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NUMERICAL SIMULATION MODEL OF THE ROTARY-VIBRATIONAL MILL WORKING PROCESS

CYFROWY MODEL SYMULACYJNY PROCESU ROBOCZEGO MŁYNA OBROTOWO-WIBRACYJNEGO

The bases for the development of the numerical model simulating the working process of a rotary-vibrational mill are elucidated in the present paper. Such mills, combining advantages of ball mills of angular motion and vibratory mills, are applied among others for production of micropowders, nanopowders, nanostructures and metallic and metallo-ceramic alloys. However, at present, there is a lack of an integrated approach to modelling operations of such mills, which would take into consideration processes occurring in between milling balls when a feed is present, as well as the dynamics of the system: mill - milling balls. Normal and tangent interactions together with moments of rolling friction between working elements of a mill - balls and a chamber in the presence of material being milled, are taken into account in the presented hereby model. The influence of the milled material is described by means of the proper selection of reological constants at the contact of this material elements. The time-history of impact and friction phenomena - determined during the simulation procedure – is utilised in the model for the estimation of the mill power demand. The simulation results were confirmed by the experimental tests.

W pracy przedstawiono zasady budowy cyfrowego symulacyjnego modelu młyna obrotowo-wibracyjnego. Młyny te, łączące zalety młynów kulowych o ruchu obrotowym i młynów wibracyjnych stosowane są m.inn. do wytwarzania mikro i nano proszków i nanostruktur. W chwili obecnej brak jest kompleksowego podejścia w modelowaniu pracy tych młynów, ujmującego zarówno procesy zachodzące pomiędzy mielnikami w obecności nadawy jak i dynamikę układu młyn – mielniki. W przedstawionym w pracy modelu uwzględniono oddziaływania normalne, styczne oraz momenty tarcia tocznego pomiędzy elementami młyna wypełnionego kulami, w obecności materiału mielonego. którego oddziaływanie opisano za pomocą odpowiedniego doboru stałych reologicznych na styku mielników z pozostałymi elementami młyna. Wyznaczony w trakcie symulacji przebieg zjawisk udarowych i tarciowych wykorzystywany jest w modelu dla wyznaczenia zapotrzebowania mocy przez młyn. Wyniki symulacji potwierdzono przez porównanie z rezultatami badań doswiadczalnych.

1. Introduction

Rotary-vibration mills [1, 5, 6, 7] exhibit advantages of ball mills of angular motion and of vibration mills, which ensure high milling quality, possibility of programming the grain-size distribution of the milling product as well as the adequate yield, at a significantly reduced power consumption. Those mills are characterised by much lower environment harmfulness as compared to gravitational and vibration mills. Due to this fact, they find wide application in a laboratory practice (there are 29 units in Poland, out of which 12 in AGH). They are applied for producing powders of materials of highly variable properties, especially for production of micro and nano-powders ceramic, chemical and organic [8] as well as metallic nano-structures [9]. Rotary-vibration mills are used in industry for production of powders of high technological requirements, such as: glaze for TV kinescopes, glaze for powder enamelling, aluminium oxides for filaments of sodium lamps, special dolomites, polycrystalline silicate, suspensions for latex production, biomaterials on the matrix of glass and hydroxy apatyte [7, 10]. An acoustic pressure level of laboratory mills is within 74÷80 dB, while of industrial ones within 80÷84 dB.

Building of the analytical model for the milling process and accompanying processes, i.e. mixing, segregation, agglomeration, coating and transportation is very difficult due to a large number of milling balls, changing

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properties of materials (during milling) and variability of interactions.

Investigations of modelling milling processes in vibration mills can be divided into three groups. The first group concerns only modelling of mutual interactions of milling balls with taking into account a material being milled. These topics are dealt with by: Inoue and Okaya [11] and Yokoyama, Tamura, Usui and Jimbo [12]. The second group concerns the mathematical description of a mill, where the milled material is expressed simply either as a part of a mass or as damping. Papers of Rose [13], Bayer and Höffl [14] and Beenken, Gock and Kurrer [15] can be considered as belonging to this group. The third group represents modelling of physical processes occurring in mills from the point of view of their automation. Those models concern only changes in the grain-size distribution of the material under milling or the kinetics of material flow through the mill without taking into account any dynamic problems of a mill physical system.

As can be seen, each group exhibits a partial approach to the problem: either the milling process is taken into consideration or the mill physical system or kinetics of the milling process with balances of flowing material either via the mill or via the milling system. Thus, a need exist of developing a new model of the mill and the milling process, which will take into account all mentioned above phenomena and which will be useful in selection of basic parameters in the designing procedure of the rotary-vibration mills.

The numerical model determining the behaviour of milling balls in the mill rolling chamber performing simultaneously intensive vertical vibrations of amplitude A and frequency ω and a slow angular motion with angular velocity ω_b, is shown in the present paper. The example of such a mill is given in Fig.1. The chamber vibrations are obtained by means of a dual mass vibrator of counter running masses, driven by a belt transmission directly from a motor, while a chamber angular motion is transmitted by a belt transmission from a moto-reducer. The mill is filled by an arbitrary number of milling balls of a spherical shape and contains a scattered feed material determining friction conditions and restitution of normal impulses at balls mutual contacts and at ball-chamber contacts.

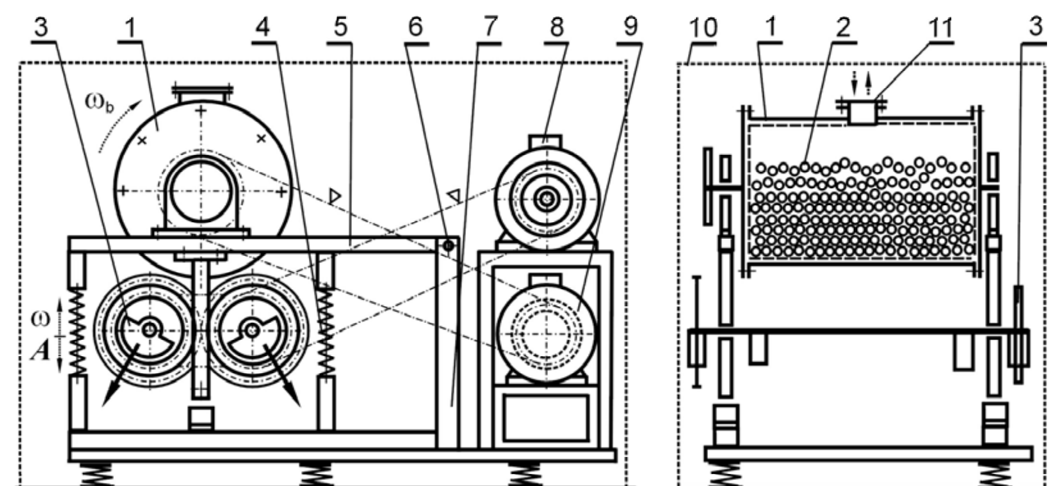


Fig. 1. Scheme of the industrial rotary-vibrational mill: 1 – chamber, 2 – milling balls, 3 – vibrator, 4 – elastic support, 5 – chamber frame, 6 – joint, 7 – mill frame with a support, 8 – vibrator drive, 9 – chamber drive, 10 – housing, 11 – charging-discharging hole

The development of the model of collisions: milling ball-milling ball and milling ball-mill, which would link the description features of the collision as a phenomenon occurring in a definite time with the timeless Newtonian description based on the notion of the coefficient of restitution R at the collision, is the most essential problem for the realisation of the complete model of such system.

The "time" description simplifies and shortens the simulation process of collisions in a multimass system allowing to solve equations of motion in a continuous way, without the necessity of interrupting the process for the determination of the new initial conditions of the system after collision.

On the other hand, the "timeless" description allows to use the coefficient of restitution R, well known experimentally, describing the restitution of normal impulses, for the determination of changes in the motion state of the system.

This approach allows also for the relatively simple reckoning of a random scatter of local properties of bodies being in the vicinity of event points, e.g. by defining of the random function R(Φ), where Φ is a generalised [4] energy density flux at a collision.

At the model development the authors utilised the description of reological properties of the contact zone – proposed in paper [16] – which enabled formulating models of normal forces and sliding friction moments (taking into consideration a mutual thrust of bodies) – for instantaneous influences. Models of friction were formulated with a dynamic approach and based on the phenomenon of the developed friction, which allowed to use the same models for the simulations of the slip and of the relative rest.

The numerical model of the rotary-vibrational mill was built on the basis of the ANSI PASCAL language. Procedures for the automatic formulation of dynamic equations of motion for the given arbitrary number of milling balls as well as functions determining the initial distribution of balls in the limited space of the mill chamber – were introduced into the model. The integration method of differential equations of motion was implemented on the basis of the Runge-Kutty-Heun algorithm in the constant-step variant [3]. The visualisation process of movements of milling balls and the mill chamber was based on functions of creations and animation of pictures in the graphical packet of the Matlab environment.

The model of milling balls movement in the rotary-vibration mill exhibits a high compatibility of the qualitative representation of the milling process with the observations in actual mills of transparent walls. There is also a good quantitative compatibility in the range of influence of the mill motion parameters on the character of milling balls movement [17].

The numerical model provides also relevant data concerning energy demands by actual systems and can constitute the basis for calculations carried out for industrial mills.

2. Interaction model in the normal direction

It was pointed out in paper [16], that a simple central collision of two bodies can be considered as a process occurring in a definite time, assuming the reological model determined by equation (1) for the description of normal forces.

The model is based on the contact rigidity theory of Hertz-Sztajerman and on the description of energy dissipation during collision – by means of the material damping model. Finely, this leads to the dependence of the normal force F on the total deformation of bodies x_w and its time derivative, shown in Fig. 2 – as the solid line.

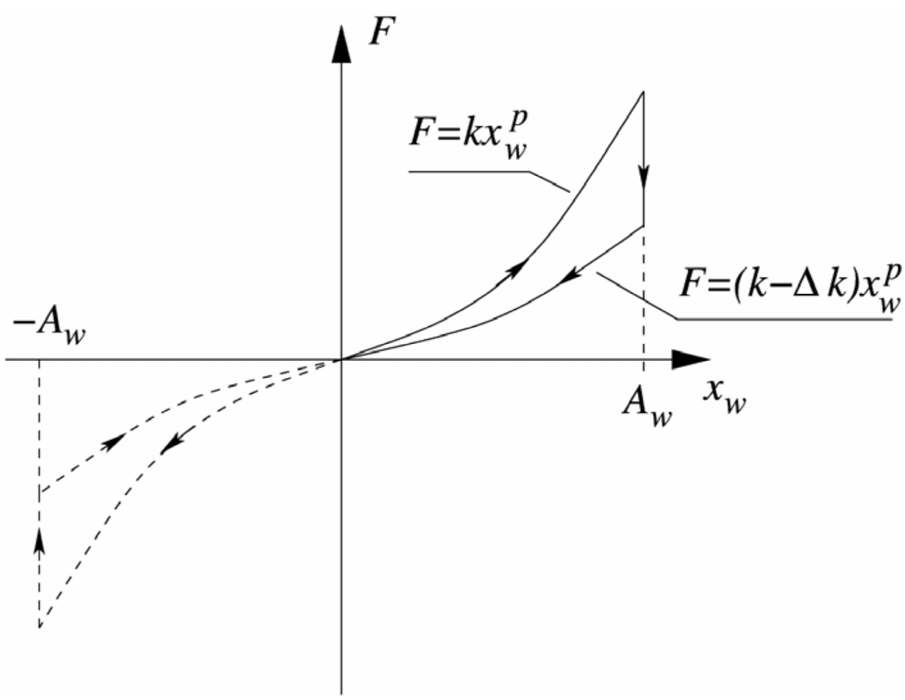


Fig. 2. Hysteresis loop describing the collision process, k, p – Hertz-Sztajerman constants

As it was shown, the following dependencies occur:

$$F_{ij} = (x_i - x_j)^p k \left[1 - \frac{\Psi}{4} + \frac{\Psi}{4} \mathrm{sgn}(\dot{x}_i - \dot{x}_j) \right] \qquad (1)$$

$$\Delta k = \frac{\Psi}{2} \tag{2}$$

$$R = \sqrt{1 - \frac{\Psi}{2}}, \tag{3}$$

where: R – coefficient of restitution at collision,
Ψ – material damping coefficient,
k – remaining notations – as in Fig 2.

Determining – from the last equation – Ψ as a function of R and substituting it to (1) we get the formulation of the contact force (4), which ensures obtaining the needed value of the coefficient of restitution R – in the simulation process:

$$F = kx_w^p \left\{1 - \frac{1 - R^2}{2}[1 - \operatorname{sgn}(\dot{x}_w)]\right\}, \tag{4}$$

where:
$x_w = x_i - x_+ j$ – total deformation of bodies participating in the collision

Such approach enables to obtain the equivalence of the collision description as the 'timeless' process, i.e. in the Newton's expression utilising the coefficient of restitution, versus the collision description as the dynamic process, occurring in a definite time and characterised by the reological properties k, ψ of the contact zone.

These formulations allow also to perform the selection of parameters in the model of normal interactions in such a way as to maintain all the time the determined experimentally R value for the colliding bodies.

3. Mathematical model of balls movements in the mill

The numerical model of the rotary-vibration mill describing the behaviour of the milling balls in the mill rolling chamber performing simultaneously vertical vibrations and rotational motion was developed on the basis of the derived model of force interactions in the normal direction and when instantaneous interactions being the result of sliding and rolling friction were taken into consideration.

Equations of plane motion of the i^{th} ball can be written in the following form:

$$\begin{cases} m\ddot{x}_i = F_{xi0} + \sum_{j=1}^{n} F_{xij} + T_{xi0} + \sum_{j=1}^{n} T_{xij} \\ m\ddot{y}_i = F_{yi0} + \sum_{j=1}^{n} F_{yij} + T_{yi0} + \sum_{j=1}^{n} T_{yij} - mg \\ J\ddot{\varphi}_i = M_{i0} + \sum_{j=1}^{n} M_{ij} + T_{i0} r \operatorname{sgn}(v_{vi0}) + \sum_{j=1}^{n} M_i(\vec{T}_{ij}) \end{cases}, \tag{5}$$

where individual components of normal forces $\vec{F}$, friction forces $\vec{T}$ and moments of rolling friction $\vec{M}$ applied to the i^{th} ball from the drum side and from other balls, are shown in Fig.3.

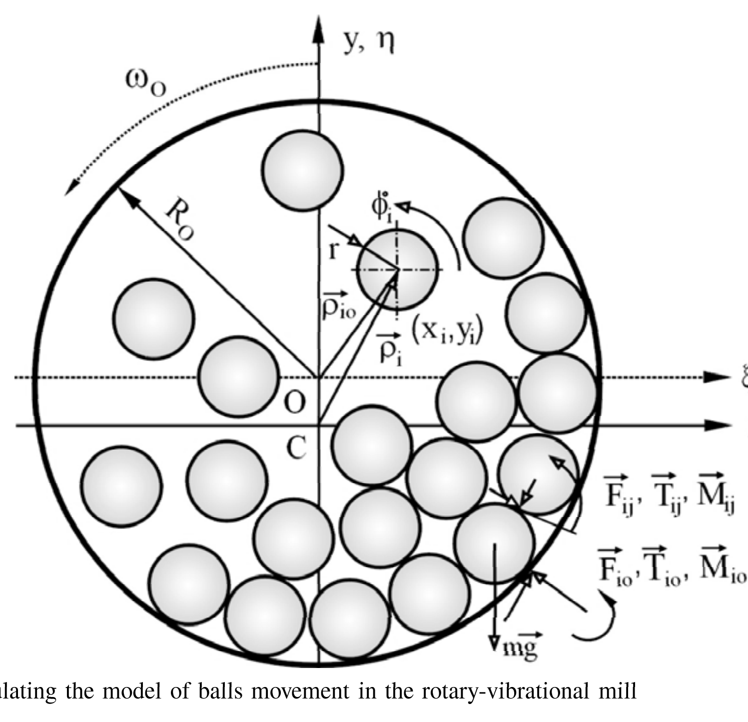


Fig. 3. Diagram used for calculating the model of balls movement in the rotary-vibrational mill

The following dependencies occur:

$$\delta_{i0} = \sqrt{x_i^2 + (y_i - y_0(t))^2} + r - R_o \tag{6}$$

$$F_{i0} = \delta_{i0}^{(p_0)} k_0 \left[1 - \frac{\Psi_0}{4}\left(1 - \operatorname{sgn}(\dot{\delta}_{i0})\right)\right] \tag{7}$$

$$\begin{cases} F_{xi0} = -F_{i0}\frac{x_i}{r_{i0}} \\ F_{yi0} = -F_{i0}\frac{y_i - y_0}{r_{i0}} \end{cases} \tag{8}$$

$$\begin{cases} T_{xi0} = F_{i0}\mu_0\frac{y_i - y_0}{r_{i0}}\operatorname{sgn}(v_{\mu i0}) \\ T_{yi0} = -F_{i0}\mu_0\frac{x_i}{r_{i0}}\operatorname{sgn}(v_{\mu i0}) \end{cases} \tag{9}$$

$$M_{i0} = -\operatorname{sgn}\left(\dot{\varphi}_i - \frac{v_{iperim}^{relat}}{r_{i0}}\right) F_{i0} f_0, \tag{10}$$

where: δ_{i0} means the total deformation (indentation) of a ball and a drum, while velocity components are as follows:

$$v_{iperim}^{relat} = -\dot{x}_i\frac{y_i - y_0}{r_{i0}} + (\dot{y}_i - \dot{y}_0)\frac{x_i}{r_{i0}} \tag{11}$$

$$v_{\mu i0} = v_{iperim}^{relat} + \dot{\varphi}_i r - \omega_0 R. \tag{12}$$

Analogically, one can write the total deformation for the i^{th} and the j^{th} ball:

$$\delta_{ij} = 2r - \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2} \tag{13}$$

and components of forces and moments:

$$F_{ij} = \delta_{ij}^{(p)} k \left[1 - \frac{\Psi}{4}\left(1 - \operatorname{sgn}(\dot{\delta}_{ij})\right)\right] \tag{14}$$

$$\begin{cases} F_{xij} = F_{ij}\frac{x_i - x_j}{2r} \\ F_{yij} = F_{ij}\frac{y_i - y_j}{2r} \end{cases} \tag{15}$$

$$\begin{cases} \vec{T}_{ij} = -\mu F_{ij}\frac{\vec{v}_{ij}^{\perp}}{\left|\vec{v}_{ij}^{\perp}\right|} \\ T_{ijx} = \frac{-\mu F_{ij} v_{ij}^{\perp}}{\sqrt{v_{ijx}^{\perp 2} + v_{ijy}^{\perp 2}}} \\ T_{ijy} = \frac{-\mu F_{ij} v_{ijx}^{\perp}}{\sqrt{v_{ijx}^{\perp 2} + v_{ijy}^{\perp 2}}} \end{cases} \tag{16}$$

$$\begin{matrix} M_i(\vec{T}_{ijy}) = T_{ijy}\left(\frac{x_j - x_i}{2}\right) - T_{ijx}\left(\frac{y_j - y_i}{2}\right) \\ M_i = 0 \qquad where: \quad i = j \end{matrix} \tag{17}$$

$$M_{ij} = -F_{ij} f \operatorname{sgn}\left(\dot{\varphi}_i - \frac{v_{iperim(ij)}^{relat}}{r}\right), \tag{18}$$

where the component value of the relative velocity (versus the contact point belonging to the j^{th} ball) perpendicular to the line joining centres of balls being in contact – equals:

$$v_{ij}^{\perp} = (\dot{\varphi}_i + \dot{\varphi}_j)r + (\dot{x}_i - \dot{x}_j)\frac{y_j - y_i}{2r} + (\dot{y}_i - \dot{y}_j)\frac{x_j - x_i}{2r} \tag{19}$$

which in projection on system axes gives:

$$\begin{cases} v_{ijx}^{\perp} = v_{ij}^{\perp}\frac{y_i - y_j}{2r} \\ v_{ijy}^{\perp} = v_{ij}^{\perp}\frac{x_j - x_i}{2r} \end{cases} \tag{20}$$

while the circulation angular velocity of the i^{th} ball by the j^{th} ball equals:

$$\frac{v_{iperim(ij)}^{relat}}{r} = \omega_{sij} = \begin{cases} \frac{\dot{y}_j - \dot{y}_i}{x_j - x_i} & when: \ \left|x_j - x_i\right| \geqslant \Delta \\ \frac{\dot{x}_j - \dot{x}_i}{y_i - y_j} & when: \ \left|x_j - x_i\right| < \Delta \end{cases} \tag{21}$$

which results from equating equations (22) and (23), determining the velocity $\vec{v}_{obw(ij)}^{wzgl}$ in two different ways:

$$\vec{v}_{iperim(ij)}^{relat} = \vec{\omega}_{sij} \times \vec{r}_s \tag{22}$$

$$\vec{v}_{iperim(ij)}^{relat} = \vec{i}\frac{\dot{x}_j - \dot{x}_i}{2} + \vec{j}\frac{\dot{y}_j - \dot{y}_i}{2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{j} \\ 0 & 0 & \omega_{sij} \\ \frac{x_j - x_i}{2} & \frac{y_j - y_i}{2} & 0 \end{vmatrix} \tag{23}$$

$i, j = 1, 2, 3$........n

4. Determination of a power loss

Components of power lost during the working process were determined on the basis of time-histories of state variables during collisions, slips, and mutual coatings of balls and chamber:

A) Interaction ball-ball.

a) Power loss due to an incomplete restitution of normal forces:

$$N_{ij} = \frac{(-1)^n}{2}\sum_{i=1}^{n} F_{ij}\frac{d}{dt}(\delta_{ij}) \tag{24}$$

b) Power loss at coating:

$$N_{0ij} = \frac{1}{2}\sum_{i=1}^{n}\left|M_{ij}\left(\dot{\phi}_i - \omega_{sij}\right)\right| \tag{25}$$

c) Power loss due to mutual slips:

$$N_{ij} = -\frac{1}{2}\sum_{i=1}^{n}\mu F_{ij} v_{ij}^{\perp} \tag{26}$$

B) Interaction ball-chamber.

a) Power loss due to an incomplete restitution of normal forces:

$$N_n = -\sum_{i=1}^{n} F_{i0} \tfrac{d}{dt}(\delta_{ij}) \tag{27}$$

b) Power loss at coating:

$$N_{0ij} = \sum_{i=1}^{n} \left| M_{i0} \left(\dot{\phi}_i - \frac{\vec{v}_{obw}^{wzgl}}{r_{i0}} \right) \right| \tag{28}$$

c) Power loss due to mutual slips:

$$N_\mu = -\sum_{i=1}^{n} T_{i0} \left| v_{i0} \right|. \tag{29}$$

Determination of type B power losses is sufficient for the power demand estimation of the mill, since values of those losses depend on type A losses, via mutual interactions in the system.

The mathematical model developed in the present paper enables to determine movements of milling balls caused by angular and vibrating motion of the drum as well as to estimate the mill power demand, while the visualisation module based on the possibilities of the graphical packet of the Matlab environment allows to reproduce the system movement on the monitor.

The simulation results of the milling balls movement in the rotary-vibrational mill chamber (of a diameter of 0.21 m), which was filled with 42 corundum balls of a diameter 19 mm, without the milled material, are illustrated in Fig. 4. The photograph of the instantaneous position of balls in the actual mill, obtained at the same geometrical and kinematic parameters of balls and the mill chamber equipped with transparent cover [6. 17], is seen in Fig. 5.

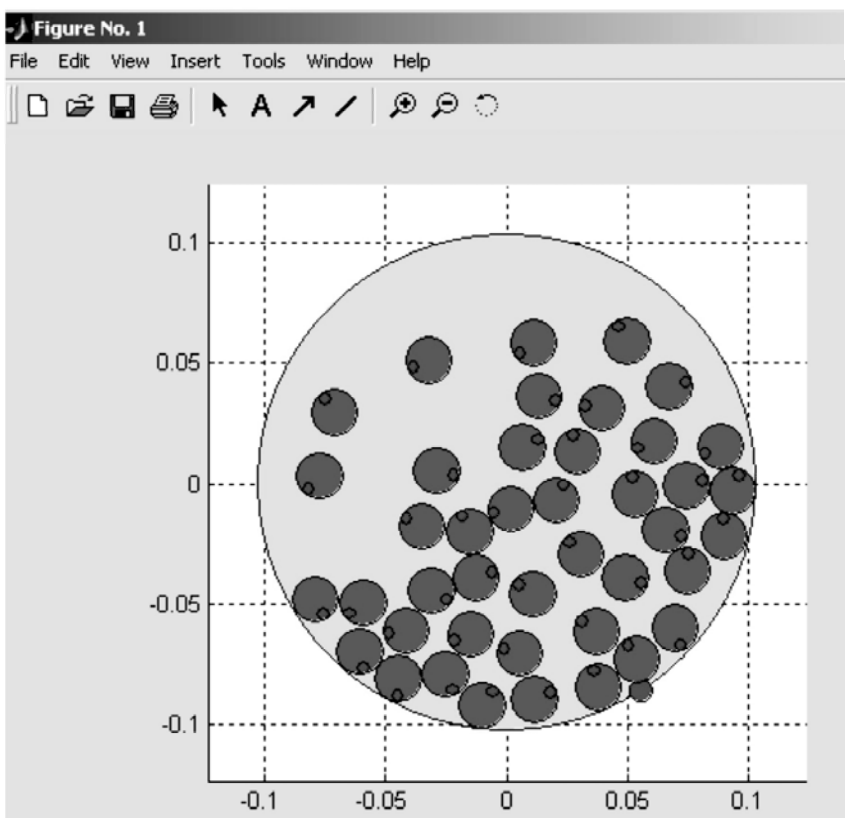


Fig. 4. Instantaneous position of balls of a diameter of 19 mm, according to the computer simulation

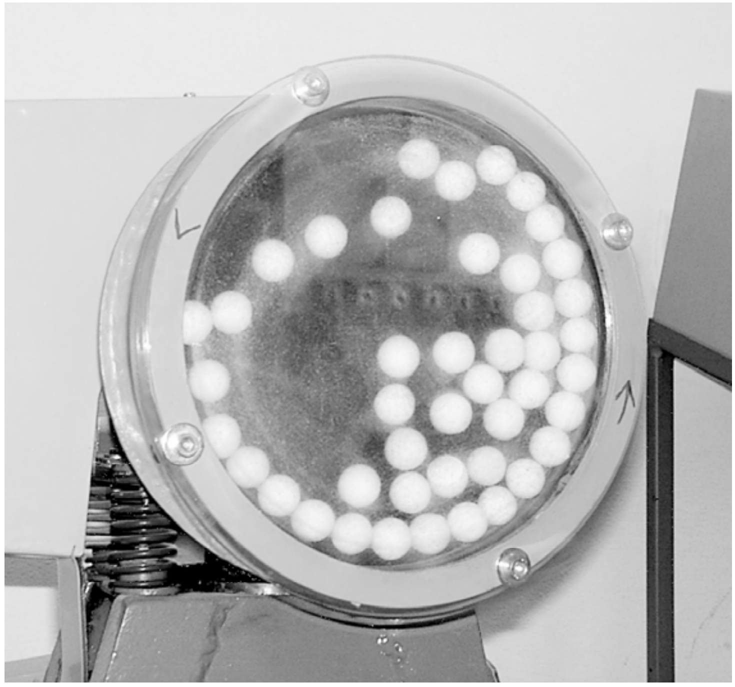


Fig. 5. Instantaneous position of 42 balls of a diameter of 19 mm in the mill, experiment

Successive phases of the simulation are seen in Fig. 6, 7 and 8, while the photograph of the position of 77 corundum balls of a diameter of 13.5 mm in the actual mill (having the transparent cover of the chamber) of the same geometrical and kinematic parameters is shown in Fig. 9 [17].

It should be emphasised, that 42 balls of a diameter of 19 mm and 77 balls of a diameter of 13.5 mm constituted the same filling ratio of the chamber, being equal to 0.40 .

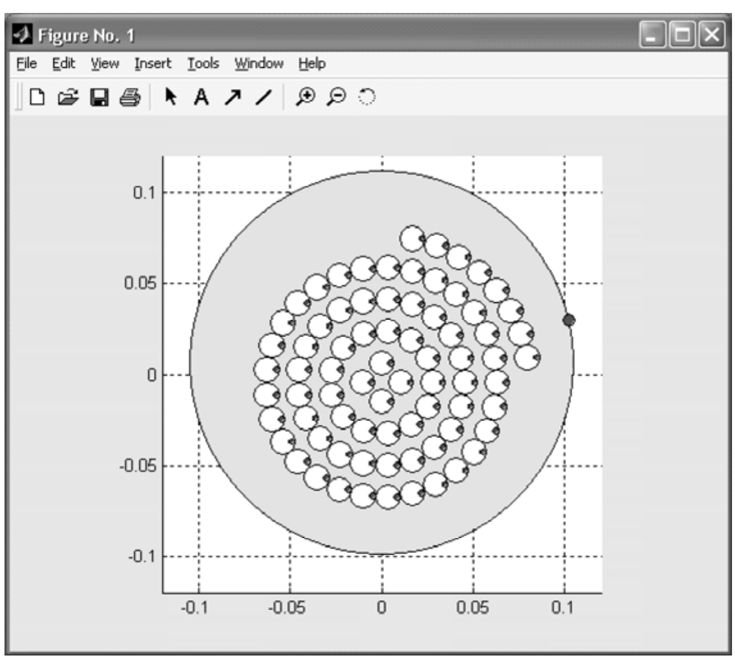


Fig. 6. Initial position of balls (ball diameter =13.5 mm)

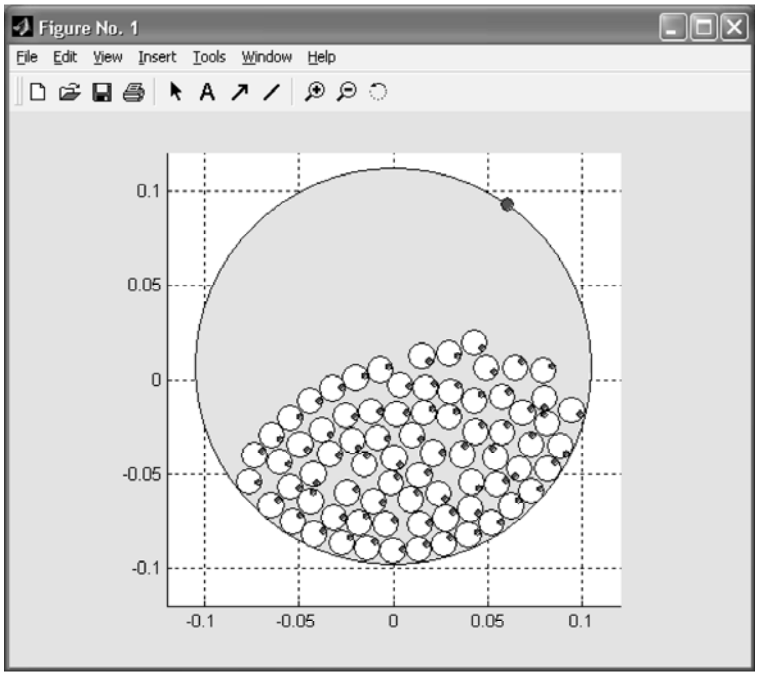


Fig. 7. Position of balls during their first contact with the mill chamber (ball diameter=13.5 mm)

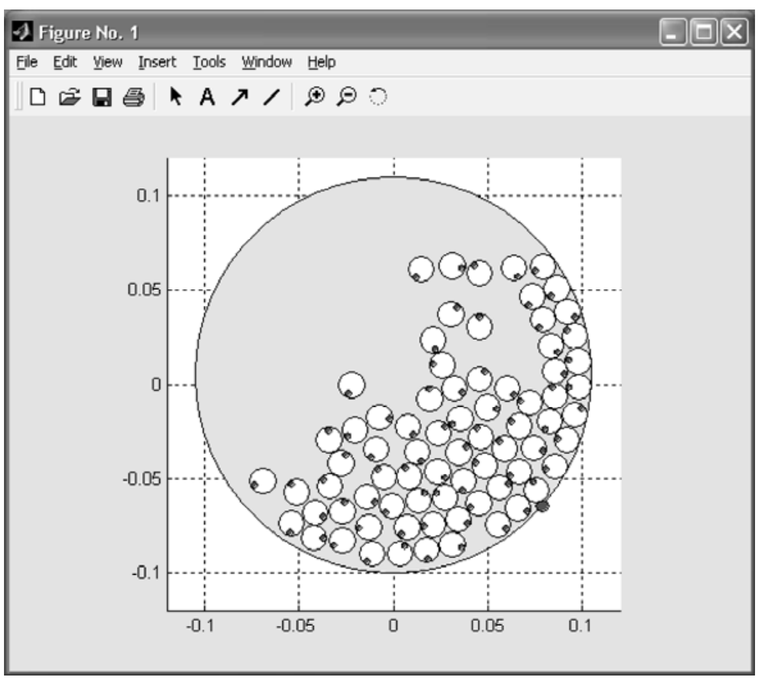
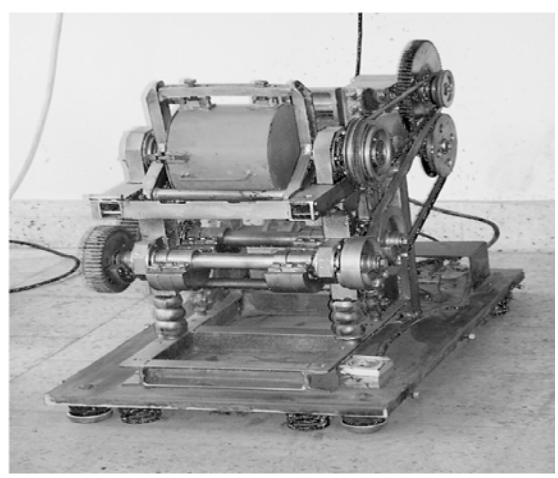


Fig. 8. Example of the position of balls during the steady state performance (ball diameter=13.5 mm)

Fig. 9. Instantaneous position of balls in the rotary-vibrational mill chamber – during operation in the Department of Manufacturing Systems AGH

The laboratory mill LAMOW-B-5/2 [6, 18] was used to verify the numerical model from the point of view of power consumption – Fig. 10. The structure of this relatively large laboratory mill of a chamber capacity being 5 dm^3, is the same as the structure of industrial mills of a chamber capacity of 20 dm^3. Industrial rotary-vibration mills of a larger chamber capacity (up to 200 dm^3) are equipped with independent drives of a vibrator and of a chamber [6].

a



b

Fig. 10. Laboratory rotary-vibrational mill LAMOW-B-5/2: a) Mill in the soundproof- insulation housing; b) Mill after the housing removal [6, 18]

The verification of the numerical model of the rotary-vibrational mill LAMOW-B-5/2 was preceded by a total identification of all essential – for the numerical model - parameters, such as: masses, moments of inertia of balls, friction coefficients, and – especially – scattering coefficients, which values are given in Table 1.

TABLE 1

Coefficients of restitution and material scattering for the material of balls and the assumed conditions [6]

Ball diameter, mm	Material of balls	Chamber material	Material between balls	Coefficient of restitution, R	Material damping coefficient, ψ
19.0	Corundum	Corundum	None	0.73	0.93 ± 0.03
19.4	Corundum	Steel	Milled sand	0.29	1.83 ± 0.06
17.1	Steel	Steel	None	0.54	1.32 ± 0.04
17.3	Steel	Steel	Milled sand	0.15	1.95 ± 0.06

The example of a computer simulation for the case of the corundum balls (19 mm) with the milled material and the same geometrical and kinematic parameters as the ones of the rotary-vibrational mill LAMOW-B-5/2 [6] is shown in Fig. 11. At that time, the mill chamber was filled with 338 balls of a mass of 4.2 kg and with 1.2 kg of high-silica sand. The verification result is given in Table 2, (bold line).

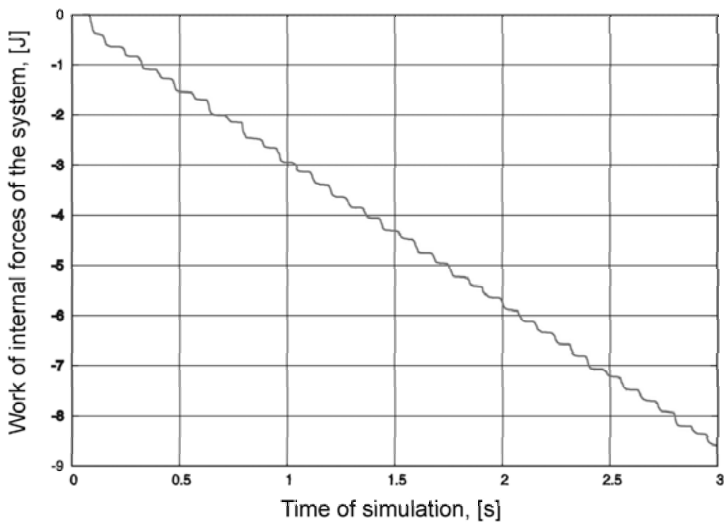


Fig. 11. Computer simulation of the internal forces work (L<0), in the system: chamber – corundum balls (of a diameter = 19 mm) – milled high-silica sand, filling ratio = 0.40

Investigations of the power consumption of the LAMOW-B-5/2 mill, was carried out for two kinds of milling balls (steel and corundum), at three levels of chamber filling ratio and corresponding to them three values of ball masses. The power consumption was measured for both materials of milling balls with and without the material being milled (high-silica sand). The verification results of the numerical model are given in Table 2.

5. Conclusions

The obtained results of the computer simulation indicate a high compatibility of the qualitative representing of the milling process with observations in actual mills of transparent walls (Fig.5 and 9) as well as the high quantitative compatibility in the domain of the influence of motion parameters on the milling balls movement character. The numerical model can also constitute a credible tool for estimating the power consumption during the milling process and can become the base for selecting parameters of industrial mills. The verification results of the model from the point of view of power consumption carried out on the large laboratory rotary-vibration mill indicate very good convergence with the experimental results carried out on the actual object. An average error in the power demand estimation between the simulation model and the experiment was approximately 10 %.

TABLE 2

Verification results of the numerical model of the LAMOW-B-5/2 mill [6]

Filling ratio	Material of balls	Ball diameter mm	Ball mass, kg	Sand mass, kg	Ball power ***model***, W	Ball power ***experiment***, W	Difference: model-experiment	
							W	%
0.20	Corundum	Φ19.0	2.1	0.60	12.9	12.6±6	0.3	2.4
0.20	Steel	Φ17.2	4.6	0.60	26.9	24.7±7	2.2	8.9
0.40	Corundum	Φ19.0	4.2	1.20	22.9	20.2±6	2.7	13.4
0.40	Steel	Φ17.2	9.2	1.20	49.6	53.4±6	3.8	7.1
0.60	Steel	Φ17.2	13.8	1.80	60.6	70.7±7	10.1	14.4
0.20	Corundum	Φ19.0	2.1	–	8.8	8.1±6	0.7	8.2
0.40	Corundum	Φ19.0	4.2	–	19.8	17.6±6	2.2	12.5
0.40	Steel	Φ17.2	9.2	–	43.1	38.4±7	4.7	12.2

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