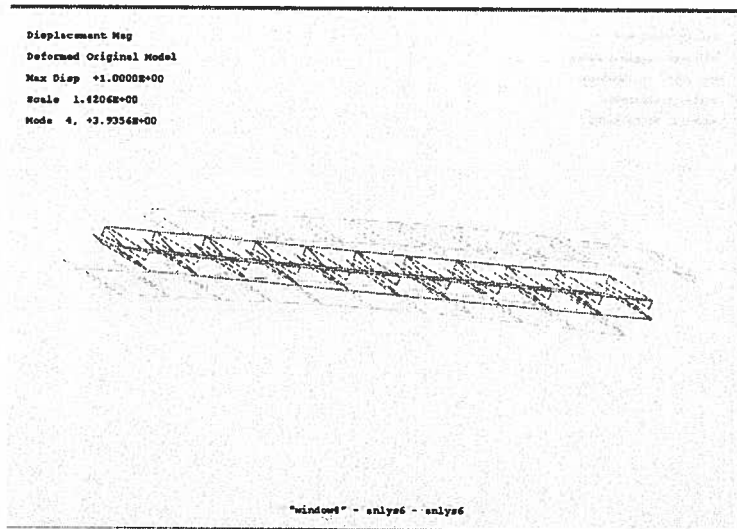


Fig. 8. Vibrations of the trough of the conveyer in relation to the vibrations of the vibroinsulating frame – in the direction of operation: s-s. $f_4 = 3.933[\text{Hz}]$

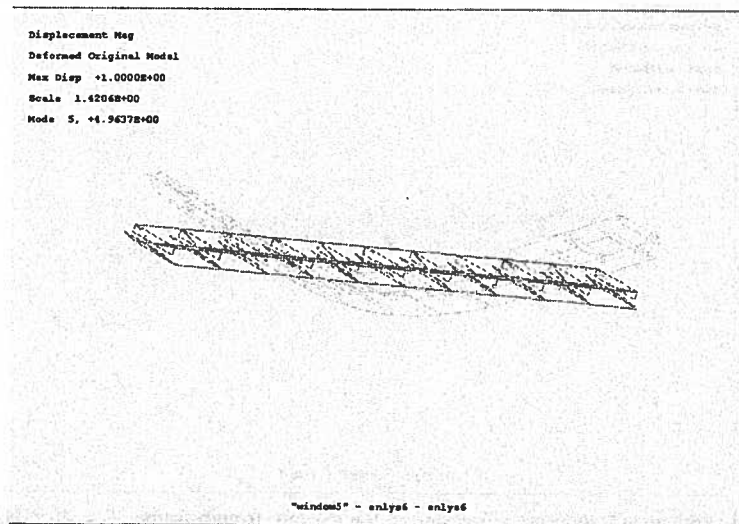


Fig. 9. The first form of symmetric transverse vibrations of the system: trough-frame. $f_5 = 4.96[\text{Hz}]$

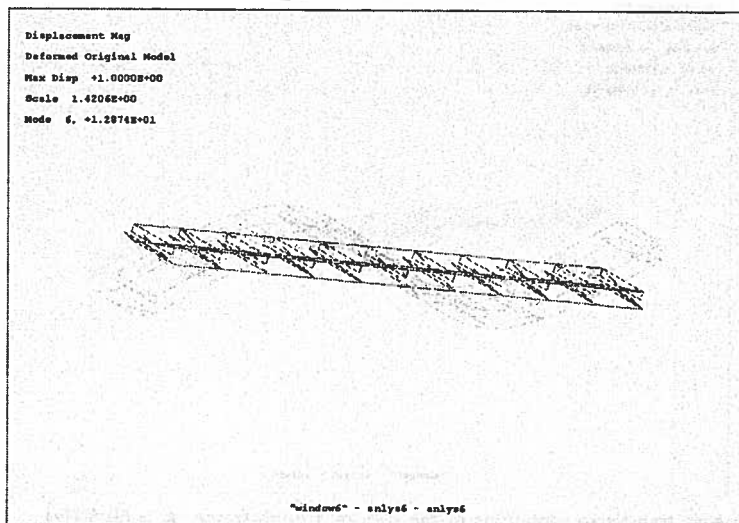


Fig. 10. The first form of anti-symmetric transverse vibrations of the system: trough-frame. $f_6 = 12.87[\text{Hz}]$.

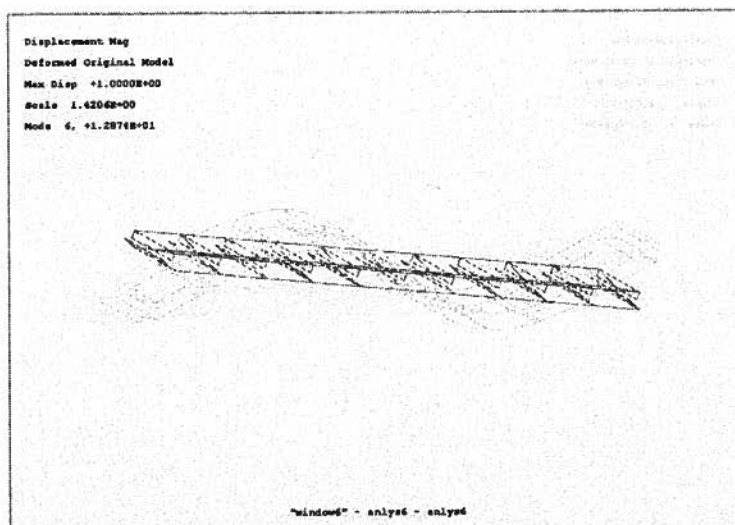


Fig. 11. The second form of symmetric transverse vibrations of the system: trough-frame. $f_7 = 24.3$ [Hz].

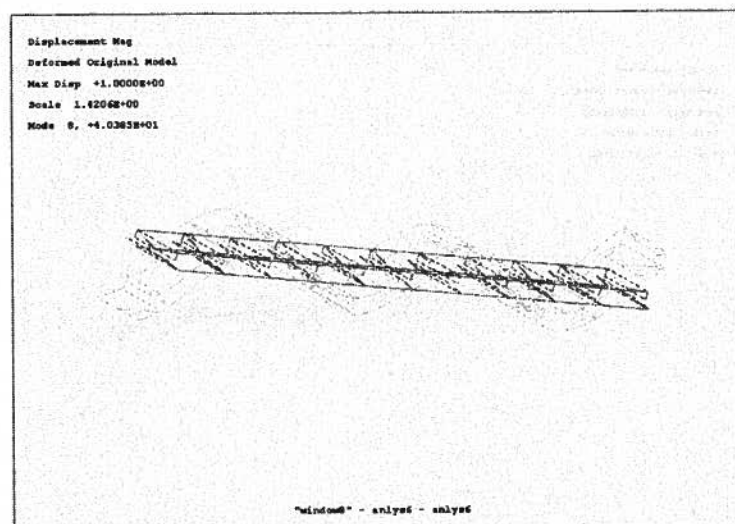


Fig. 12. The second form of anti-symmetric transverse vibrations of the system: trough-frame. $f_8 = 40.3$ [Hz].

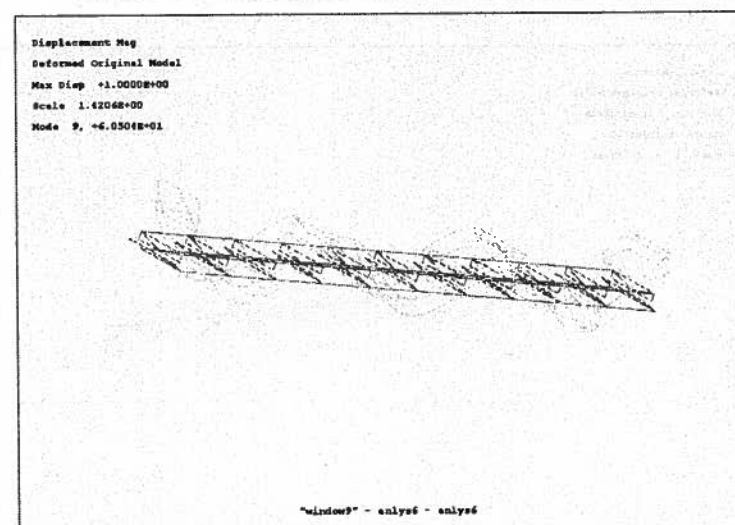


Fig. 13. The third form of symmetric transverse vibrations of the system: trough-frame. $f_9 = 60.5$ [Hz].

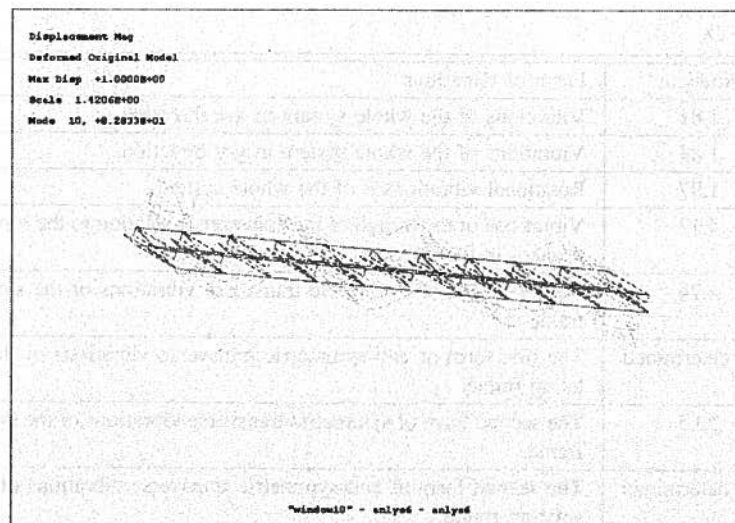


Fig. 14. The third form of anti-symmetric transverse vibrations of the system: trough-frame. $f_{10} = 82.8[\text{Hz}]$.

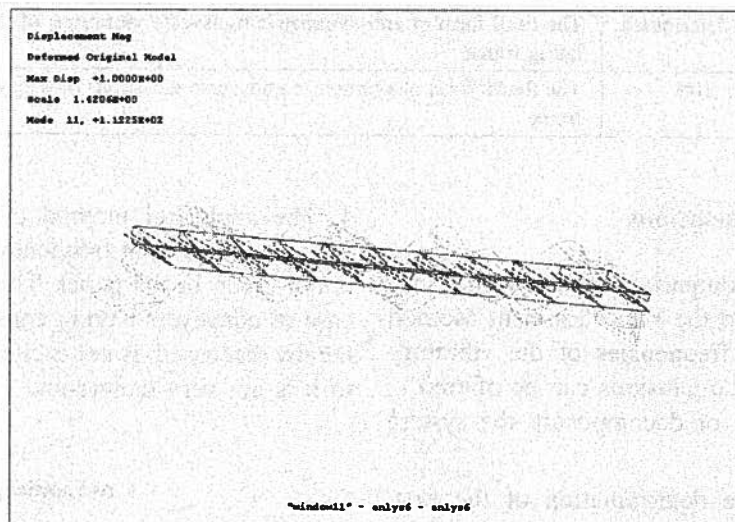


Fig. 15. The fourth form of symmetric transverse vibrations of the system: trough-frame. $f_{11} = 112.2[\text{Hz}]$.

It can be assumed with the sufficient accuracy that the first four graphs (Fig. 5–8) present vibration forms related to deflections of leaf springs and springs while the next graphs (Fig. 9–15) illustrate transverse vibrations of the system. We must realise that this does not mean that the first forms of vibrations are always related to deflection of leaf springs. At adequately selected parameters the frequency of transverse vibrations can be lower than the frequency of vibrations related to deflection of springs, however, this situation does not interfere with the possibility of applying the proposed hereby method. As has been proved and shown in the graphs the axial

compliance of the leaf springs can be omitted for the first 8 forms of transverse vibrations of the system – for the typical parameter values of the vibratory conveyor. The forms of vibrations at higher frequencies are related to axial deflections of leaf springs. However, when the trough is supported on rocking levers and springs instead of leaf springs, the axial deflection of the element connecting the trough with the frame can be omitted for the higher frequencies too.

Natural frequencies of the system determined by the simulation and analytical methods are tabulated, respectively in the table below:

	Simulating	Analytic	Form of vibrations
f_1 [Hz]	1.72	1.81	Vibrations of the whole system in x-x direction.
f_2 [Hz]	1.82	1.89	Vibrations of the whole system in y-y direction.
f_3 [Hz]	2.06	1.97	Rotational vibrations β of the whole system.
f_4 [Hz]	3.93	4.17	Vibrations of the trough of the conveyer in relation to the vibrations of the vibroinsulating frame – in the direction of operation: s-s.
f_5 [Hz]	4.96	4.74	The first form of symmetric transverse vibrations of the system: trough- vibroinsulating frame.
f_6 [Hz]	12.87	Not determined	The first form of anti-symmetric transverse vibrations of the system: trough- vibroinsulating frame.
f_7 [Hz]	24.3	23.5	The second form of symmetric transverse vibrations of the system: trough- vibroinsulating frame.
f_8 [Hz]	40.3	Not determined	The second form of anti-symmetric transverse vibrations of the system: trough- vibroinsulating frame.
f_9 [Hz]	60.5	58.5	The third form of symmetric transverse vibrations of the system: trough- vibroinsulating frame.
f_{10} [Hz]	82.8	Not determined	The third form of anti-symmetric transverse vibrations of the system: trough- vibroinsulating frame.
f_{11} [Hz]	112	108	The fourth form of symmetric transverse vibrations of the system: trough- vibroinsulating frame.

5. Conclusions

On the basis of the comparison of the results obtained by the analytical and the Finite Element Method – in the range of natural frequencies of the vibratory conveyers – the following conclusions can be offered:

1. There is the possibility of decomposing the system into two separate models:

– Discrete model for the determination of the first, most often the lowest, natural frequencies resulting mainly from deformations of elastic elements of the conveyer.

– Continuous model of the beam type (composed of the summation of masses and transverse stiffness of trough and frame) supported on elastic foundation, without the possibility of rotation – for frequencies at which transverse compliance of the conveyer is significant.

2. The natural frequencies obtained for these partial models are corresponding, with the high accuracy, to the results of the computer analysis performed by means of the Finite Element Method.

3. The models presented in the paper do not comprise higher forms of transverse vibrations (of frequency above 100 Hz), at which the influence of axial deflection of leaf springs is quite significant.

4. The analytical method of the determination of the transverse vibration frequency of anti-symmetric system is not given in the paper. This form of vibration, in the case of conveyers having construction similar to the one hereby discussed, is not excited and the resonance related to it is not very dangerous.

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