A R C H I V E S O F M E T A L L U R G Y A N D M A T E R I A L S
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PROBABILITY OF PARTICLE-BUBBLE COLLISION IN PNEUMO-MECHANICAL FLOTATION CELL

PRAWDOPODOBIEŃSTWO ZDERZENIA ZIARNA Z PĘCHERZYKIEM W PNEUMO-MECHANICZNEJ KOMORZE FLOTACYJNEJ

The particle – air bubble collision is one of three elementary processes which determine the rate of bubble mineralization in flotation. It is the result of bubble – particle hydrodynamic interactions and depends mainly on the ratio of the particle size to the bubble size. The efficiency of the process is measured by the probability of particle-bubble collision.

In the practice of upgrading in the cell with mechanical agitation of the pulp both the diameter of a particle and of air bubbles has a certain distribution. Assuming that the diameter of particle d_p and the diameter of bubble d_b are random variables, the probability of collision is the function of the quotient of independent random variables D_p and D_b. Applying the theorems of probability calculus concerning the function of random variables, a general formula of probability density function of the quotient of two random variables D_p/D_b was presented. The family of gamma distributions is the most often applied and giving the best agreement with the experiment for the distribution of the D_p random variable. In this paper it was assumed that it is Rayleigh's distribution which characterizes well the distribution of particle size in the narrow size fraction. Simirlarly, for the distribution of the D_b random variable, the three-parameter log-normal distribution is applied, apart from the distribution applied in granulometry. These are, however phenomenological approaches. In this paper the distribution obtained as a result of heuristic considerations has been used for the air bubble distribution.

The air getting into the flotation cell is subject to dispersion in the turbulent vortexes of the liquid. Assuming that the newly formed surface of bubbles possesses energies corresponding to Boltzmann's distribution, the author obtained Rayleigh's distribution for the air-bubble diameter. The parameter of this distribution depends upon the surface tension of the flotation solution, gas flow-rate and power transmitted into the flotation cell. Calculating the most probable value of the quotient of D_p/D_b random variable, the expression for the probability of bubble-particle collision in the cell with mechanical pulp agitation was obtained. This probability depends on surface tension of the solution, gas flow-rate, gas hold-up, turbulent energy dissipation, volume concentration of the solid state in the cell and the average particle size.

Zderzenie ziarna z pęcherzykiem jest jednym z trzech procesów elementarnych od których zależy tempo mineralizacji pęcherzyka we flotacji. Jest ono wynikiem oddziaływań hydrodynamicznych pęcherzyk – ziarno i zależy w głównej mierze od stosunku wielkości ziarna do wielkości pęcherzyka. Efektywność procesu mierzy się prawdopodobieństwem zderzenia ziarna z pęcherzykiem.

W praktyce wzbogacania w komorach z mechaniczną agitacją mętów zarówno średnica ziarna jak i średnica pęcherzyków mają pewne rozkłady wielkości. Zakładając, że średnica ziarna d_p i średnica pęcherzyka d_b są zmiennymi losowymi wówczas prawdopodobieństwo zderzenia jest funkcją ilorazu niezależnych zmiennych losowych D_p i D_b. Korzystając z twierdzeń rachunku prawdopodobieństwa, odnoszących się do funkcji zmiennych losowych, przedstawiono ogólny wzór na funkcję gęstości rozkładu ilorazu dwóch zmiennych losowych D_p/D_b. Dla rozkładu zmiennej losowej D_p najczęściej używanymi i dającymi najlepszą zgodność z doświadczeniem jest rodzina rozkładów gamma. W tym artykule założono, że jest to rozkład Rayleigha dobrze charakteryzujący rozkład wielkości ziarna w wąskiej klasie ziarnowej. Podobnie dla rozkładu zmiennej losowej D_b oprócz rozkładów stosowanych w granulometrii stosuje się trójparametrowy rozkład log-normalny. Są to jednak podejścia fenomenologiczne. W tej pracy dla rozkładu wielkości pęcherzyków zastosowano rozkład uzyskany na gruncie rozważań heurystycznych.

Powietrze dopływające do komory flotacyjnej ulega zdyspergowaniu w turbulentnych wirach cieczy. Opierając się na założeniu, że nowo tworzona powierzchnia pęcherzyków ma energie, których rozkład odpowiada rozkładowi Boltzmanna, uzyskano rozkład Rayleigha dla wielkości pęcherzyków. Parametr tego rozkładu jest zależny od napięcia powierzchniowego roztworu flotacyjnego, wydatku powietrza oraz mocy przekazywanej do układu flotacyjnego. Wyliczając najbardziej prawdopodobną wartość ilorazu zmiennej losowej D_p/D_b uzyskano wyrażenie na prawdopodobieństwo zderzenia pęcherzyk-ziarno

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w komorze flotacyjnej z mechaniczną agitacją mętów. Prawdopodobieństwo to jest zależne od napięcia powierzchniowego roztworu, wydatku powietrza, napowietrzenia mętów, turbulentnej dyssypacji energii, koncentracji objętościowej fazy stałej w komorze flotacyjnej oraz średniej wielkości ziaren nadawy.

1. Introduction

Bubble mineralization is the result of three elementary events (microprocesses): particle-bubble collisions, adhesion of particles to bubble surfaces and detachment of particles from bubbles. They are the result of bubble – particle interactions. The efficiency of respective microprocesses is measured by the probability of their occurrence. The particle-bubble collision is the result of hydrodynamic interactions. Its probability depends, among others, upon the size of particles and bubbles, particle density, physicochemical properties of the medium, character of bubble motion. Adhesion, after the prior collision, is conditioned by the surface interactions, resulting from the particle hydrophobic properties. The probability of the occurrence here is mainly controlled by a proper agent regime. As a result of adhesion, the particle-bubble aggregate is formed which passes to the froth layer under the condition that it is stable aggregate. Stability of the aggregate depends on the detachment probability of particle from a bubble. The microprocess of particle detachment refers mainly to large and high density particles and relatively low hydrophobic properties. The probability of this event, apart from the above factors, depends upon the intensity of turbulence of the medium, characterized quantitatively by the turbulent energy dissipation.

In the kinetics equation of the first order, obtained on the grounds of the stochastic model, two independent constants occur: permanent adhesion rate constant λ_o and detachment rate constant μ_o. Ultimate recovery R_∞ and the first order rate constant k are connected with the constants of permanent adhesion and detachment by the relations [1]:

$$R_\infty = \frac{\lambda_o}{\lambda_o + \mu_o} \tag{1a}$$

$$k = \lambda_o + \mu_o \tag{1b}$$

The value of ultimate recovery is proportional to the negative value of thermodynamic potential [2]. Therefore, for the sake of a complete characteristics of the course of flotation process, it should be considered as a kinetic and thermodynamic phenomenon.

These independent constant of the cyclic flotation model λ_o and μ_o have statistical interpretation and are expressed by the probabilities of particle-bubble collision P_c, particle adhesion to the bubble P_a and detachment of the particle from the bubble P_d.

2. Probabilities of particle collision, adhesion and detachment

The probability of collision is defined as a relation of the number of floatable particles colliding with a bubble in a unit of time $\frac{\Delta l_c}{\Delta t}$ to the number of floatable particles $(n_o - l_a)$ contained in a unit of volume of the flotation cell in a given moment t [3]:

$$P_c = \frac{1}{n_o - l_a}\frac{\Delta l_c}{\Delta t} \tag{2}$$

where: n_o – initial number of floatable particles contained in the flotation cell, Δl_c – number of particles colliding with bubbles in the time Δt, l_a – number of particles attached to bubbles to the time t, $(n_o - l_a)$ – number of floatable particles, remaining in the cell, not coagulated with the bubble to the time t.

Probability of adhesion P_a is the relation of the total number of the particles $\frac{\Delta l_t}{\Delta t}$, which were attached to the bubbles flowing through a unit of surface of the cross-section of the flotation cell in a unit of time, to the number of particles colliding with the bubbles in a unit of time [4]:

$$P_a = \frac{\frac{\Delta l_t}{\Delta t}}{\frac{1}{4}S_b\frac{\Delta l_c}{\Delta t}} = \frac{\Delta l_t}{\frac{1}{4}S_b\Delta l_c} \tag{3}$$

where: S_b – bubble surface area flux, Δl_t – number of particles attached to the bubbles in the time Δt.

The probability of detachment P_d of a particle from a bubble is the ratio of the number of particles detached from the bubble in the unit of time $\frac{\Delta l_d}{\Delta t}$ to the total number of particles which were attached to the bubbles $\frac{\Delta l_t}{\Delta t}$ in the unit of time:

$$P_d = \frac{\frac{\Delta l_d}{\Delta t}}{\frac{\Delta l_t}{\Delta t}} = \frac{\Delta l_d}{\Delta l_t} \tag{4}$$

where: Δl_d – number of particles detached from bubbles in the time Δt.

The probability of permanent adhesion P_s is the ratio of the number of particles attached durably to the bubble in the unit of time $\frac{\Delta l_s}{\Delta t}$ to the total number of particles attached to the bubble in the unit of time $\frac{\Delta l_t}{\Delta t}$:

$$P_s = \frac{\frac{\Delta l_s}{\Delta t}}{\frac{\Delta l_t}{\Delta t}} = \frac{\Delta l_s}{\Delta l_t} \tag{5}$$

where: δl_s – number of particles attached durably to the bubbles in the time Δt. Since $\Delta l_s = \Delta l_t - \Delta l_d$, then, respectively,

$$P_s = \frac{\Delta l_t - \Delta l_d}{\Delta l_t} = 1 - \frac{\Delta l_d}{\Delta l_t} = 1 - P_d \qquad (6)$$

This is therefore, the probability of non-detachment of a particle from bubbles under the effect of external forces. The above mentioned probabilities are functions of random variables, such as particle diameter, bubble diameter, induction time, contact angle, etc., which influence the distribution of flotation rate constant.

This paper presents the distribution of the quotient of random variables of the particle size D_p and bubble size D_b, affecting the value of probability of particle-bubble collision, and the author calculated the most probable value of collision probability in the flotation cell with mechanical pulp agitation. It was preceded by a review of the collision probability models.

3. Model of collision probability

A collision in the process of flotation denotes a particle approach to the bubble surface in such a distance when surface interaction are activated [5].

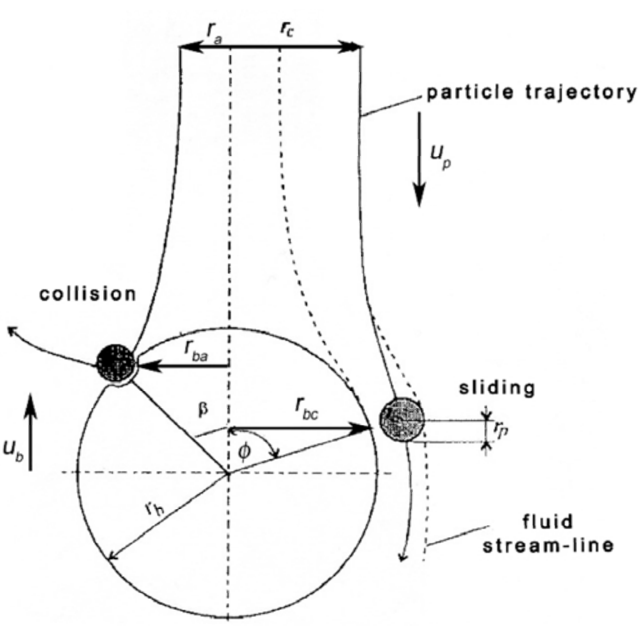

Fig. 1. Particle streamline flow around a bubble: r_p – particle radius, r_b – bubble radius, r_c – collision radius, r_a – adhesion radius, β – adhesion angle, φ – collision angle (Schulze [40])

The mechanism of particle-bubble collision depends upon the character of liquid motion against the bubble [6]. At the streamline flow of liquid around the bubble, the collision results from the particle motion along the appropriate current line of the liquid, close enough to the central line crossing the bubble centre (Fig.1). The collision occurs when the distance of the current line is smaller than r_c, where r_c is the so-called radius of collision.

To calculate the probability of interceptional collision the author applied the geometrical definition of probability [7] which, for this case is expressed by the formulae:

$$P_c = \frac{\pi\, r_c^2}{\pi\, r_b^2} = \left(\frac{r_c}{r_b}\right)^2 \qquad (7a)$$

or

$$P_c = \left(\frac{r_c}{r_b + r_p}\right)^2. \qquad (7b)$$

In the above definition of probability of interceptional collision it was assumed that the particles are distributed uniformly in the liquid volume. To calculate radius of collision, Navier – Stokes' equation should be solved from which the function of liquid current (potential of liquid current) is obtained. This is the equation of the current line upon which the particle moves. The main factor affecting the probability of interceptional collision is constituted by the character of liquid motion around the bubble since the function of liquid current, describing this motion, changes according to the bubble size, or more precisely, to Reynolds number for the bubble [8]. At the same time, it is assumed that a particle moves with the terminal falling velocity (down-wards) whereas a bubble moves with the terminal rising velocity. For very little (Stokes' flow) and very large (potential flow) Reynolds numbers the equation is solved analytically for a bubble. On the other hand, for the intermediate values of Reynolds number, encountered most often in flotation systems, the equation is solved numerically, or some authors apply empirical functions of liquid current.. When the function of liquid current is given, the equation for rc is derived, and the probability of interceptional collision is calculated from equation (7a) or (7b).

Sutherland was the first who presented the formula for the probability of interceptional collision [9]. Considering the potential motion of liquid stream around the bubble, while neglecting the force of gravity and inertia acting upon the particle, he obtained a formula for the radius of collision: .

$$r_c = \sqrt{\frac{3 d_p d_b}{4}}. \qquad (8)$$

The probability of interceptional collision, according to formula (7a) is:

$$P_c = 3\frac{d_p}{d_b} \qquad (9)$$

where: d_p – particle diameter, d_b – bubble diameter.

Formula (9) is valid for very large bubbles, not applied practically in flotation upgrading, and for high velocities around the bubble, i.e. for $Re_b >> 1$ [5].

Next, Gaudin studied streamline liquid motion around very little bubbles ($Re_b < 1$) and, neglecting the force of inertia for the probability of interceptional collision, he obtained the formula [10]:

$$P_c = \frac{3}{2}\left(\frac{d_p}{d_b}\right)^2 \tag{10}$$

Yoon verified Gaudin's model and found out that it could be proved only for very little bubbles, below 100 μm [11,12].

Flint and Howarth studied the dynamics of motion of particles and bubbles and observed that those motions could be characterized by two dimensionless factors [13]:

$$K = \frac{\rho_p d_p u_b}{9\mu\, d_b} \quad G = \frac{(\rho_p - \rho_l) d_p^2\, g}{18\mu\, u_b}$$

where: u_b – terminal rising velocity of the bubble, ρ_p – particle density, ρ_l – liquid density, g – acceleration due to gravity, μ – coefficient of dynamic viscosity of liquid.

For little particles, when the force of inertia is neglected, probability of interceptional collision equals:

$$P_c = \frac{G}{1+G} = \frac{d_p^2}{d_p^2 + \frac{18\mu}{(\rho_p - \rho_c)g} u_b}, \tag{11}$$

at the same time probability of collision is expressed by formula (11) both for Stokes' flow and potential flow. Since rising velocity u_b decrease when the bubble size decreases, therefore, as it results from equation (11), for little particles probability of collision is higher for little bubbles.

Anfruns and Kitchener proposed the hydrodynamic model of probability of interceptional collision, taking into consideration only the forces of gravity and viscosity and neglecting the force of particle inertia and drag of liquid. For the conditions of Stokes' motion, they solved analytically the equation of particle motion against the bubble, obtaining the equation of motion path, and, according to it, they calculated probability of collision [14]:

$$P_c = \frac{\left(1 + \frac{d_p}{d_b}\right)^2}{1 + \frac{u_p}{u_b}} \left[\frac{u_p}{u_b} + \frac{2\psi_c^o}{\left(1 + \frac{d_p}{d_b}\right)^2}\right] \tag{12}$$

where: Ψ_c^o – value of the function of liquid current of boundary path for the collision angle $\alpha = 90°$ and the distance from the bubble centre $r = (d_p + d_b)/2$.

Weber and Paddock, for the intermediate values of Reynolds number for bubbles, presented the probability of collision as a sum of two components [15]:

$$P_c = P_{cg} + P_{ci} \tag{13}$$

where: P_{cg} – the probability of gravitational collision resulting from the finite falling velocity of the particle in the gravitational field which causes the deviation of the particle path from the line of liquid current, whereas P_{ci} is the probability of interceptional collision.

Both probabilities depend upon Reynolds number for the bubble. Weber and Paddock obtained expressions for both probabilities while solving numerically Navier – Stokes' equation. Probability P_{cg} is presented by the following formula [15]:

$$P_{cg} = \left(1 + \frac{d_p}{d_b}\right)^2 \frac{u_p}{u_b} \sin^2\phi_c \tag{14}$$

where: φ_c – maximum angle of collision above which there is no particle-bubble collision.

To calculate probability P_{ci} Weber and Paddock apply the Stokes-like function of liquid current and, depending upon the value of Reynolds number for bubbles, they obtain the follow-ing expressions [15]:

$$P_{ci} = \left(1 + \frac{2}{1 + \left(\frac{37}{Re_b}\right)^{0,85}}\right)\left(\frac{d_p}{d_b}\right) \quad \text{for } Re_b < 200 \tag{15}$$

and

$$P_{ci} = \frac{3}{2}\left(1 + \frac{\frac{3}{16} Re_b}{1 + 0,249 Re_b^{0,56}}\right)\left(\frac{d_p}{d_b}\right)^2 \quad \text{for } 200 < Re_b < 300 \tag{16}$$

These expressions only forecast the values P_c for the wide range of particle and bubble size because they have never been empirically verified.

Jiang and Holtham took into consideration the relative motion of the particle and bubble of the sizes from the $d_p = 6 - 120\mu$m and $d_b = 50 - 860\mu$m ranges [8]. From the equations of particle motion in relation to the bubble and taking into account the force of gravity, buoyancy, drag of particle motion and boundary layer, they obtained a general expression for the probability of collision [8]:

$$P_c = a_c \left(\frac{d_p}{d_b}\right)^n, \tag{17}$$

At the same time, the values of coefficients a_c and n depend upon the bubble size but do not depend on the particle size.

Dobby and Finch expressed the probability of collision by Reynolds number Re_b and Stokes number S_k [16]:

$$P_c = \left[1,627\mathrm{Re}_b^{0,06} S_k^{0,54} \left(\frac{u_b}{u_p}\right)^{0,16} P_{Co}\right] \tag{18}$$

where: P_{Co} – constant, $S_k = \frac{1}{9}\frac{\rho_p}{\rho_b}\left(\frac{d_p}{d_b}\right)^2 \mathrm{Re}_b$ $\mathrm{Re}_b = \frac{u_b \rho_c d_b}{\mu}$.
Stokes number is the ratio of the force of inertia to the force of drag of bubble motion. Formula (18) is valid for $\mathrm{Re}_b^{0,06} S_k^{0,54} \left(\frac{u_b}{u_p}\right)^{0,16} > 0,614$, i.e. for $20 < \mathrm{Re}_b < 300$, $S_k < 0{,}8$ and $\frac{u_b}{u_p} < 25$

Yoon and Luttrell [17] applied the same approach as Sutherland [9]. Neglecting the forces of inertia they assumed at the same time that the particle path is identical as the line of liquid current. They considered various conditions of liquid motion at the bubble surface, taking into consideration the functions of liquid current, appropriate to the conditions of motion. In this respect, their model is a generalization of Sutherland's model. For Stokes' flow, i.e. very small Reynolds numbers ($\mathrm{Re}_b <<$ 1), they obtained the formula identical to the formula given by Gaudin [10]. Consequently, for very large Reynolds numbers, i.e. the potential flow, they obtained the formula identical to Sutherland's model. For intermediate Reynolds number, i.e. those which exist in majority of flotation cells, they assumed the empirical function of current, obtained from the analysis of a large number of current lines of the liquid flowing around the bubble. Applying the definition of probability, expressed by the formula (7b), they obtained the following equation for the probability of interceptional collision [17]:

$$P_c = \left(\frac{3}{2} + \frac{4}{15}\mathrm{Re}_b^{0,72}\right)\left(\frac{d_p}{d_b}\right)^2 \tag{19}$$

Fig.2 presents the dependence of collision probability upon the bubble size for d_p = 11,4 μm drawn according to dependence (19).

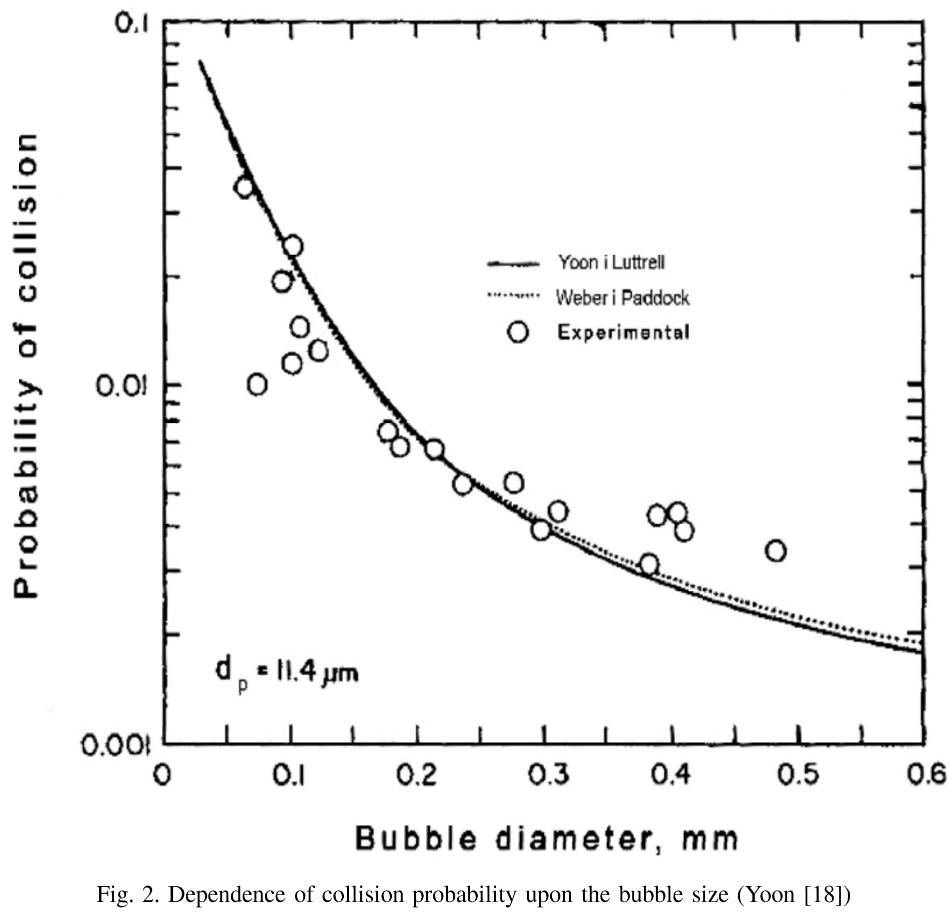

Fig. 2. Dependence of collision probability upon the bubble size (Yoon [18])

As it results from formula (19), the probability of collision changes as d_b^{-2} for small bubbles ($\mathrm{Re}_b << 1$). It depends on d_b in lesser and lesser degree. For very large bubbles Pc changes as [18,19].

Schulze [5] proposed the model in which, analogically as in Weber's and Paddock's model [15], the probability of collision is the sum of three components, i.e. the probability of interceptional collision P_{ci}, the probability resulting from the finite settling velocity of the particle in the gravitational field P_{cg} and the probability considering the particle inertia, resulting from the finite dimensions of the particle P_{cin}, while [5]:

$$P_{ci} = \frac{2\psi_c}{\left(1 + \frac{u_p}{u_b}\right) u_b \mathrm{Re}_b^2} \tag{20}$$

$$P_{cin} = \frac{1}{\left(1 + \frac{u_p}{u_b}\right)} \left(1 + \frac{d_p}{d_b}\right)^2 \left(\frac{S_k}{S_k + e}\right)^b . \tag{21}$$

In the above equations Ψ_c is the function of liquid current for the boundary path (boundary angle of collision above which no particle-bubble collision occurs), while e and b are the coefficients dependent on Re_b. The force of inertia, analogically as the force of gravity, causes a deviation of the particle from the liquid current line, resulting in the bubble-particle collision [20]. For the probability P_{cg} Schulze applied the formula given by Weber and Paddock (formula 14). Respectively, the total probability of collision is:

$$P_c = P_{ci} + P_{cg} + \left[1 - \frac{P_{ci}}{\left(1 + \frac{d_p}{d_b}\right)^2}\right] P_{cin}. \tag{22}$$

By means of considering the force of inertia, Schulze's model can be considered to be the most complete [21].

All the above formulas are based on the analysis of bubble motion, rising in the liquid with terminal velocity and particle motion settling with terminal velocity along the appropriate current line. Such a situation exist in flotation columns. At the liquid turbulent motion, which occurs in flotation cells with intensive agitation, the mechanism of collision is different [6]. According to Gaudin [10], this is the motion of liquid deprived of viscosity together with the particle performing inert rotational movements. The rising bubble is the spherical obstacle for the liquid vortex. The particle – bubble collidion occurs at the contact with this obstacle.

The following formula for the number of particle-bubble collisions in the volume unit of suspension and the unit of time, for low turbulent energy dissipation, is given by Saffman and Turner [22]:

$$Z_{pb} = \sqrt{\frac{8\pi}{15}} N_p N_b \left(\frac{d_p + d_b}{2}\right)^3 \left(\frac{\varepsilon}{\nu}\right)^{\frac{1}{2}} \tag{23}$$

where: N_p and N_b – number of particles and bubbles in the volume unit, respectively, ν – coefficient of liquid kinetic viscosity, $\varepsilon = P/M$ – turbulent energy dissipation, P – power of agitation (power introduced into the flotation cell), M – mass of suspension in the flotation cell.

Dependence (23) has been derived for small particles and bubbles closed in the liquid vortex in the section of flotation cell with low energy dissipation.

Schubert and Bischofberger modified Abrahamson's model [24] for the process of coagulation and derived the following formula for the number of bubble-particle collisions [23]:

$$Z_{pb} = 5,0 N_p N_b \left(\frac{d_p + d_b}{2}\right)^2 \left(\bar{u}_p^2 + \bar{u}_b^2\right)^{\frac{1}{2}} \tag{24}$$

where: u_p and u_b – respectively the velocities of particle and bubble against the liquid, equal to:

$$\bar{u}_i^2 = \frac{0,4\varepsilon^{\frac{4}{9}} d_i^{\frac{7}{9}}}{\nu^{\frac{1}{3}}} \left(\frac{\rho_i - \rho_c}{\rho_c}\right)^{\frac{2}{3}} \tag{25}$$

while ρ_i is the particie density (ρ_p) or bubble density (ρ_b) and $i = p$ or b.

Pyke and coworkers [25] and Duan and coworkers [26] proposed the formula for the number of collision for flotation cells with mechanical agitation of the pulp:

$$Z_{pb} = \frac{7,5}{\pi} \frac{J_g\ A}{d_b V} \left[\frac{0,33\varepsilon^{\frac{4}{9}}}{\left(\frac{\mu}{\rho_s}\right)^{\frac{1}{3}}} \left(\frac{\rho_l - \rho_b}{\rho_l}\right)^{\frac{2}{3}} \left(\frac{0,01}{u_b}\right)\right] \tag{26}$$

where: V – volume of the flotation cell, A – area of the transverse section of the cell, J_g – superficial gas velocity.

In expression (26) it is assumed that the diameter of bubbles is much larger than the diameter of particles.

According to Koch and Schwarz [27], in order to consider the tendency of small particles to the streamline motion around the bubble, the number of particle-bubble collisions should be multiplied by the probability of interceptional collision to obtain the complete participation of all particles of both above mentioned mechanisms in the process of collision. Finch and Dobby revealed that for the collision mechanism, resulting from the streamline motion, the number of bubble-particle collision is [28]:

$$Z_{pb} = 0,25 S_b = \frac{3J_g}{2d_b}. \tag{27}$$

Consequently, the total and permanent adhesion rate constant is expressed by universal formulae:

$$k = Z_{pb} P_c P_a \tag{28}$$

$$\lambda_o = Z_{pb} P_c P_a (1 - P_d) \tag{29}$$

$$\mu_o = Z_{pb} P_c P_a, \tag{30}$$

while Z_{pb} is determined depending upon the mechanism of collision, i.e. upon the type of flotation machine.

As it can be seen from the presented models, that the probability of collision can be performed as a general formula, analogical to formula (17).

4. Probability of collision as the function of quotient of two random variable D_p and D_b

In all above mentioned formulas probability of collision is the function of quotient of particle diameter d_p to bubble diameter d_b. In the industrial flotation systems the feed has a certain distribution of particle size. In pneumo-mechanical flotation cells in which air is dispersed by turbulence of the medium, generated by the rotor motion, the produced bubbles posses also a certain distribution of diameter. Therefore it can be said that both the particle diameter and the bubble diameter are random variables of determined distributions.

Let the random variable of particle size D_p have the probability density function $f_p(d_p)$, whereas the random variable of bubble size D_b have the probability density function $f_b(d_b)$. When we assume the independence of random variables D_p and D_b, the random variable U, being the quotient of variables D_p and D_b, i.e.

$$U = \frac{D_p}{D_b}, \tag{31}$$

has the probability density function $f_1(u)$, determined by the following formula [29]:

$$f_1(u) = \int_0^\infty d_b f_b(d_b) f_p(u d_b) \delta d_b. \tag{32}$$

In relation to this, the probability of collision, according to formula (17), can be expressed by the random variable U:

$$P_c = a_c U^n \tag{33}$$

As it can be seen from the above presented models coefficient a_c is dependent on the hydrodynamic conditions in the flotation cell and assumed model of particle-bubble collision [22,23,25,26,27,28].

4.1. Distributions of random variables D_p and D_b

Several types of distributions of random variables are used to characterize the distribution of particle size in a sample. The gamma family of distribution is the most often applied and achieving the best compatibility with empirical distributions. These distributions can be expressed by one common formula in the form of the so-called generalized gamma distribution [29]. The random variable D_p has a generalized gamma distribution if its probability density function is determined by the following formula [29]:

$$f_p(d_p) = \frac{|\alpha| \lambda_p^{b/\alpha}}{\Gamma(b/\alpha)} d_p^{b-1} \exp\left(-\lambda_p d_p^\alpha\right) \quad \text{for} \quad d_p > 0, \alpha b > 0, \lambda > 0 \tag{34}$$

where: α, b and λ_p – distribution parameters.
Special cases of this distribution are:

a) gamma distribution when $\alpha = 1$

$$f_p(d_p) = \frac{\lambda_p^b}{\Gamma(b)} d_p^{b-1} \exp\left(-\lambda_p d_p\right) \tag{35a}$$

b) Weibull's distribution, known in mineral processing as the R-R-B distribution, when $b = \alpha$

$$f_p(d_p) = b \lambda_p d_p^{b-1} \exp\left(-\lambda_p d_p^b\right) \tag{35b}$$

c) Rayleigh's distribution when $b = \alpha = 2$

$$f_p(d_p) = 2 \lambda_p d_p \exp\left(-\lambda_p d_p^2\right) \tag{35c}$$

In the laboratory practice, the investigations are often carried out on a narrow size fraction of a given material. The precise size analysis, performed by means of the laser diffractometry, indicates that the distribution of particle size in a narrow size fraction can be approximated with equal accuracy by Weibull's or Rayleigh's distribution [30].

In order to characterize the distribution of bubble size, such functions are applied which are analogical to those in the distributions of the dispersed phase in dispersive liquid-liquid and gas-liquid systems in which liquid is the dispersed phase [31, 32, 33, 34, 35, 36]. These are distributions known from granulometry. Apart from gamma and Weibull's distributions, which are two-parameter, the authors applied a three-parameter log-normal distribution of the form [36]:

$$f_b(d_b) = \frac{dV_b}{\delta d_b} = \frac{\sigma_b}{\sqrt{\pi}} \frac{d_{b\max}}{d_b(d_{b\max} - d_b)} \exp\left[-\sigma_b^2\left(\ln\frac{a_b d_b}{d_{b\max} - d_b}\right)^2\right] \tag{36}$$

where:

$$V_b = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\sigma_b} \exp\left(-u^2\right) du \tag{37}$$

represents the total volume of bubble in the range $(0,d_b]$, and

$$u = \ln\frac{a_b d_b}{d_{b\max} - d_b}. \tag{38}$$

The parameters of distribution (36) are: d_{bmax} – maximum bubble size, σ_b – dimensionless parameter connected with standard deviation, a_b – dimensionless empirical parameter.

When the model dependence (36) is fitting to the empirical dependence, distribution parameters are obtained and, accordingly, average value of Sauter's diameter of the bubble [36]:

$$d_{b32} = \frac{d_{b\max}}{1 + a_b \exp\left(\frac{1}{4\sigma_b^2}\right)}. \tag{39}$$

All the above approaches to approximation of the distribution of bubble size are the phenomenological character. The distribution obtained in heuristic considerations, is based upon an assumption that air flowing into the flotation cell is subject to dispersion in turbulent vortexes of the liquid and the newly formed bubble surfaces have energies whose correspond to Boltzmann's distribution. If the surface energy of bubbles is $w = \pi d_b^2 \sigma$, then the probability density function of bubble diameter is expressed by Rayleigh's distribution [37]:

$$f_b(d_b) = 2\lambda_b d_b \exp\left(-\lambda_b d_b^2\right) \tag{40}$$

while a distribution parameter is equal to:

$$\lambda_b = \left[\frac{\left(36\sigma^2 V_g^3 \rho_l g\right)^{1/3} + \eta P \sqrt[3]{d_i}}{6,77 V_g \sigma \sqrt[3]{d_i}}\right]^2 \tag{41}$$

where: σ – surface tension of flotation solution [N/m], V_g – gas flow-rate [m^3/s], ρ_l – liquid density [kg/m^3], g – acceleration due to gravity [m/s^2], P – power of rotor motor [W], η – efficiency of the motor mechanical system, d_i – internal diameter of capillary bringing the air to the cell [m].

The product $\eta P = P_n$ denotes the net power transmitted to the flotation system. The following quotient

$$\bar{\varepsilon} = \frac{\eta P}{M} \tag{42}$$

in which M is the mass of flotation suspension presents the turbulent dissipation of energy to the flotation system.

The mass of flotation suspension (of three-phase system) is expressed by the following formula:

$$M = [\rho_s c + \rho_g \varepsilon_g + \left(1 - c - \varepsilon_g\right)\rho_l]V \tag{43}$$

where: ρ_s – density of solid phase, ρ_g – den sity of gas phase, ρ_l – density of liquid phase, c – volume concentration of solid phase, ε_g – gas holdup.

If is taken into consideration that $\rho_g << \rho_s$ and ρ_l then

$$M = [\rho_s + \left(1 - c - \varepsilon_g\right)\rho_l]V = [(\rho_s - \rho_l)\, c + \left(1 - \varepsilon_g\right)\rho_l]V. \tag{44}$$

After considering expression (44), turbulent energy dissipation to the flotation system is equal:

$$\bar{\varepsilon} = \frac{\eta\, P}{[(\rho_s - \rho_l)c + \left(1 - \varepsilon_g\right)\rho_l]V} \tag{45}$$

Power transmitted into the flotation system is connected with the rotational speed of the rotor N, and its diameter D by means of the following equation [38]:

$$P_n = N_o N^3 D^5 \rho_l \tag{46}$$

where: N_o – power number assuming the constant value for a given type of the flotation cell, regardless its size. This a criterion number.

By means of equations (45) and (46) it is possible to relate the parameter of Rayleigh's distribution with energy dissipation ε, gas holdup in the flotation cell ε_g, solid phase concentration c or rotational speed N and rotor diameter D.

5. Distribution of *U* random variable and probability of collision

According to the consideration of the previous chapter, the distribution of random variable D_b is expressed by Rayleigh's distribution (equation 40). For the narrow size fraction of the feed the distribution of random D_p can be also characterized by Rayleigh's distribution. Respectively, for this case, according to formula (32), the probability density function of random variable U is expressed by the following formula:

vspace-2mm

$$f_1(u) = 4\lambda_b \lambda_p u \int_0^{\infty} d_b^3 \exp\left[-\left(\lambda_b + \lambda_p u^2\right) d_b^2\right] \delta d_b \tag{47}$$

In the table of integers the following integer is given [39]:

$$\int_0^\infty \xi^{2m+1} \exp\left(-\beta\xi^2\right) d\xi = \frac{m\,!}{2\beta^{m+1}}, \quad \beta > 0 \qquad (48)$$

For the case of the integer of formula (47) $m = 1$, $\beta = (\lambda_b + \lambda_p u^2)$. Taking into account formula (48) the probability density function of variable U is:

$$f_1(u) = \frac{2\lambda_b \lambda_p u}{\left(\lambda_b + \lambda_p u^2\right)^2} \qquad (49)$$

Function $f_1(u)$ fulfills the standardizing condition for the density function:

$$\int_0^\infty f_1(u) du = 1 \qquad (50)$$

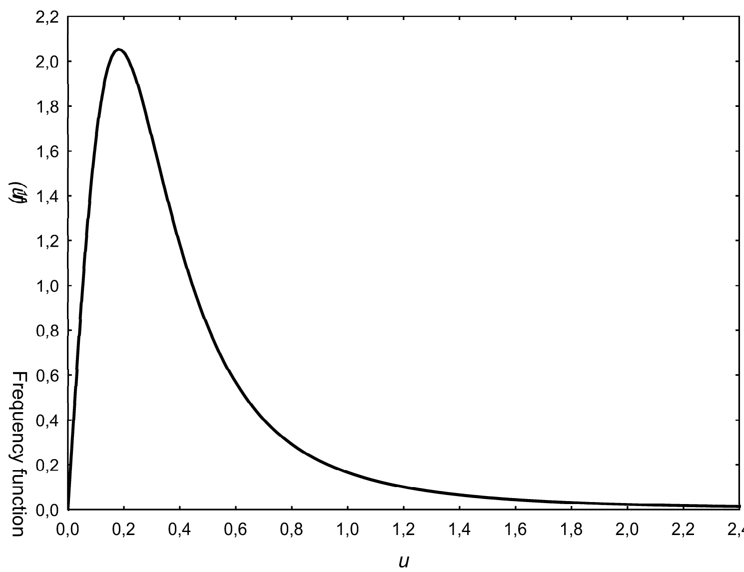

Fig. 3. Probability density function of random variable U

Fig.3 represents the probability density function $f_1(u)$ for λ_b = 0,295 and λ_p = 2,95. The point, in which function $f_1(u)$ reaches its maximum, is the most probable value of quotient $d_p/d_b = u$. Comparing the derivative of function $f_{1(u)}$ to zero, the value u_{pr} is calculated:

$$u_{pr} = \sqrt{\frac{\lambda_b}{3\lambda_p}} \qquad (51)$$

Consequently, the most probable value of collision probability in the flotation cell is expressed by the formula:

$$P_c = a_c \left(\frac{d_p}{d_b}\right)^2 = \frac{a_c}{3}\frac{\lambda_b}{\lambda_p} = \frac{a_c}{3\lambda_p}\left[\frac{\left(36\sigma^2 V_g^3 \rho_l g\right)^{\frac{1}{3}} + \eta P \sqrt[3]{d_i}}{6,77 V_g \sigma \sqrt[3]{d_i}}\right]^2 \qquad (52)$$

Substituting ηP of equation (45), the following dependence is obtained:

$$P_c = \frac{a_c}{3\lambda_p}\left[\frac{\left(36\sigma^2 V_g^3 \rho_l g\right)^{\frac{1}{3}} + \bar{\varepsilon}\left[(\rho_s - \rho_l)\, c + \left(1 - \varepsilon_g\right)\rho_l\right] V \sqrt[3]{d_i}}{6,77 V_g \sigma \sqrt[3]{d_i}}\right]^2 \qquad (53)$$

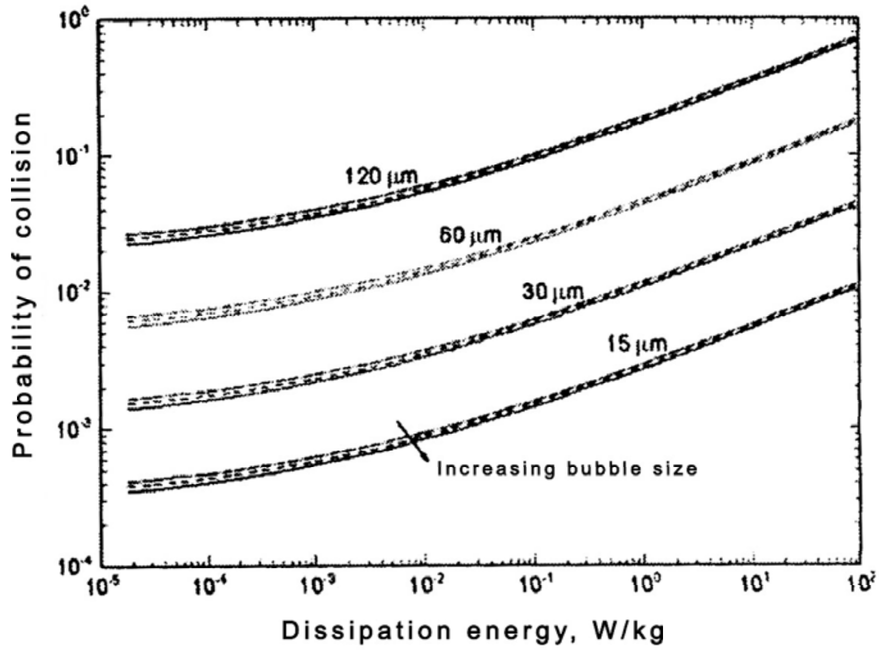

Fig. 4. Dependence of collision probability on energy dissipation for four particle size and three bubble size 1.0, 1.05 and 1.1 mm (Koch and Schwarz [27])

Substituting for ηP of formula (46) into equation (52), we obtain the dependence of probability of collision on the number of rotational speed and rotor diameter:

$$P_c = a_c \left(\frac{d_p}{d_b}\right)^2 = \frac{a_c}{3}\frac{\lambda_b}{\lambda_p} = \frac{a_c}{3\lambda_p}\left[\frac{\left(36\sigma^2 V_g^3 \rho_l g\right)^{\frac{1}{3}} + N_o N^3 D^5 \rho_l \sqrt[3]{d_i}}{6,77 V_g \sigma \sqrt[3]{d_i}}\right]^2 \tag{54}$$

As it results from formulae (53) and (54), the probability of collision increases with the increase of turbulent energy dissipation, volume concentration of the solid phase and rotational speed of rotor while it falls with the increase of gas flow-rate, gas holdup and surface tension of the flotation solution. Fig.4 presents the dependence of probability of collision upon the turbulent energy dissipation for four particle diameters and three bubble diameters [27]. The probability rises with the increase of energy dissipation, according to equation (53).

6. Final remarks

The probability of collision is one of the three main elementary processes conditioning mineralization of air bubbles in flotation. If the diameters of particle and bubble are strictly determined, then, according to the known formulae, this probability is proportional to the quotient of particle diameter to bubble diameter. In the case when both the particle and bubble diameters have certain distributions, as it happens in the flotation of geometrical heterogeneous material in the flotation cell with mechanical pulp agitation, then the calculation of collision probability encounters obstacles due to the indeterminacy of particle and bubble diameters. It seems to be plausible to calculate the most probable value. It can be done applying the generally known theorems of probability calculus, concerning the distributions of function of random variable. The most probable value depends on the parameters of distributions of particle and bubble diameters. At the flotation of the material whose particle size distribution was the result of grinding, the real effect upon the most probable value of collision probability exist by means of the distribution of bubble diameter. If the probability density function of bubble diameter is obtained by means of the heuristic approach, as it happens in this paper, we obtain the analytical expression of dependence of probability of collision on physicochemical properties of the flotation solution, aeration rate of the pulp, the concentration of the solid phase in the pulp and turbulent dissipation of energy in the flotation cell.

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