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FRACTURE TOUGHNESS OF MATERIALS AT THE PRESENCE OF PLASTIC DEFORMATION

ODPORNOŚĆ NA PĘKANIE MATERIAŁÓW PRZY OBECNOŚCI ODKSZTAŁECEŃ PLASTYCZNYCH

In this paper the problem of fracture toughness is reanalyzed. It is shown that fracture toughness is not a material property. It depends on a shape and size of structural elements. The measures of in- and out-of-plane constraint are defined and their influence on fracture toughness is demonstrated. The idea of "local approach" to fracture is shortly described and some results obtained within this approach concerning the fracture toughness determination are presented. The idea of fracture energy is presented and this quantity is computed both for linear and non-linear materials using the step-like crack growth model.
Keywords: fracture toughness, in- and out-of-plane constraint, fracture energy

W pracy przeanalizowano problematykę odporności na pękanie. Pokazano, że odporność na pękanie nie jest stałą materiałową. Zależy ona także od kształtu i wymiarów elementów konstrukcyjnych. Zdefiniowano miary więzów płaskich i w kierunku grubości oraz pokazano ich wpływ na odporność na pękanie. Idea „lokalnego podejścia" do procesu pękania jest krótko przypomniana i przytoczono niektóre rezultaty uzyskane przy stosowaniu tej idei do określania rzeczywistej odporności na pękanie. Przedstawiono też rezultaty oceny energii pękania dla materiałów plastycznych przy zastosowaniu tzw. Skowego modelu propagacji pęknięć.

1. Fracture mechanisms

An extension of a macro-crack under monotonic loading may proceeds either by cleavage or ductile transgranular fracture mechanisms at low or room temperatures. It is a result of complex processes at the micro or nano scale. There are many, sometimes contradictory theories aiming at the qualitative and quantitative descriptions of the micro-separation (micro-cracks and voids) nucleation and growth in the area in front of a macro-crack. These micro-separations nucleate, grow and coalesce with a macro-crack. The mechanisms of the micro-separation formation depend on a type of material and its microstructure, temperature, stress, strain and strain rate. It is not a purpose of this article to review all the available publications in this field. An extended summaries on this subject can be found in [1-4], published recently.

We will concentrate on the fracture mechanisms in steels. For these materials the cleavage fracture mechanism is often observed at low temperatures (ferritic steels). In the cleavage-ductile transition temperature range the micro-crack nucleation results from the inhomogeneity in plastic deformation between the matrix and inclusion. A rough rule is that dominating nucleation kernels are larger particles (e.g. carbides). At these kernels the micro-separations are first nucleated. However, when they are very infrequent, smaller but more abundant kernels might dominate the process. For small particles the strain to initiate the micro-crack is inversely proportional to the particle size. The hard kernel is broken due to the sufficiently high tensile stress when the elastic energy released from the particle by interfacial separation is at least equal to the surface energy created. If the stress in front of the nucleated micro-separation is high enough it grows as a micro-crack, if not it grows as a void. A number of observations reported in the literature [2] have shown that at low temperatures, cleavage fracture is essentially nucleation controlled. In many publications cleavage fracture is considered as the stress controlled process. It is certainly so for the cleavage fracture at the lower plateau temperature range where micro-cracks

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are nucleated not due to the fracture of hard kernels but nucleation may occur at any location of the high stress concentration. Such concentration may appear as a result of the dislocation pile-up, twin's interactions, near grain vertices etc. In the cleavage-ductile transition temperature range the plastic strains are necessary to nucleate the micro-crack [2-3], however the high opening stresses are necessary to propagate it.

Ductile fracture due to the void nucleation, growth and coalesce (VNGC) is sometimes considered as a plastic strain controlled process. Micro-separations in metallic polycrystalline materials are nucleated, as a rule, after severe local plastic flow. Thus the void's nucleation is strain controlled. The void's growth process depends both on the strain and hydrostatic stress levels. It follows from the McClintock [5] and Rice and Tracy [6] simple models. These models are based upon a number of simplifying assumptions which do not necessarily reflect the actual behavior of real materials. However, some experiments on low alloy steel [7] have shown that results by Rice and Tracy were rather well obeyed. On the other hand one must remember that the high level of the stress triaxiality impedes plastic flow. Sometimes the VNGC fracture mechanism is accompanied by the slip along slip facets mechanism. This mechanism is more frequent close to the specimen surfaces along the shear lips.

2. Mechanical fields in front of crack in elastic – plastic materials

In 1968 Hutchinson [8], Rice and Rosengren [9], published their solution for the stress, strain and displacement fields in front of the crack which thereafter will be called the HRR solution. It is valid for the nonlinear material, characterized by the Ramberg – Osgood (RO) constitutive equation, under assumption of small strain and for the plane stress or plane strain models of the structural element. Thus, it can be considered as a solution for the stationary crack and plastic material according to the deformation theory of plasticity.

$$\sigma_{ij}(\theta = 0) = \sigma_o \left(\frac{J}{\alpha \varepsilon_o \sigma_o I_n r} \right)^{\frac{1}{1+n}} \tilde{\sigma}_{ij}(\theta, n), \qquad (1)$$

where σ_o is the yield stress, $\varepsilon_o = \sigma_o / E, \alpha$ and n are the RO coefficient and power exponent respectively, r is a distance from the crack tip, θ is an angle between x_1 coordinate and the **r** vector (the coordinate system is fixed at the crack tip and x_1 is located in the crack plane, perpendicularly to the crack front, x_2 is perpendicular to the crack plane), I_n and $\tilde{\sigma}_{ij}(n, \theta)$ are functions of n and should be computed [10], they may be found also in the handbooks. J is a function of external loading, crack length a and geometry of the element. It is called the J – integral [11], it is path independent for the stationary cracks and it should be computed numerically or approximated by various more or less exact formulae [12]. According to Eq.(1) the stresses (as well as strains) are singular. Other terms of the asymptotic expansion have been neglected. Because of the stress singularity the fracture criterion can not be expressed in terms of stress tensor components and according to Broberg [13] it can be written in the form

$$J_\alpha(\sigma_{appl}, a, geometry) = J_{\alpha C}, \qquad (2)$$

where α = I, II, II depending on the mode of loading.

However, the J_{IC} value (there are no reliable methods to measure J_{IIC} and J_{IIIC}) *is not a material constant*. It depends on the shape and size of the specimen or structural member. If it is measured according to standards, e.g. [14] the value of J_{IC} achieves one of the smallest values among the others, measured on specimens not satisfying the standard's requirements. Thus, when it is used in the fracture criterion the result obtained is always conservative, safe but not economical. If the constraint is low, e.g. for short cracks, the fracture toughness can be several times higher (even four, five times), Fig. 1.

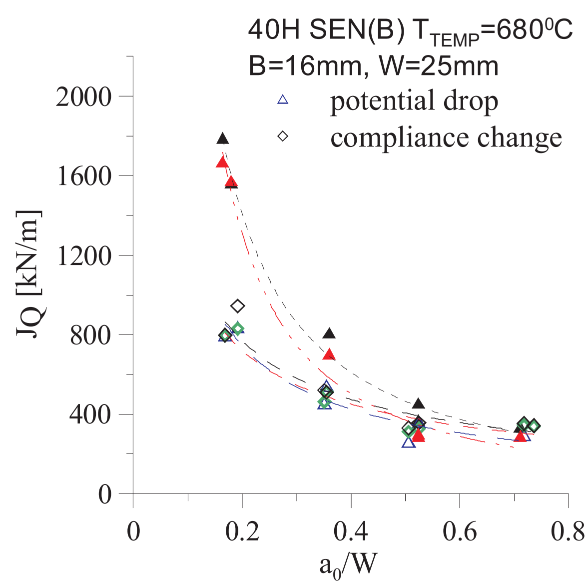

Fig. 1a Fracture toughness as a function of crack length [15]

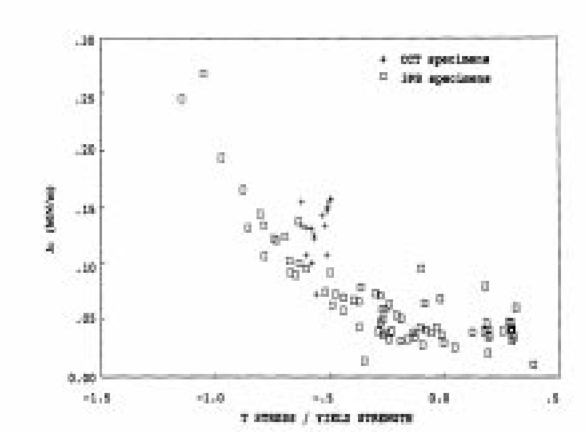

Fig. 1b Fracture toughness as a function of T-stress [16]

The value of the $J_{\rm I}$ – integral represents difference of the potential energy of external forces of two almost identical specimens containing crack of the lengths a and $a + da$. It is similar but not identical quantity to the Energy Release Rate (ERR), $G_{\rm I}$. They would be identical for elastic materials. Due to the relation (Rice [17]) between the energy released and deformation energy the critical value of the J – integral represents also the amount of energy dissipated up to the moment of fracture and during the fracture process. In another words it "contains" both energy dissipated for plastic deformation and the process of fracture. It will be discussed in more details in Section 4.

The standard specimens are characterized by the high constraint. They are classified as the in-plane and out-of-plane constraint. In-plane constraint depends on the in-plane specimen dimensions, e.g. a/W where W is the specimen width. The out-of-plane constraint depends on the specimen thickness, B. The geometrical dimensions of the specimen (machine member) are certainly not convenient measures of the constraint to use in the engineering practice. The Q-parameter, introduced by O'Dowd and Shih (OS) [18], is probably the most popular, in the literature, and useful measure of the in-plane constraint

$$\sigma_{ij}(\theta = 0) = \sigma_o \left(\frac{J}{\alpha \varepsilon_o \sigma_o I_n r} \right)^{\frac{1}{1+n}} \tilde{\sigma}_{ij}(\theta, n) + Q\sigma_0 \delta_{ij}. \quad (3)$$

The Q-parameter (or Q-stress) is not a second term of the asymptotic series representing the stress distribution in front of the crack. It simply replaces the all neglected terms of this series. It corrects the hydrostatic stress components σ_{11}, σ_{22} only, computed by Eq. 1. The Q-parameter weakly depends on the distance from the crack tip. O'Dowd and Shih proposed to neglect this dependence. Thus, it was postulated to compute this quantity at the distance $r = 2J/\sigma_o$ [18]. The Q-parameter must be computed numerically using the finite element method [18]. If the normalized distance $r = 2J/\sigma_o$ is introduced into Eq. 3 the stress level in front of the crack depends on the material properties and the Q-parameter only. In turn, the Q-parameter depends on the external loading, material properties and the in-plane dimensions of the structural element.

An alternative to OS theory was introduced by Yang, Chao and Sutton (YCS) [19] According to YCS the stress field in front of the crack can be computed from the relation:

$$\frac{\sigma_{ij}}{\sigma_0} = A_1 \left[\left(\frac{r}{L} \right)^{s_1} \tilde{\sigma}_{ij}^{(1)}(\theta, n) + A_2 \left(\frac{r}{L} \right)^{s_2} \tilde{\sigma}_{ij}^{(2)}(\theta, n) + + A_2^2 \left(\frac{r}{L} \right)^{s_3} \tilde{\sigma}_{ij}^{(3)}(\theta, n) \right], \quad (4)$$

where:

$$A_1 = \left(\frac{J}{\alpha \varepsilon_0 \sigma_0 I_n L} \right)^{-s_1}; \quad s_1 = -\frac{1}{1+n}; \quad \tilde{\sigma}_{ij}^{(k)}(n, \theta),$$

$$k = 1, 2, 3;$$

L is a characteristic length (e.g. the specimen thickness) the exponent s_2 is a quantity, which value can be found in the YCS's [20] or Nikishkov's [21] papers, $s_3 = 2s_2 - s_1$; the coefficient A_2 should be computed numerically [20], [22] in an arbitrary point in front of the crack tip located within the distance range $1 \leqslant r\sigma_0/J \leqslant 5$. Other quantities are known from the HRR solution [8][9] and were defined below Eq. 1.

Eq. 3 consists of three terms, which are sufficient to estimate the stress field in front of the crack with a very good accuracy. However, to estimate these stress components only two coefficients, which depend on the external loading are necessary: A_1, which is the function of J and A_2, which does not depend on the distance from the crack tip. Almost the same numerical effort must be undertaken to compute Q or A_2. Both parameters Q and A_2 can be used as the in-plane constraint measures. Other quantities which can serve as an alternative measures of the in-plane constraint are: σ_m/σ_o where σ_m is hydrostatic stress (e.g. [23]) or σ_m/σ_e (e.g. [24]), where σ_e is effective stress or σ_{22}/σ_o (e.g. [25]). However, the Q-parameter is equivalent both to σ_m and σ_{22}.

The analysis of the out-of-plane constraint influence on fracture toughness in not well established in the case of the non-brittle fracture. It is because of the false opinion that the out-of-plane constraint is not so important in the fracture toughness analysis. This opinion might be true in the case of brittle fracture. It is not true in the case of ductile fracture (see next section). As a measure of the out-of-plane constraint we will adopt the quantity introduced to the fracture mechanics by G u o [26], [27], [28].

$$T_z = \frac{\sigma_{33}}{\sigma_{11} + \sigma_{22}}. \qquad (5)$$

This function may be interpreted as the generalized Poisson ratio and for plastic materials it is equal to zero for plane stress and 0.5 for plane strain. The average through the thickness value of T_z, Tm, depends on the specimen thickness (also on the distance from the crack tip, power exponent in the R-O constitutive relation and external loading). T_z function should be computed numerically, although some analytical approximations are available [21]. Guo [26-28].proposed an approximated formula to compute the stress field in front of the crack in the 3D structural element.

$$\sigma_{ij} = \sigma_o \left(\frac{J_{far}}{\alpha \varepsilon_o \sigma_o I_n(T_z) r} \right)^{\frac{1}{1+n}} \tilde{\sigma}_{ij}(\theta, n, T_z), \qquad (6)$$

where J_{far} is a far field J-integral, computed along the contour drawn in the region dominated by plane stress. Functions $I_n(T_z), \tilde{\sigma}_{ij}(\theta, n, T_z)$ can be computed numerically and the suitable program is available [10].

3. Local approach to fracture – fracture toughness, constraint analysis

It is rather general opinion in the fracture mechanics community that in order to assess the influence of the constraint on fracture toughness the local approach to fracture should be used. Using local approach one should concentrate attention on the physical processes leading to the ultimate failure and taking place in a small region in front of a crack. These processes are either stress or strain controlled or both. The local approach originates from the B e r e m i n paper [29]. B e r e m i n defined the probability of failure which depends on the distribution of the micro-voids or micro-cracks nucleation kernels:

$$P_f = 1 - \exp\left[-\left(\frac{\sigma_w}{\sigma_u} \right)^m \right], \qquad (7)$$

where the so called W e i b u l l stress is defined as

$$\sigma_w = \left(\sum_j \left(\sigma_{22}^j \right)^m \frac{V_j}{V_o} \right)^{1/m} = \left(\frac{1}{V_o} \int_V \sigma_{22}^m dV \right)^{1/m}, \qquad (8)$$

where V_j is the volume of the jth material unit in the crack tip plastic zone experiencing a maximum opening stress σ_{22}^j, the reference volume V_o is often taken to be representative of the material structure, V is a volume of the process zone. It is an important quantity. However, different authors define this volume differently. The W e i b u l l distribution parameters should be determined according to some procedures. One of them was proposed in the ESIS P6-98 document. This procedure requires the experimental investigations on the cylindrical notched specimens and associated numerical computations. Many authors do not accept this procedure when the fracture mechanics analysis is to be carried out, since the stress field in front of the crack is different than that within the cylindrical specimen. Others, so called calibration methods, are proposed, e.g. [30]. In most papers σ_u parameter is considered as a value of σ_w computed at the 63.2% probability of fracture and Beremin suggested that this quantity should be a material constant.

To assess the influence of the constraint on fracture toughness it is assumed that, for given material, the probability of fracture is the same independently of the shape and size of structure [31]. Using this assumption G a o and D o d d s [31] derived formula for fracture toughness of materials and structures with a small plastic zone in front of the crack (small scale yielding)

$$K_{J_c}^{T \neq 0} = K_{J_c}^{T=0} \left(\frac{B_{T=0}}{B_{T \neq 0}} \right)^{1/4} F\left(n, \nu, E/\sigma_o, T/\sigma_o, m\right), \qquad (9)$$

where $K_{J_c}^T$ denotes the value of the SIF at the onset of fracture, computed from the value of J_c using the formula $K = \sqrt{JE}$. The function $F(-)$ should be determined for various ranges of material properties, ν, E, σ_o, n, W e i b u l l modulus m and the T-stress. In [31] this function was determined numerically for the small scale yielding

$$F(n, \nu, E/\sigma_o, m) = 1 + \sum_{i=1}^{N_1} \left[\sum_{j=0}^{N_2} b_{ij}(n, \nu, E/\sigma_o) m^j \right] \left(\frac{T}{\sigma_o} \right)^i, \qquad (10)$$

where the coefficients b_{ij} were given in [31]. Eq. 9 takes into account both the in-plane constraint (T-stress) and out-of-plane constraint – the specimen thickness, B.

Although many problems and uncertainties await for the solution and clarification in the statistic, local approach to the fracture toughness assessment, the method has already found practical application. It is used to determine certain constants in the empirical formula to compute fracture toughness taking into account the in-plane constraint, proposed in the FITNET or SINTAP [32], [33] procedures

$$K^c_{mat} = K_{mat}\left[1 + \alpha(-Q)^k\right] \quad for \quad Q \leqslant 0. \tag{11}$$

Using statistic, local approach S h e r r y et al published [34] Lookup Tables for α and k with respect to T/σ_o (T-stress plays similar role for elastic solution like Q-parameter for plastic one – Eq. 3) and a range of W e i b u l l modulus m.

Another local approach to fracture is based on the assumption that the fracture to happen requires that the stress components (hydrostatic or opening) should exceed a certain critical value within a finite volume in front of the crack. This hypothesis was proposed by R i t c h i e , K n o t t and R i c e (RKR) [35] for cleavage fracture. However, it is often observed that the cleavage fracture is preceded by plastic deformation (Fig. 2).

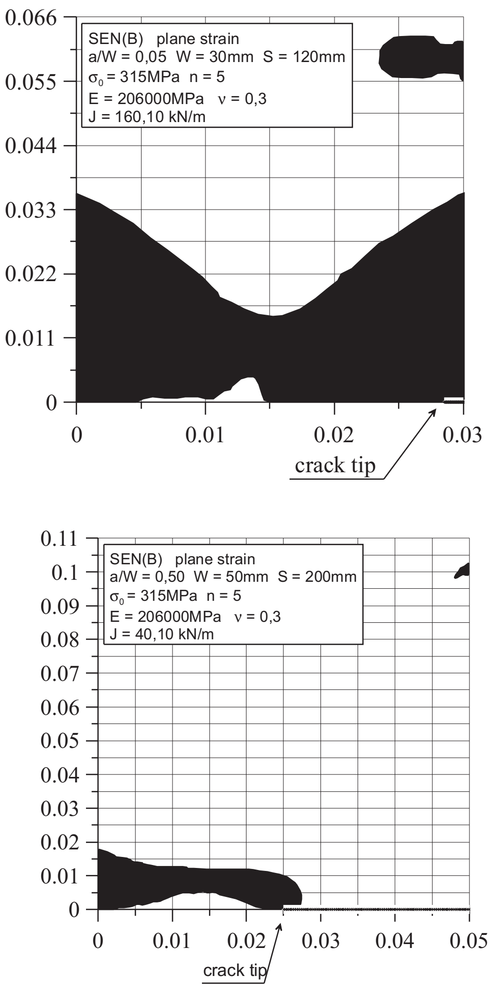

Fig. 2 The plastic zones in front of the cracks at the critical moments for two specimens in the Sumpter, Forbes experiment [16]

Using RKR hypothesis and Eq. 3 O ' D o w d [36] derived formula for fracture toughness as a function of the Q-parameter.

$$J_C = J_{IC}\left(1 - \frac{Q}{\sigma_c/\sigma_0}\right)^{n+1}, \tag{12}$$

where σ_c is the critical stress, characteristic for given material. It is usually considered as an adjustable parameter in the theory, although O ' D o w d suggests how to measure it. O ' D o w d analysis was made assuming that strains are small in front of the crack. Using, slightly modified RKR hypothesis and assuming that strains are large in front of the crack, which was justified by the numerical computations, N e i m i t z et al [37] derived the alternative equation

$$J_C = J_{IC}\left[\left(\frac{1}{\varphi_{ref}}\right)^{\frac{1}{1+n}} - \frac{(Q - Q_{ref})}{\tilde{\sigma}_{22}}\left[\frac{E}{\alpha\sigma_0 I_n}\right]^{\frac{-1}{1+n}}\right]^{1+n} \varphi_{ref}, \tag{13}$$

where φ_{ref} is a normalized distance between the stress maximum and the crack front $\varphi = r\sigma_o/J$. This value should be computed numerically for the reference specimen, which can be associated with the standard specimen. All other quantities were defined in Eqs (2) and (3). One may notice that the σ_c value does not appear in Eq.(13).

Derivation was based on the specific features of the stress field in front of a crack. Such a field can only be analyzed numerically. The main conclusions following from the numerical computations are summarized below

- The hydrostatic stress components reach maximum at certain distance in front of the crack. The maximum value of the opening stress component is not sensitive to the external loading changes but it is sensitive to the in-plane characteristic specimen size a/W. It depends on the deformation properties of the material, n and σ_0/E. This happens when the plastic zone covers an essential part of the ligament in front of the crack tip. Numerical results can be approximated by the formula [37]:

$$\frac{\sigma_{22}^{\max}}{\sigma_o} = c\,(a/W, \sigma_o/E) + \frac{d\,(a/W, \sigma_o/E)}{n^2}, \tag{14}$$

where

$$c = \left[0.38 - 59\left(\frac{\sigma_o}{E}\right) + 29073\left(\frac{\sigma_o}{E}\right)^2 \right.$$
$$\left. -3833137\left(\frac{\sigma_o}{E}\right)^3 \ln\left(\frac{a}{W}\right) \right.$$
$$\left. + \left[3.3 + 76.8\left(\frac{\sigma_o}{E}\right) - 15214\left(\frac{\sigma_o}{E}\right)^2 + 505043\left(\frac{\sigma_o}{E}\right)^3\right]\right] \tag{15}$$

$$d = \left[2.33\left(\frac{a}{W}\right)^{-0.29}\right]\left(\frac{\sigma_o}{E}\right)^{(-0.034\ln(a/W)-0.51)}. \qquad (16)$$

From these results it can be shown that for the small values of $n \leqslant 5$ the maximum stress depends very weakly on a/W in the range of the later 0.05-0.8. However, it changes essentially with σ_o/E. For less hardening material, $n > 10$, the value of the maximum of the saturated opening stress depends considerably on a/W and it is almost independent of σ_o/E.

- The stress maximum is located at the distance $r = \varphi J/\sigma_0$ from the crack tip. It can be shown that the coefficient φ is a function of $n, \sigma_0/E$ and the in-plane constraint measure Q when the maximum stress saturates to a constant value. In fact, this function does not change with the external loading but changes with the ratio a/W. When the stress maximum still changes with external loading the coefficient φ depends on the $\sigma_{22}^{\max}$ also.

Neimitz and Galkiewicz [15] used another model within the scope of the local approach to fracture. They estimated fracture toughness for ductile fracture mechanism in terms of Q and Tm parameters. The model was based on the assumption that at the critical moment the representative void in front of the crack reach a critical size and this size does not depend on the shape and size of the structural element.

$$J_c^{dv} = J_{IC}\frac{d_n(T_m = 0.5, Q = 0)}{d_n(T_m < 0.5, Q \neq 0)}\frac{\sinh\left[\frac{\sqrt{3}\sigma_m(T_m=0.5,Q=0)}{\sigma_0}\right]}{\sinh\left[\frac{\sqrt{3}\sigma_m(T_m,Q)}{\sigma_0}\right]}, \qquad (17)$$

where d_n is a parameter from the well known relation between J- integral and crack tip opening displacement, δ_T, ($\delta_T = d_n J/\sigma_o$), σ_m is a hydrostatic stress, which can be computed from Eq. (3) or (4) for plane strain or for 3D numerically, or using approximate formula [15] which utilizes T_z parameter.

4. Fracture energy, influence of constraint

The physical interpretation of fracture energy, adopted from Rice [38] in this paper, is as follows:

"Contributions to fracture energy, Γ, come entirely from microstructural features not represented in the continuum model of the extending crack" and "...the surface energy term, Γ, must include not only the usual surface energy due to the ultimate de-cohesion, but also the energy dissipated in inhomogeneous plastic sliding occurring prior the separation along non-favorably oriented portions of the fracture surface." One can write directly from the first law of thermodynamics;

$$\frac{\partial(W - (U_e + U_p)}{\partial a} = 2\Gamma \rightarrow \frac{\partial(W - U_e)}{\partial a} = 2\Gamma + \frac{\partial U_p}{\partial a}, \qquad (18)$$

where U_e and U_p are elastic and plastic strain energies respectively and W is work of external forces. The left hand side of the second of Eq. (18) is a crack tip driving force and the two terms at the right hand side represent dissipation: the fracture energy on unit area and dissipation due to the plastic deformation respectively. In this section we will concentrate on the fracture energy Γ. It was derived from Eq. (18) in [39], both for fixed grips and dead load situation, that fracture energy is equal:

$$-\frac{1}{2}\int_{A'} T_i^{(1)}u_i^{(2)}dA = \Gamma A' \qquad (19)$$

for linear materials and

$$-\frac{1}{1+n}\int_{A'} T_i^{(1)}u_i^{(2)}dA = \Gamma A' \qquad (20)$$

for nonlinear materials. In (19) and (20) $T_i^{(1)}$ is an opening stress distribution in front of the crack before the crack jump over the distance Δa and $u_i^{(2)}$ is the crack face displacement over the distance Δa after crack jump, A' is a new crack area. Rice [38] received very similar to Eq. (19) result:

$$\Gamma = \lim_{A'\to 0}\left(\frac{1}{2A'}\int_{A'} T_i^{(1)}u_i^{(2)}dS\right). \qquad (21)$$

The difference between Eqs (19) and (21) is in the limit operation. In [39] it was shown that there were no formal or physical reasons that would force us to assume that the length of the crack jump approaches zero (A' or $\Delta a \to 0$) and that the crack growth is a continuum process.

If one introduces the elastic stress and displacement distributions into Eq (19), a well known relationship between the ERR and the SIF derived by Irwin [40] will be obtained. Irwin did not assume that $\Delta a \to 0$. His derivation required that the stress field is singular in front of the crack tip; in fact the singularity of the strain energy density should be as r^{-1}. It will be shown, in this section, that the non-singular terms, when included into the analysis provide an interesting information on fracture process, provided one assumes that crack extends in a jump-like way. In the analysis to follow the jump-like crack growth model will be used. However, we suggest that the length of the crack jump is not an arbitrary quantity it should represent an average distribution of the micro-cracks or voids nucleation kernels.

If Eq.(4) and the three-terms formula for displacement distribution along the crack faces [19] is introduced in Eq. (20), the formula to estimate the energy of fracture can be obtained in the form [39]:

$$\Gamma = \frac{1}{n+1}\alpha\varepsilon_0\sigma_0 L A_1^{n+1}\sum_{k=1}^{3}\sum_{j=1}^{3} \left\{A_2^{(j+k-2)}\left(\frac{\Delta a}{L}\right)^{\Delta s(j+k-2)}\beta_{ij}\sum_{i=1}^{t}\frac{f_i(m_j)}{(i-1)!(s_k+1)}\right\}, \tag{22}$$

where:

$$f_1(m_j) = 1; \quad f_2(m_j) = m_j; \quad f_3(m_j) = m_j(m_j-1); \quad f_4(m_j) = m_j(m_j-1)(m_j-2) \tag{23}$$

$$\Delta s = s_2 - s_1$$

$$m_j = m_1 + \Delta s(j-1)$$

$$m_1 = ns_1 + 1 = \frac{1}{1+n} = -s_1$$

$$\beta_{ij} = \tilde{\sigma}_{22}^{(i)}(\theta, n)\hat{u}_2^{(j)}(\theta, n)$$

t is an arbitrary number depending on the accuracy of the final result, one wish to receive. We suggest four terms.

One may notice that if $j = k = 1$ the fracture energy is independent of Δa:

$$\Gamma = \beta_{11}\frac{J}{I_n}\frac{1}{n+1}\left[\frac{1}{s_1+1} - \frac{m_1}{s_1+2} + \frac{m_1(m_1-1)}{2(s_1+3)} - \frac{m_1(m_1-1)(m_1-2)}{6(s_1+4)}\right]. \tag{24}$$

In Table 1. an example numerical results are shown concerning the amount of the fracture energy with respect to $J = J_{IC}$ when only the first term is used from the above series (22).

Column 9 in Table 1. shows what part of the J_{IC} value is a fracture energy. Higher the value n the less amount of energy is available for fracture energy. If fracture takes place the most of the dissipated energy is spent at plastic deformation. It follows directly from Eq. (20) that when $n \to \infty, \Gamma \to 0$. It is so called Rice's paradox. It tells us that for perfectly plastic materials the fracture process, which creates a new surface, is not possible. If the energy conservation law (18) is rewritten as below

$$\frac{\partial(W - U_e)}{\partial a} - \frac{\partial U_p}{\partial a} = 2\Gamma \tag{25}$$

the left hand side of Eq. (25) decides how much energy is available for fracture. In most metals and metallic alloys an excess of the strain energy is more readily dissipated at plastic deformation. Thus, not always the cleavage fracture, which is controlled by the stress field, may take place. Using Eq. (22) we will test how the in-plane constraint, expressed here by the parameter A_2, influences the amount of energy available for fracture at different n and the length of the crack jump Δa.

For the sake of the analysis the experimental data reported by Chao and Ji [41] has been utilized: $n = 5$, $L = 27{,}5$ mm, $A_2 = -0.6$ for $a/W = 0.7$ or $A_2 = -1.0$ for $a/W = 0.05$. The energy available for fracture was normalized by the ERR for linear material and $\Delta a = 0$.

TABLE

The values of all the quantities that are necessary to compute the fracture energy Γ using Eq.(24)

n	$\tilde{\sigma}_{22}^{(1)}(\theta=0,n)$	$\hat{u}_{22}^{(1)}(\theta=\pi,n)$	I_n	β_{11}/I_n	bracket [-]	1/(n+1)	product of column 5×6	Γ/J_{IC}
1	2	3	4	5	6	7	8	9
3	1.94	-2.7	5.507	-0,95	1.142	0.25	1.08	0.27
5	2.217	-2.368	5.023	-1.04	1.074	0.16	1.11	0.18
7	2.366	-2.18	4.766	-1.08	1.048	0.14	1.13	0.16
10	2.497	-2.015	4.539	-1.11	1.031	0.09	1.14	0.1
20	2.684	-1.75	4.215	-1.137	1.014	0.048	1.15	0.055

The analysis was also made for $n = 10$. Example of the results are shown in Figs. 3 and 4. Several observations can be made analyzing Figs. 3 and 4:

- One may notice that increase of n from 5 to 10 reduces considerably the amount of energy available for fracture.
- For longer potential jumps less energy is available to make such a jump.
- The shorter the initial crack length (A_2 is smaller) more energy is dissipated for plastic deformation and less energy is left for a new surface. Thus, if distribution of the kernels of the microcrack nucleation is sparse there may not be enough energy available to propagate the microcrack and jump-like cleavage fracture may not be observed.

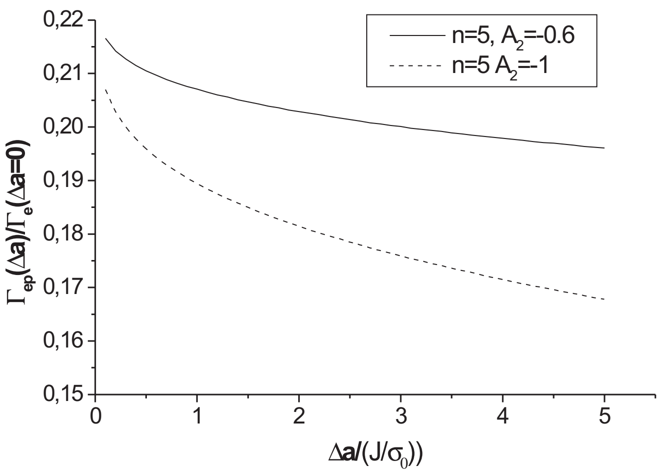

Fig. 3 The amount of energy available for fracture for $n = 5$

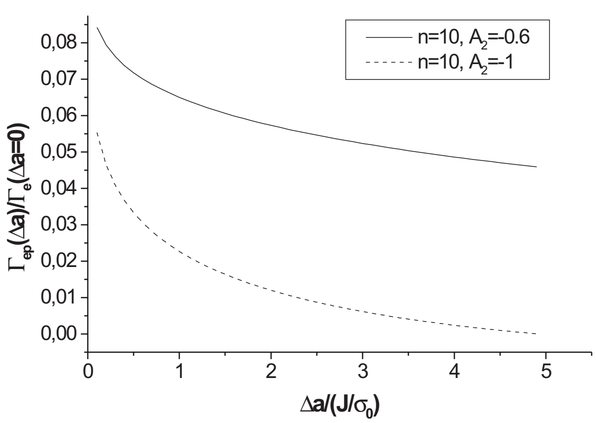

Fig. 4 The amount of energy available for fracture for $n = 10$

- One may also notice that for large n values the jump-like crack growth is not likely unless A_2 is close to zero or the length of the jump is short (dense distribution of nucleation kernels). Thus, the analysis for materials characterized by $n \geqslant 12$ is usually not possible or the cleavage can not take place.
- If one can estimate the length of the jump, e.g. from the micro structural studies and nucleation kernels distribution, for given value of Jc, the fracture energy can be roughly estimated from plots like those in Figs (3) and (4).

Eq. (22) contains three unknowns: J_{IC}, Γ and Δa. In [39] the two additional equations have been derived which allow to compute the two of the three quantities: $J_{IC}, \Delta a, \Gamma$, if one of them is known; e.g. if one can estimate Δa from the analysis of microstructure or J_{IC} from the experiment.

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