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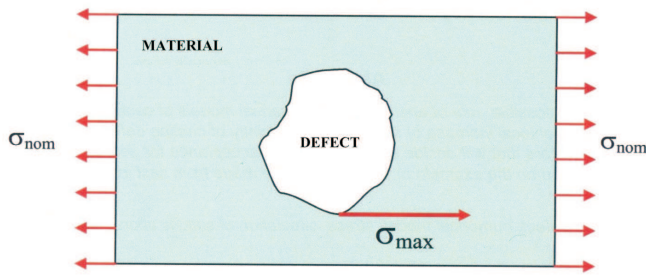


Fig. 1. A void-type defect in band stretched with nominal stress  $\sigma_{nom}$

The model presented here, satisfying the Huber-Mises-Hencky (HMH) yield criterion, is useful when a concentrated defect of irregular shape occurs, e.g. an internal or external shrinkage cavity. In the case of porosity it usually gives overstated values of the stress  $\sigma_{max}$  and if this is the case, then the model described below is recommended for use.

## 2. Numerical model of porosity-type defect in material satisfying the Garson-Tvergaard criterion surrounded by material satisfying the HMH yield criterion.

In this case, the defect has also a distinct contour but its interior is filled with porous material of a modulus of elasticity lower than the modulus of elasticity of a homogeneous material (Fig. 2).

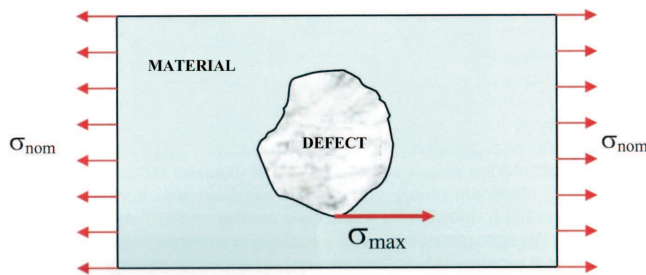


Fig. 2. A porosity-type defect in band stretched with nominal stress  $\sigma_{nom}$

In this case, the Gurson-Tvergaard yield criterion applies [1], [2]. It is available in some FEM programmes, but for computations it is necessary to give the value of percent share of porosity inside the selected area. The degree of porosity can be determined by simulation of the casting solidification process, using e.g. *Magma* programme.

## 3. Analytical model of ellipsoidal void-type defect in linear-elastic material.

In this model, an arbitrary defect shape is reduced to the form of ellipse which envelopes the defect (Fig. 3).

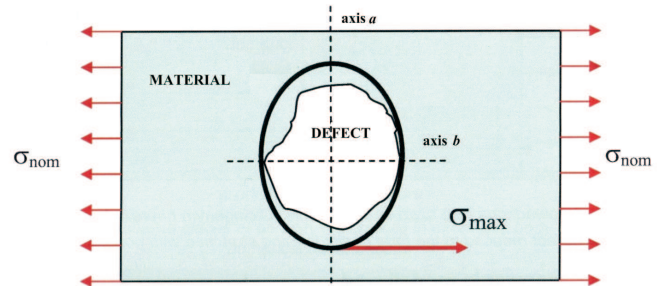


Fig. 3. Defect reduced to the form of ellipse in band stretched with nominal stress  $\sigma_{nom}$

The value of maximum stress  $\sigma_{max}$  is calculated from the well-known formula:

$$\sigma_{max} = \sigma_{nom} \left( 1 + 2 \frac{a}{b} \right) \quad (1)$$

## 4. Analytical model of slot-type defect in linear-elastic or elastic-plastic material.

Also in this case, the function of the defect can be described with the equations of fracture mechanics. Depending on the cast alloy type, a linear-elastic or elastic-plastic model of material is adopted, applying as a criterion the critical stress intensity factor  $K_{IC}$  or J integral. The additional difficulty in the case of cast iron is determination of a real value of the nominal stress, as it is necessary to introduce a correction coefficient allowing for the graphite type (respective guidelines can be found in available reference literature). The defect is seen as a slot with dimensions of the longest diagonal drawn perpendicular to the direction of the nominal stress  $\sigma_{nom}$  (Fig. 4), while the stress intensity factor  $K_I$  is calculated from the following equation [5]:

$$K_I = \sigma_{nom} \sqrt{\pi \cdot a} \cdot F \quad (2)$$

where: F – the dimensionless coefficient depending on a band – slot system geometry, a – length of slot.

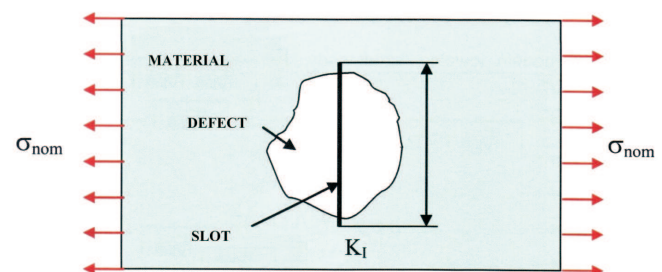


Fig. 4. Defect reduced to the form of slot in band stretched with nominal stress  $\sigma_{nom}$

## 3. Example of practical application of the model of defects

As an example of the practical application of the above described models, the body of a pressure valve cast from EN GJS 400-15 iron was used (Fig. 5);

the casting was designed to withstand the pressure of 50 atm acting on its walls.

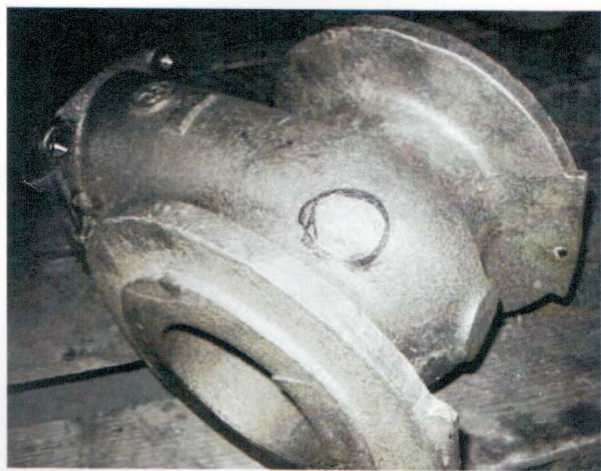


Fig. 5. Cast valve body

The analysis required making a 3D valve drawing with the gating system and risers. As a result of simulation of the casting pouring and solidification process it was possible to predict the places where the void-type defects could occur along with their shape, dimensions and percent content (Fig. 6). Among many places which were indicated by the programme as areas threatened with the occurrence of various forms of porosity, for further analysis

were selected those areas where the stress caused by performance loads assumed the highest value. This operation required construction of a numerical model of the valve body adapted to calculations made by FEM.



Fig. 6. One of the possible places of porosity occurrence in cast valve body determined by numerical computations

Based on the analysis of the state of stress, the stress components were determined and examined to eliminate those which, because of low value or specific orientation in respect of the defect location, had no significant effect on the generation of stress accumulation phenomena.

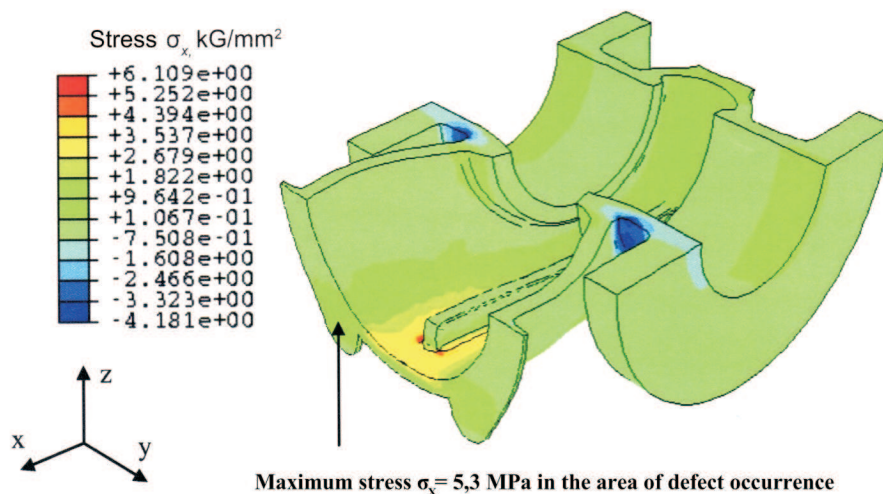


Fig. 7. Values of the stress state component  $\sigma_{nom}(x)$  determining stress increase in the vicinity of defect



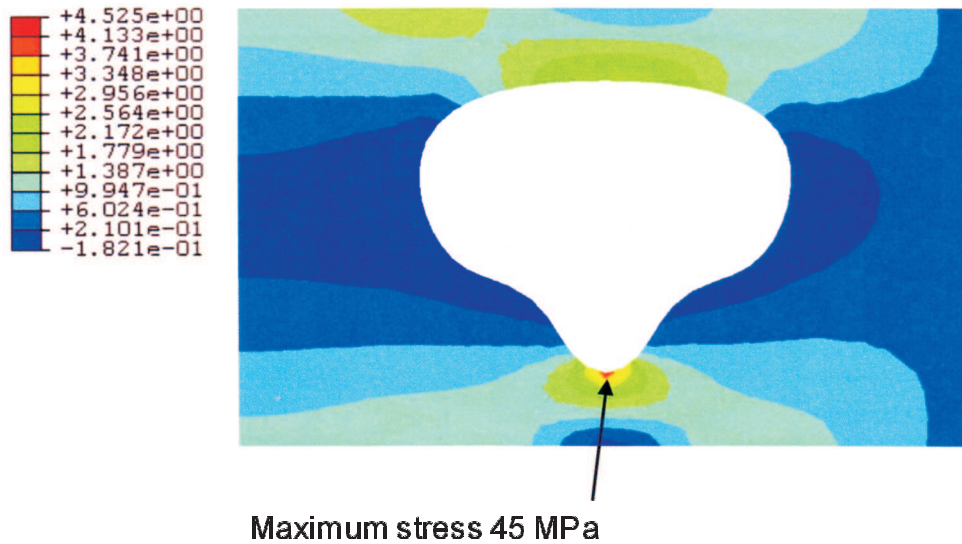


Fig. 8. A void-type defect in band stretched with nominal stress  $\sigma_{nom}(x) = 5.3$  MPa

Combining the stress value with the defect geometry and location (Fig. 8) and further with the percent content of porosity according to the above described four models has allowed the determination of maximum stress  $\sigma_{max}$  and stress intensity factor  $K_I$  in the crack tip (Table 1).

TABLE 1

The computed stresses  $\sigma_{nom}$ ,  $\sigma_{max}$  and stress intensity factors compared with material constants for the cast iron of EN GJS 400 – 15 grade

Model	Nominal stress $\sigma_{nom}$	Maximum stress $\sigma_{max}$	Stress Intensity Factor	$R_{0.2}$	$K_{IC}$
	MPa	MPa	MPa·m <sup>0.5</sup>	MPa	MPa·m <sup>0.5</sup>
1	5.3	45.2		240	45
2		35.1			
3		15.9			
4		2.5			

#### 4. Fatigue model

In a fatigue model, the shape of defect is reduced to a slot of certain length positioned perpendicular to the direction of maximum load, as shown in Figure 4.

The rate of fatigue crack propagation is defined as an increase in crack length falling to one cycle [3]:

$$\frac{dl}{dN} = f(\Delta K, R) \quad (3)$$

where:

$l$  – the length of slot,

$N$  – the number of cycles,

$R$  – the stress ratio,

$\Delta K$  – the variability range of stress intensity factor expressed with the following formula:

$$\Delta K = \beta \Delta \sigma \sqrt{\pi l} \quad (4)$$

$\beta$  – the factor taking into account the finite dimensions of a body,

$\Delta \sigma$  – the range of stress variability.

Function  $f$  and constants  $m$  and  $C$  are determined experimentally (no analytical solution has been developed so far). For the second, linear phase of the crack growth, the function can be written as:

$$\log\left(\frac{dl}{dN}\right) = m \log(\Delta K) + \log(C) \quad (5)$$

Allowing for the properties of a logarithmic function, the Paris equation is obtained:

$$\frac{dl}{dN} = C (\Delta K)^m \quad (6)$$

To determine the number of cycles  $N_f$  followed by failure of the structural element, it is necessary to make appropriate transformations and integrate the above equation, allowing additionally for a formula expressing  $\Delta K$ :

$$N_f = \frac{1}{C} \int_{l_0}^{l_k} \frac{dl}{(\beta \Delta \sigma \sqrt{\pi l})^m} \quad (7)$$

where  $l_0$  is the initial length of crack, and  $l_k$  is the final length associated with the critical length  $l_{kr}$ , determined from the selected criteria of fracture mechanics. The solution of the above equation requires numerical integration.

## 5. Summary

The defect in the form of porosity or internal cavity need not be always the reason for casting rejection on account of poor quality. The results of calculations carried out on four mathematical models have proved that the defect-induced increase in stress and stress intensity factor has the values much lower than its critical counterparts.

The proposed models cover, in principle, all possible variations of this type of material discontinuity present in the casting walls. The selection of a best model depends on the shape of the defect and stress direction. Defects of an elongated shape should rather be examined as slots using the methods of fracture mechanics. If the shape is irregular, better accuracy is obtained on numerical models which exactly reflect the contour of the discontinuity, giving better accuracy of calculations. In this case, high grid density on the edge of the defect and isoparametric non-linear elements of reduced integration should be applied. It is also necessary to determine the type of defect and, if it is porosity, use the Gurson-Tvergaard yield criterion. If the shape is regular and the cavity is of compact nature, a model can be applied where it is assumed that the defect has an outline of the ellipse. The study of fatigue life requires experimental determination of the defect growth rate (Paris test) to determine the necessary constants or accept data stated in the

literature, e.g.  $C = 2.2 \cdot 10^{-10}$ ,  $m = 4.5$  [4], adopting also an interval for the load changes. In any case, the starting point for analysis is the determination by numerical methods (FEM) of the stress tensor components in casting without any defects exposed to the effect of performance loads. The location, shape and dimensions of the defect can be determined experimentally or predicted from appropriate algorithms adapted to the computer programmes used currently for the simulation of casting solidification process.

## REFERENCES

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