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MATHEMATICAL MODELLING OF IMPULSE COMPACTION PROCESS OF MOULDING SANDS – SIMULATION AND EXPERIMENTAL RESEARCH

MODELOWANIE MATEMATYCZNE PROCESU IMPULSOWEGO ZAGĘSZCZANIA MAS FORMIERSKICH – BADANIA SYMULACYJNE I EKSPERYMENTALNE

The paper presents a mathematical model of the impulse compaction process of moulding sands, based on a description of the impulse head dynamics and the model of deformation and compaction process of moulding sands. It was found that for a simulation examination of the developed model, a necessary and sufficient condition is knowing the coefficients $k_t(\delta)$ and $k_c(\delta)$ which characterise viscous and elastic properties of moulding sand. The relationships $k_t(\delta)$ and $k_c(\delta)$ can be determined on the ground of experimental measurements of speed V_L of ultrasonic wave propagation in the examined moulding sand as a function of its compaction degree δ.

Experimental examinations of the impulse compaction process were carried out on moulding sands with various contents of bentonite and water. Comparison of simulation and experimental results of impulse compacting was used for verification of the developed mathematical model.

The presented here simulation results of the mathematical model permit determining the influence of selected parameters of the impulse compaction process on its efficiency that can be evaluated on the ground of total pressures p_c and, what is more important, pressures p_u resulting from compacting the sandmix.

Analysis of the presented results of simulation and experimental research confirm that the formulated mathematical model describes very well the process of impulse compaction of moulding sands.

Keywords: moulding sand, impulse compaction, rheological model, mathematical model, simulation research, experimental research

Zaprezentowano model matematyczny procesu impulsowego zagęszczania mas formierskich, sformułowany w oparciu o opis dynamiki głowicy impulsowej oraz model procesu deformacji i zagęszczania mas formierskich. Stwierdzono, że do badań symulacyjnych opracowanego modelu matematycznego jest konieczna i wystarczająca znajomość współczynników $k_t(\delta)$ i $k_c(\delta)$, charakteryzujących własności lepkie i sprężyste masy formierskiej. Zależności $k_t(\delta)$ i $k_c(\delta)$ można określić na podstawie wyników badań eksperymentalnych pomiarów prędkości V_L rozchodzenia się fali ultradźwiękowej w badanej masie formierskiej w funkcji stopnia jej zagęszczenia δ.

Do badań eksperymentalnych procesu impulsowego zagęszczania stosowano masy formierskie o różnej zawartości procentowej bentonitu i wody. Porównanie wyników badań symulacyjnych i eksperymentalnych procesu impulsowego zagęszczania, posłużyło weryfikacji opracowanego modelu matematycznego.

Na podstawie przedstawionych w pracy wyników badań symulacyjnych opracowanego modelu matematycznego procesu impulsowego można określić wpływ wybranych parametrów procesu impulsowego zagęszczania na jego efektywność, którą ocenić można na podstawie zmian nacisków całkowitych p_c oraz, co ważniejsze, nacisków p_u będących wynikiem zagęszczania masy.

Analiza przedstawionych wyników badań symulacyjnych i eksperymentalnych potwierdziła, iż sformułowany model matematyczny bardzo dobrze opisuje proces impulsowego zagęszczania mas formierskich.

1. Introduction

The most beneficial effects of compacting conventional bentonite moulding sands are obtained by means of dynamic compaction methods, in particular of the impulse method that utilises effect of compressed air pressure wave on the moulding sand. The impulse compaction process is performed in a very short time of several to a dozen milliseconds which permits reaching high speeds of sandmix deformation and in consequence very good results of its compacting.

In practice, the basic technological parameters of the produced moulds are improving which results in a possibility of representing complex shapes of

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models, increased dimensional exactness and good quality of the mould surface.

Designing the impulse moulding machines and obtaining optimum results of the impulse compaction process requires knowing a mathematical model of the process and results of its simulation examinations. Modelling the impulse compaction process requires considering jointly the two mathematical models: of the impulse head as well as of the process of dynamic deformation and compaction of moulding sand.

2. Modelling the impulse compaction process of moulding sands

For many years, in the Laboratory of Basic Automation of the Institute of Production Engineering and Automation of Wroclaw University of Technology have been conducted works on mathematical modelling as well as simulation and experimental examinations of the dynamic compaction process of moulding sands. These works resulted in developing a mathematical model that completely describes the impulse compaction process of moulding sands [1].

The grounds for developing the model were:

- mathematical model of dynamics of the impulse head,
- mathematical model of the deformation process of moulding sand, based on rheological description of mechanical properties of disintegrated media.

2.1. Model of dynamics of impulse head

Layout of the impulse head used in experimental examinations of the impulse compaction process is shown in Fig. 1.

To create a mathematical model of dynamics of the impulse head used in experimental compacting, a modification of the simplified E.W. Gerc's model was applied [2, 3].

Dynamics of the impulse head was described using the following simplifying assumptions:

- air is a perfect gas,
- thermodynamic processes are of quasi-static nature,
- no heat exchange occurs between gas in the head chambers and the environment,
- frictional resistance in seals is negligibly small,
- air temperature in the head chambers is constant.

Considering the above simplifying assumptions, dynamics of the impulse head can be described by the following system of differential equations:

$$m_t \cdot \frac{d^2 y}{dt^2} = A_1 \cdot (p_0 - p_2) - c \cdot (y + y_0) - m_t \cdot g \quad (1)$$

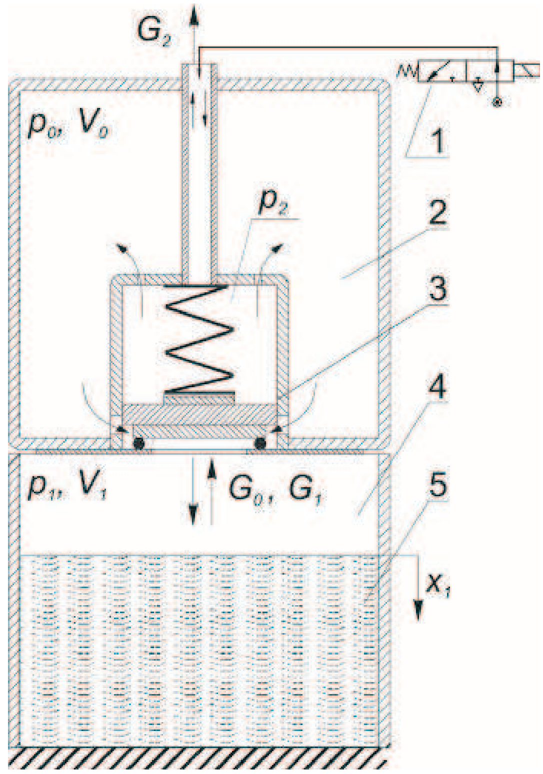

Fig. 1. Layout of the impulse compacting head, controlled by a self-acting impulse valve: 1 - distribution valve; 2 - accumulator tank; 3 - self-acting impulse valve;. 4 - technology space, 5 - moulding sand, p_i - absolute pressure in i$^{\text{th}}$ chamber, G_i - outflow rate from i$^{\text{th}}$ chamber, V_i - capa city i$^{\text{th}}$ chamber, x_i - coordinate of top layer of compacted sandmix column

$$\frac{dp_0}{dt} = \frac{-\kappa \cdot G_0 \cdot R \cdot T_0}{V_0} \quad (2)$$

$$\frac{dp_1}{dt} = \frac{\kappa}{(h + x_1)} \cdot \left(\frac{G_1 \cdot R \cdot T_1}{A_1} - p_1 \cdot \frac{dx_1}{dt} \right) \quad (3)$$

$$\frac{dp_2}{dt} = \frac{\kappa}{sz - y} \cdot \left(p_2 \cdot \frac{dy}{dt} - \frac{G_2 \cdot R \cdot T_2}{A_2} \right) \quad (4)$$

where:

m_t	–	mass of the impulse valve piston,
y	–	coordinate of the valve piston position,
A_1	–	cross-section area of the moulding box,
A_2	–	cross-section area of the impulse valve piston,
p_0, p_1, p_2	–	absolute pressure respectively in: accumulator tank chamber, working space above moulding sand and return chamber of the impulse valve,
c	–	spring constant,
y_0	–	preliminary (assembly) spring deflection,
g	–	acceleration of gravity,
κ	–	adiabatic exponent,

G_0, G_1, G_2	–	air outflow rate respectively from: accumulator tank chamber, working space above moulding sand and return chamber of the impulse valve,
R	–	gas constant of air,
T_i	–	air temperature in i-th chamber,
V_0	–	accumulator tank volume,
h	–	distance between the head and top layer of sandmix column,
x_1	–	coordinate of top layer of compacted sandmix column,
sz	–	impulse valve travel.

Air outflow rate G_i from the i-th chamber is described by the relationship:

$$G_i = K \cdot \alpha_i \cdot f_i \cdot p_i \cdot \sqrt{\frac{1}{R \cdot T_i}} \cdot \varphi(\varepsilon_i) \tag{5}$$

where:
α_i – air flow rate coefficient,
f_i – throttle flow area,

$$\varepsilon_i = \frac{p_a}{p_i}, \tag{6}$$

$$K = \sqrt{\frac{2 \cdot \kappa}{\kappa - 1}}, \tag{7}$$

p_i – pressure in i-th chamber,
$p_{a,1}$ – atmospheric pressure,

$$\varphi(\varepsilon_i) = \begin{cases} 0,2588 & \text{for } 0 < \varepsilon_i \leqslant 0.53 \\ \sqrt{\varepsilon_i^{2/\kappa} - \varepsilon_i^{\kappa+1/\kappa}} & \text{for } 0.53 < \varepsilon_i \leqslant 1 \end{cases} \tag{8}$$

Meaning of individual equations from (1) to (8) is as follows:
- equation (1) describes movement of the impulse valve piston,
- equations (2) to (4) present models of thermodynamic gas processes respectively in: accumulator tank chamber, working space above moulding sand and return chamber of the impulse valve,
- equations (5) to (8) describe relationships related to air flow from i-th chamber.

2.2. Rheological model of moulding sand

For indentifying rheological properties of moulding sands, the time method was applied that permits assessing the nature of properties of dynamic physical systems [4, 5]. Figure 2 shows a stepwise characteristics $h(t) = p_c(t)$ determined for a moulding sand with 5% of bentonite and humidity W=2.5%, demonstrating changes of total pressure $p_c = f(t)$ in the moulding sand caused by stepwise changing load [4].

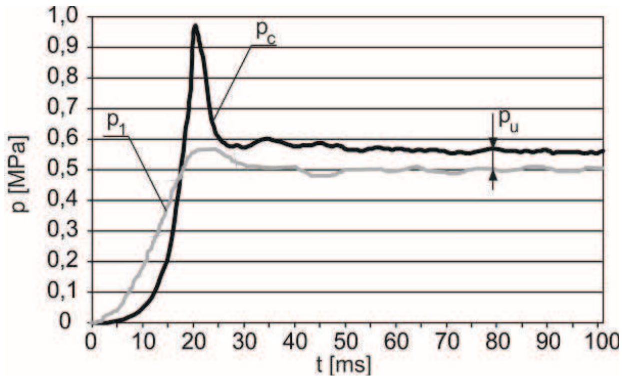

Fig. 2. Stepwise characteristics of moulding sand with 5% of bentonite and humidity W=2.5%

The obtained stepwise characteristics of the impulse compacted moulding sand permits the statement that, from the viewpoint of description of physical system dynamics, a moulding sand can be treated as an oscillating object [4, 5]. Therefore, rheological properties of a moulding sand can be described by the model being a series combination of viscoelastic models describing elementary layers of the compacted moulding sand, see Fig. 3.

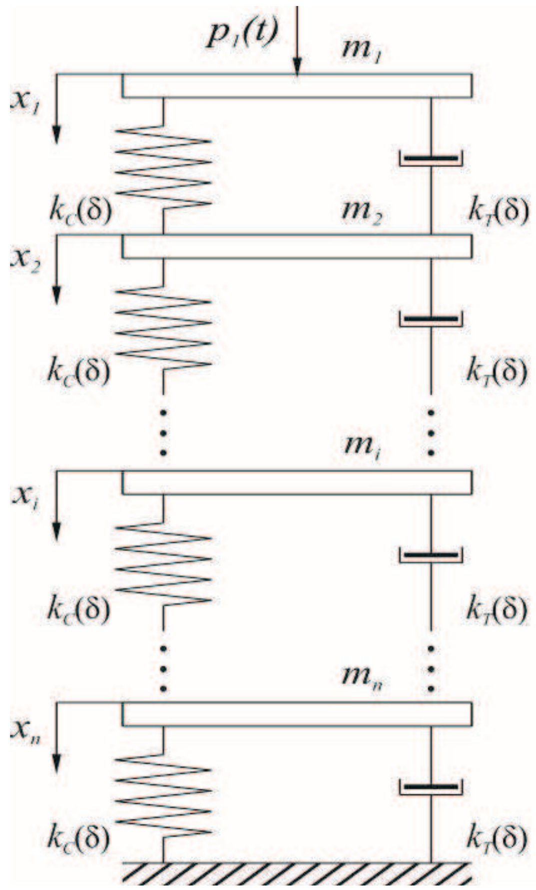

Fig. 3. Laminar rheological model of moulding sand

2.3. Mathematical model of impulse compaction of moulding sands

Considering the model of the impulse head dynamics and the rheological model of a moulding sand, the sandmix deformation process can be described by the following system of differential equations:

$$\left.\begin{array}{l} m_1 \cdot \ddot{x}_1 + k_T(\delta) \cdot (\dot{x}_1 - \dot{x}_2) + k_C(\delta) \cdot (x_1 - x_2) = p(t) \cdot A_1 + m_1 \cdot g \\ m_2 \cdot \ddot{x}_2 + k_T(\delta) \cdot (\dot{x}_2 - \dot{x}_1) + k_T(\delta) \cdot (\dot{x}_2 - \dot{x}_3) + k_C(\delta)(x_2 - x_1) + k_C(x_2 - x_3) = m_2 \cdot g \\ \vdots \\ m_i \cdot \ddot{x}_i + k_T(\delta) \cdot (\dot{x}_i - \dot{x}_{i-1}) + k_T(\delta) \cdot (\dot{x}_i - \dot{x}_{i+1}) + k_C(\delta) \cdot (x_i - x_{i-1}) + k_C(\delta) \cdot (x_i - x_{i+1}) = m_i \cdot g \\ \vdots \\ m_n \cdot \ddot{x}_n + k_T(\delta) \cdot (\dot{x}_n - x_{n-1}) + k_T \cdot \dot{x}_n + k_C(\delta) \cdot (x_n - x_{n-1}) + k_C(\delta) \cdot x_n = m_n \cdot g \end{array}\right\} \quad (9)$$

Analysis of the compaction process indicates that the pressures in individual phases of the process accept the following forms:

- In nonstationary states, pressures in moulding sands are equal to the sum of internal pressures connected with deformation and internal friction and those resulting from compacting the sandmix.
- In stationary states, pressures in moulding sands are equal to the sum of squeezing pressure and the pressure resulting from compacting the sandmix (pui), which expresses the obtained strength.

Therefore, the total pressure in the impulse compacted moulding sand can be described by the following relationship:

$$p_{Ci}(\delta) = \frac{k_C(\delta) \cdot [x_i(t) - x_{i+1}(t)] + k_T(\delta) \cdot [\dot{x}_i(t) - \dot{x}_{i+1}(t)]}{A} + p_{Ui}(\delta) \quad (10)$$

where: p_{ci} – total pressure in i-th layer of the moulding sand,
$k_c(\delta)$ – elasticity coefficient of the moulding sand,
$k_t(\delta)$ – viscosity coefficient of the moulding sand,
p_{ui} – pressure in i-th layer, resulting from compacting the moulding sand,
x_i – coordinate of i-th layer,
A_1 – cross-section area of the moulding box.

Meaning of individual equations (9) and (10) is as follows:

- The system of differential equations (9) describes the deformation process in 1-st, 2-nd, i-th and n-th layer of the moulding sand.
- The equation (10) describes the pressure in i-th layer of the moulding sand as a function of time.

3. Simulation and experimental research

The simulation and experimental examinations of the impulse compaction process of moulding sands, carried out in the Laboratory of Basic Automation, were first of all aimed at verification of the developed mathematical model.

The simulation examinations were performed in the Matlab-Simulink environment, and the experimental examinations were carried out on the stand shown in Fig. 4.

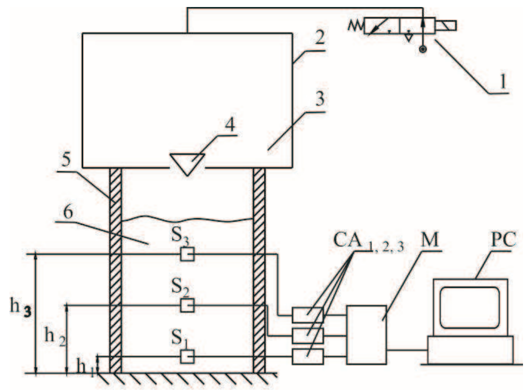

Fig. 4. Layout of test stand for examining impulse compaction of moulding sands and measuring circuit: 1 - distribution valve, 2 - impulse head, 3 - accumulator tank, 4 - impulse valve, 5 - moulding box, 6 - moulding sand, $S_{1,2,3}$ - piezoelectric sensors Kistler type 601A, $CA_{1,2,3}$ - charge amplifier Kistler type 5015A, M - data acquisition module Keithley KUSB 3100, PC - computer

The laboratory stand for examining the impulse compaction process of moulding sands consists of the impulse head (2) developed in the Laboratory of Basic Automation, equipped with a self-acting impulse valve (4), the moulding box (5) filled with moulding sand (6) and the measuring circuit for recording pressures in moulding sands. For measurements of dynamically changing pressures p_{ci} in the moulding sand, 3 piezoelectric sensors S_1, S_2, S_3 were used, located at three heights of the compacted sandmix column: h_1=50mm, h_2=100mm and h_3=150mm, as measured from the moulding board. The additional components of the measuring circuit are 3 charge amplifiers CA_1 to CA_3, data acquisition module M and a personal computer PC.

3.1. Determining the coefficients of rheological model

The ground for applying the presented mathematical model of the impulse compaction process in simulation research is knowledge of the coefficients $k_C(\delta)$ and $k_T(\delta)$ determining elastic and viscous properties of the moulding sand as a function of its compaction degree δ. The coefficients $k_C(\delta)$ and $k_T(\delta)$, determining rheological properties of moulding sands, can be determined on the ground of the experimental relationships [6, 7]:

$$k_T(\delta) = a_1 \cdot \exp[a_2 \cdot V_L(\delta)] \tag{11}$$

$$k_C(\delta) = b \cdot_1 \exp[b_2 \cdot V_L(\delta)] \tag{12}$$

where:
a_1, a_2, b_1, b_2 – factors,
$V_L(\delta)$ – speed of ultrasonic wave propagation in the examined moulding sand in function of the compaction degree.

The relationship $V_L(\delta)$, required for approximation of the coefficients $k_C(\delta)$ and $k_T(\delta)$, was determined by measuring propagation speed of longitudinal ultrasonic wave depending on the moulding sand compaction degree δ. This relationship can be approximated by an exponent function as follows:

$$V_L = c_1 \cdot \exp(c_2 \cdot \delta) \tag{13}$$

where:
c_1, c_2 – factors.

Figure 5 shows measurement results of the ultrasonic wave speed $V_L = f(\delta)$ as well as relationships for the elasticity $k_C = f(\delta)$ and viscosity $k_T = f(\delta)$ coefficients obtained by substituting the approximating function $V_L = f(\delta)$ to the equations (11) and (12). The examinations were carried out for the moulding sand with 5% of bentonite and humidity W=2.8%.

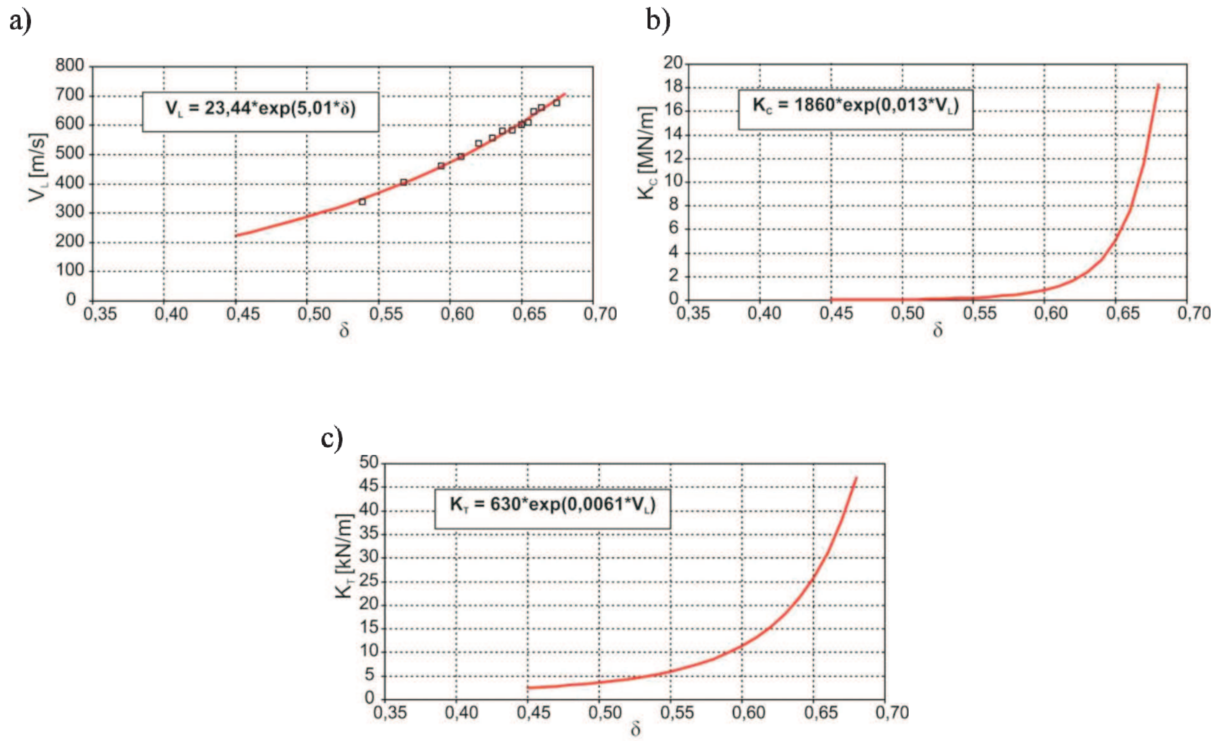

Fig. 5. Measurement results of ultrasonic wave speed $V_L = f(\delta)$ (a) as well as relationships $k_C = f(\delta)$ (b) and $k_T = f(\delta)$ (c) for moulding sand with 5% of bentonite and humidity W=2.8%

3.2. Results of simulation and experimental examinations

Simulation and experimental examinations of the impulse compaction process of moulding sands included measurements of total pressures p_{ci} in function of the process duration time. Analysis of the p_{ci} values in any layer of the moulding sand can be used for evaluating efficiency of the applied impulse process [8].

Figures 6 and 7 show experimental and simulation examination results of impulse compaction of moulding sands with 6% of bentonite and humidity 2.8% or 3.5%. The examinations were carried out at the initial pressure p_0=0.6MPa. The dynamic pressure gauges were placed at the heights h_1=50mm, h_2=100mm, h_3=150mm in a moulding box 200mm high (H_1=200mm).

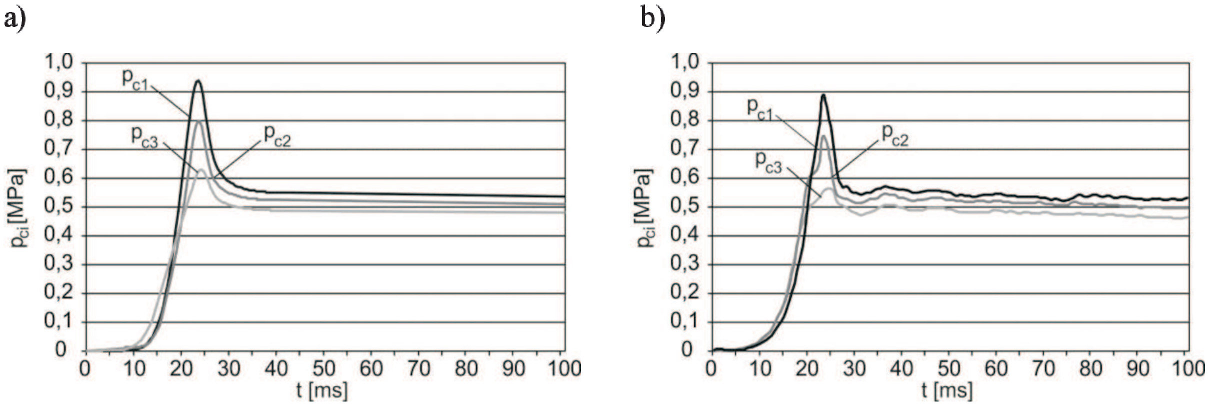

Fig. 6. . Changes of total pressure p_{ci}: simulation results (a) and experimental results (b) for the moulding sand with 6% of bentonite and humidity W=2.8%

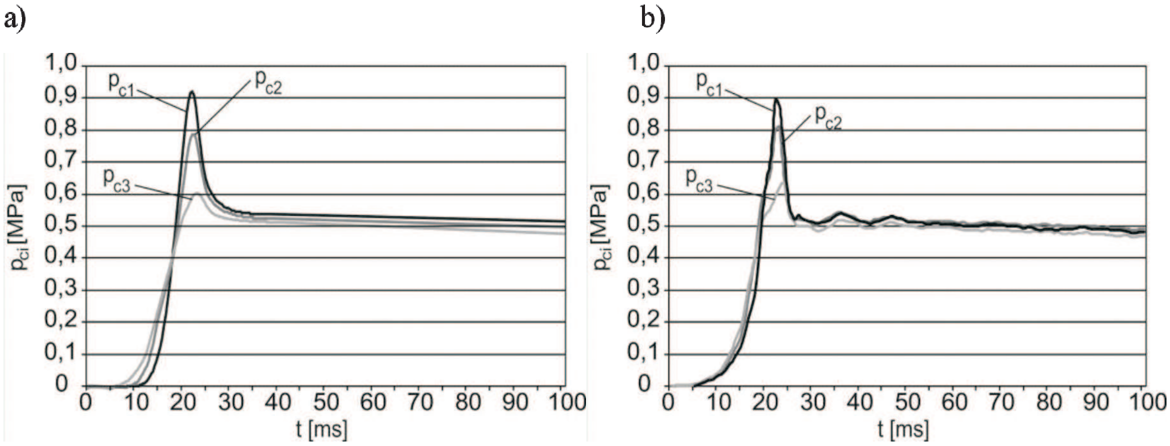

Fig. 7. Changes of total pressure p_{ci}: simulation results (a) and experimental results (b) for the moulding sand with 6% of bentonite and humidity W=3.5%

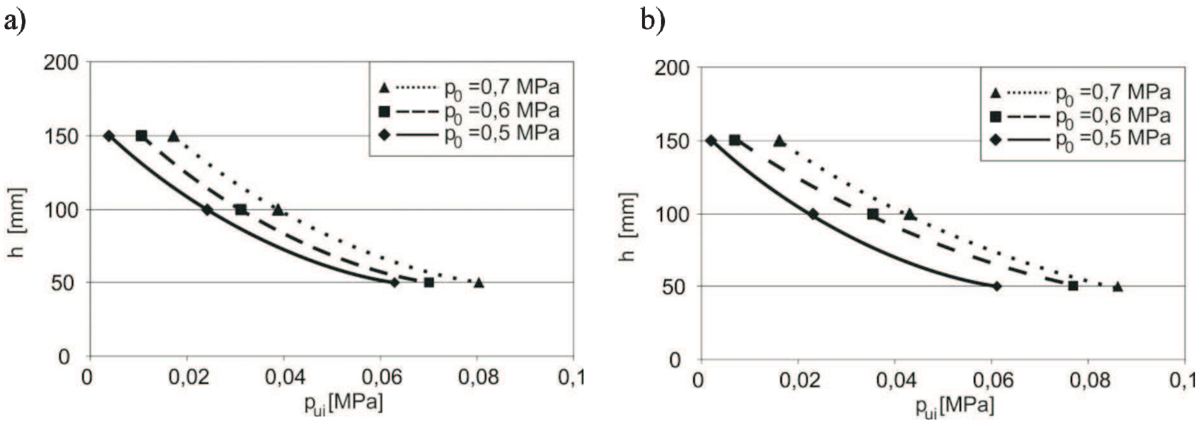

Fig. 8. Changes of pressure p_{ui} in function of sandmix column height for initial compressed air pressures p_0 = 0.5; 0.6; 0.7 MPa: simulation results (a) and experimental results (b)

Figure 8 shows changes of pressure pui resulting from compacting the moulding sand in function of the compacted sandmix layer. The examinations were performed for the moulding sand with 5% of bentonite and humidity 2.5% for three values of initial pressure p_0 in the accumulator tank. The moulding sand was placed in a moulding box 200mm high.

Figures 9 and 10 show changes of total pressures p_{ci} and of pressures p_{ui} in function of height of the sandmix column placed in a moulding box 350mm high (H_2=350mm). The experimental and simulation examinations were performed for the moulding sand with 5% of bentonite and humidity 2.5% for initial pressure p_0=0.6MPa.

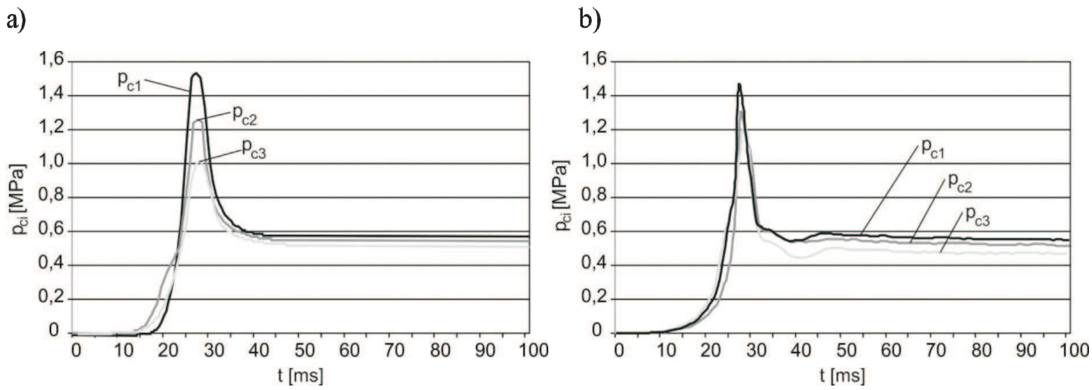

Fig. 9. Changes of total pressure p_{ci}: simulation results (a) and experimental results (b) for initial compressed air pressure p_0=0.6MPa and height of the moulding box H_2=350mm

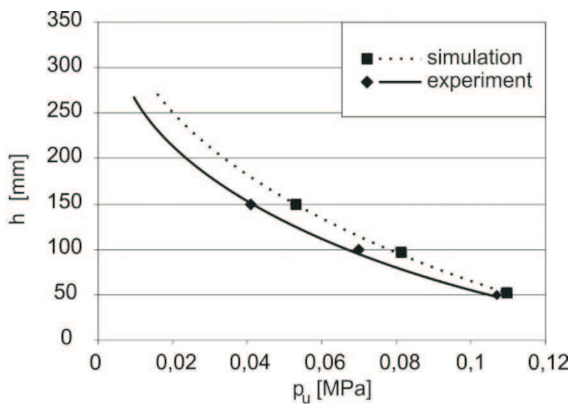

Fig. 10. Changes of pressure p_{ui} in function of height of the sandmix column for initial compressed air pressure p_0=0.6MPa and height of the moulding box H_2=350mm: simulation results and experimental results

It can be found on the grounds of the presented simulation and experimental results that the formulated mathematical model very well describes the impulse compaction process.

4. Conclusion

Obtaining optimum results of impulse compaction of moulding sands is possible thanks to proper selection of designs and parameters of the impulse moulding machines, which can be aided by knowing the course of the impulse compaction process. In the paper, a mathematical model of impulse compaction of moulding sands is presented, that permits determining changes of total pressures p_{ci} in any volume of the mould. The base for developing the model is a mathematical model of the impulse head and a model of the sandmix deformation process, formulated on the ground of the presented rheological model. To carry out simulation examinations of the mathematical model, it is only required to know the coefficients determining viscous and elastic properties of the moulding sand. Analysis of the simulation and experimental results justifies the statement that the presented mathematical model very well describes the real course of the impulse compaction process of moulding sands and can be applied for designing and optimising the impulse compaction process.

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