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DYNAMIC PROBLEMS INVOLVED IN OPERATION OF SELECTED ELEMENTS OF CRANE INSTALLATIONS USED IN THE FOUNDRY ENGINEERING

PROBLEMY DYNAMICZNE W EKSPLOATACJI WYBRANYCH ELEMENTÓW SUWNIC PRACUJĄCYCH W PRZEMYŚLE ODLEWNICZYM

In his paper is pre presented an effort of realistic assessment of loading of some element of a traveling crane. Dynamic analysis of operation of a system – crane beam – crane – crab has been made for possible phases of a crane operation. This made possible for analytic formulae presentation which determine real loads of particular elements of a considered system. Obtained relations may be useful for dimensioning and exploitation safety assessment of such structures.
Keywords: cranes, dynamic, loads

W artykule podjęto próbę oceny rzeczywistych wartości obciążeń wybranych elementów suwnicy. Przeprowadzono analizę dynamiczną pracy układu belka podsuwnicowa – suwnica – wózek dla możliwych faz pracy suwnicy, która pozwoliła na podanie formuł analitycznych z których możliwe było określenie rzeczywistych obciążeń poszczególnych elementów rozważanego układu. Uzyskane zależności mogą być przydatne do oceny stanów nośności oraz użytkowalności elementów konstrukcyjnych suwnic.

1. Introduction

Strength calculations of crane components (particularly the crane beams) are done in accordance with the principles set forth in the technical standards PN-90/B-03200, taking into account the aspect of fatigue endurance. In order to determine the stress level in the context of fatigue endurance, the characteristic load values have to be considered (with no loading factors) while the dynamic factors have to be taken into account. The method of finding the dynamic factor in various stages of the crane's duty cycle is outlined below, based on extensive theoretical and experimental data gathered by the research teams from AGH-UST in Cracow. [1] [5]

2. Loading of riding wheels in a travelling trolley during the hoisting stage

The dynamic analysis is carried out for the crane shown schematically in Fig. 1, during the weight hoisting (unsteady state conditions). The key elements of the physical system include a steel structure of the crane bridge 1; a traversing trolley supplied with the hoisting and lifting mechanisms 2; a grab with the weight being hoisted 3; branches of load-bearing ropes arranged in parallel 4.

The system shown in Fig. 1 is replaced by the mechanical model shown in Fig. 2.

The model in Fig 2 involves several simplifications:

– rotating masses of the drives in the hoist mechanism are reduced onto the shaft of the drum in this mechanism and treated as a single lump mass;
– the mass of the steel structure of the crane bridge is reduced to the point of trolley location, and treated jointly with the trolley mass;
– vibration damping and wave phenomena accompanying the strain propagation along the ropes are neglected;
– deformations along the ropes and in tooth gears in the mechanisms are neglected; Stiffness of the steel structure of the crane bridge at the point where the trolley is located is represented by a helical spring with the precisely controlled elasticity coefficient.

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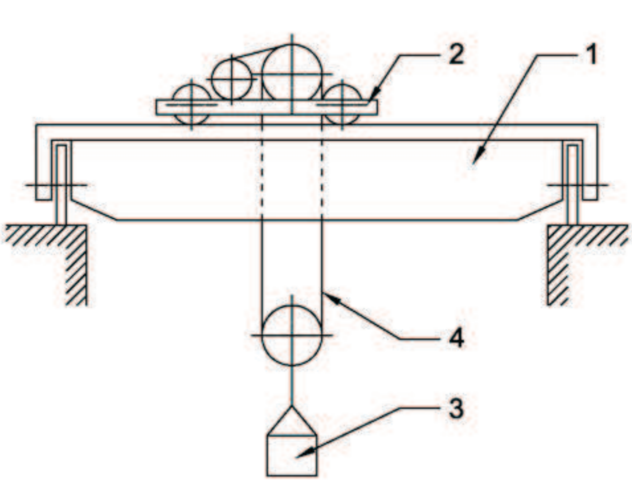

Fig. 1. Schematic diagram of the crane; 1 – steel structure of the crane bridge, 2 – traversing trolley supplied with the hoistingand lifting mechanisms, 3 – grabwith the weight being hoisted, 4 – branches of load-bearing ropes arranged in parallel

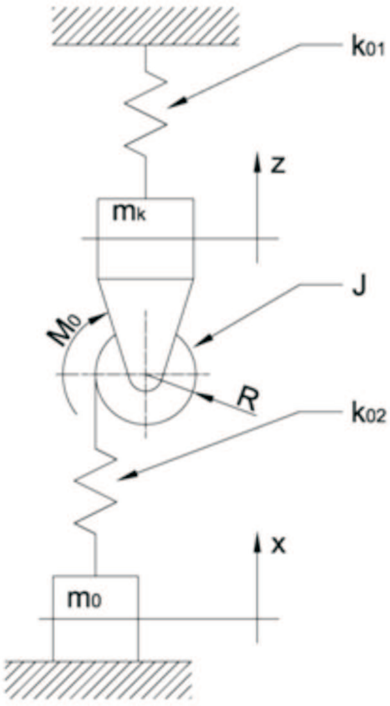

Fig. 2. Mechanical model of the crane: k_{01} – stiffness of load-bearing ropes; k_{02} – stiffness of steel structure of the bridge; m_k – mass of the bridge; m_0 – hoisted mass; M_0 – motor torque; J – reduced rotating masses; R – drum radius

Equations of motion for the vibrating system are written, based on the Lagrange's equations of the second type [3] [5]:

$$\begin{aligned} &J\ddot{\varphi} + \tfrac{1}{i_w^2} \cdot k_{02}\left[R \cdot \varphi + i_w z - i_w x\right] = M_0, \\ &m_0 \ddot{x} - \tfrac{1}{i_w} k_{02}\left[R \cdot \varphi + i_w z - i_w x\right] + m_0 g = 0, \\ &m_k \ddot{z} + \tfrac{1}{i_w} k_{02}\left[R\varphi + i_w z - i_w x\right] + k_{01} = 0. \end{aligned} \qquad (1)$$

where:

m_0 – mass of the hoisted weight; x – coordinate describing the motion of the mass m_0; m_k – mass of the steel structure of the crane bridge and the trolley reduced to the point of trolley position; z – coordinate describing the motion of the mass m_k, J – inertia moment of the rotor in the driving motor and mobile elements of the driving system in between the motor and the drum reduced onto the shaft of the hoist drum; φ – angular coordinate governing the rotating motion with the moment J; k_{02} – rope stiffness factor; k_{02} – stiffness factor of the steel bridge structure at the point of trolley position, M_0 – torque delivered by the motor in the hoisting mechanism, reduced to the drum shaft; R – drum radius; i_w – transmission ratio of a pulley block; g – acceleration of gravity.

The equation governing the torque M_0 delivered on the shaft in a asynchronous electric motor is assumed to be the linear function of angular velocity of the motor shaft $\overline{M_0} = \overline{M_1} - \overline{\alpha} \cdot \overline{\omega}$; where: $\overline{M_1}, \overline{a}$ – constants; $\overline{\omega}$ – instantaneous angular velocity of the shaft. This assumption is valid for the steady – state range of nn asynchronous motor operation. Assuming the transmission ratio between the drum and the motor to be i_m and mechanical efficiency η_m, we obtain the torque reduced to the drum shaft:

$$M_0 = M_1 - \alpha \dot{\varphi},$$

where:

$$M_1 = \overline{M_1} i_{lm} \eta_m; a = \overline{a} i_m^2 \cdot \eta_m; i_m = \frac{\overline{\omega}}{\dot{\varphi}}$$

$\dot{\varphi}$ – angular velocity of the drum

The angular coordinate φ is replaced in Eq. (1) with a linear coordinate related to φ by the formula:

y= φR

After these transformations, the system of equations (1) is rewritten as:

$$\begin{aligned} &\tfrac{J}{R}\ddot{y} + \tfrac{1}{i_w^2} R k_{02}\left(y + i_w z - i_w x\right) = R_1 - \tfrac{a}{R}\dot{y} \\ &m_0 \ddot{x} - \tfrac{1}{i_w} k_{02}\left(y + i_w z - i_w x\right) + m_0 g = 0 \\ &m_k \ddot{z} + \tfrac{1}{i_w} k_{02}\left(y + i_w z - i_w x\right) + k_{01} z = 0. \end{aligned} \qquad (2)$$

The motion of the mechanical system is analysed from the moment when the tension in ropes becomes associated exclusively with the weight of the sheave block and the ropes. The coordinate x is calculated from the base level on which the weight rests before it is lifted; the coordinate z is associated with the position of the steel structure of the bridge and the trolley, after bending induced by their own weight. In the initial state of motion, when the ropes become tense, the coordinates y and z change, while the coordinate x remains unchanged. That is why the second equation in the system (2) is no longer applicable at this stage and the system of equations (2) is reduced to two equations in the form:

$$\begin{aligned} &\tfrac{J}{R}\ddot{y} + \tfrac{1}{i_w^2} R k_{02}\left(y + i_w z\right) = M_1 - \tfrac{a}{R}\dot{y}, \\ &m_k \ddot{z} + \tfrac{1}{i_w^2} k_{02}\left(y + i_w z\right) + k_{01} z = 0. \end{aligned} \qquad (3)$$

For the initial conditions:

$$y_{(0)} = 0, z_{(0)} = 0, \dot{y}_{(0)} = \dot{y}_{01}, \dot{z}_{(0)} = 0.$$

At the moment the tension in ropes reaches $m_0 g$ (leaving aside the weight of the sheave pulley and of the ropes), the weight being hoisted begins to move upward, the coordinate x begins to change and the motion of the mechanical system will be governed by the system of three equations (3), for the initial conditions: $y_{(0)} = y_0, x_{(0)} = 0, z_{(0)} = z_0$, $\dot{y}_{(0)} = \dot{y}_{02}, \dot{x}_{(0)} = 0, \dot{z}_{(0)} = \dot{z}_0$.
The system of equations (3) is solved using the Laplace's transformation. Here we show the solution for $z(t)$.

$$z_{(t)} = n_0 + \sum_{i=i}^{0} n_i e^{q_i t} \qquad (4)$$

where:

$q_{i(1,2,3,4)}$ – are the roots of the equation

$$q^4 + \tfrac{a}{J} q^3 + \left(\tfrac{k_{01}+k_{02}}{m_k} + \tfrac{R^2 k_{02}}{J \cdot i_w^2}\right) q^2 + \tfrac{a}{J}\tfrac{k_{01}+k_{02}}{m_k} q + \tfrac{R^2 k_{02}}{J i_w^2} \cdot \tfrac{k_{01}}{m_k} = 0$$

$$n_i = \frac{-\frac{M_1 R k_{02}}{J m_k i_w}\left(q_i - q_i\right)}{q_i\left(q_i - q_1\right)\left(q_i - q_2\right)\left(q_i - q_3\right)\left(q_i - q_4\right)}, (i = 1, 2, 3, 4).$$

$$n_0 = \frac{M_1 i_w}{R\left(k_{01} + k_{02}\right)}$$

The dynamic force acting upon the structure at that stage is obtained from the formula:

$$P_{d(t)} = k_{01} z_t + Q_w + \frac{1}{2} Q_M \qquad (5)$$

where:

Q_w – weight of the trolley; Q_M – weight of the bridge

$$k_{01} = \frac{2 k_m k_c}{k_m + 2 k_c},$$

k_m – elasticity coefficient of the bridge at the trolley location; k_c – elasticity coefficient of the crane beam.

The dynamic overloading factor δ is obtained from the formula:

$$\delta = \frac{p_{d_{\max}}}{Q_w + \frac{1}{2} Q_M} \qquad (6)$$

For simplicity, small terms in Eq (4) are neglected, assuming that $k_{01} + k_{02} \approx k_{01}$, which is derived after relevant transformations. Finally, we get:

$$z_{(t)} = \frac{-M_1 i_w}{R\left(k_{01} + k_{02}\right)} \left\{ 1 + \sqrt{\frac{W^2 + \left(\frac{R k_{02}}{M_1 i_w^2} \omega_0 y_{01}\right)}{\left(W - \omega_0^2\right)^2 + \left(\omega_0 \frac{a}{J}\right)}} \sin \omega_o t \right\} \qquad (7)$$

where:

$$\omega_0 = \sqrt{\frac{k_{01} + k_{02}}{m_k}}; W = \frac{R^2 k_{02}}{J i_w^2}$$

The maximal dynamic loading of the steel bridge at the time the weight begins to be hoisted is equal to:

$$P_{d_M} = k_{01} \cdot z_{\max} = \frac{M_1 i_w k_{02}}{R\left(k_{01} + k_{02}\right)}$$

$$\left\{ 1 + \frac{R k_{02}}{i_w} \sqrt{\frac{\frac{R^2}{J} + \frac{k_{01} k_{02}}{m_k}\left(\frac{y_{01}}{M_1}\right)^2}{\left(\frac{R^2 k_{02}}{J i_w^2} - \frac{k_{01}+k_{02}}{m_k}\right)^2 + \frac{k_{01}+k_{02}}{m_k}\left(\frac{a}{J}\right)^2}} \right\} \qquad (8)$$

3. Determination of the dynamic overload factor for the unsteady ride of the trolley

A vibrating mechanical system of the crane during the simultaneous operation of the hoisting and riding mechanism comprises a steel bridge on which the trolley will travel, equipped with a grab unit and the handled material. The mechanical model of this system is shown in Fig. 3 [4] [5]. Underlying the equation of motion of the elements in the model are the following simplifying assumptions:

- the metal structure of the trolley is ideally rigid;
- deformations of the hoisting mechanism and the trolley- ride mechanism are negligibly small in relation to those of the crane beam and of the bridge;
- wheel slippage during the start-up and the braking phase is neglected;
- the weight lifting (or falling) rate is constant;
- the trolley accelerates (and brakes) with the same force P_0;
- the angle α of the weight deviation from the vertical is small;

The dynamic analysis uses the following designations:

Q_1 – weight of the trolley with the mechanisms; Q_2 – weight of the grab unit with the material handled; P_0 – acceleration (or braking) force applied to the wheel rim; $h_{(t)}$ – the height of the weight suspension at the instant t; h_1 – the distance between the trolley's cog (centre of gravity) and the centre of rotation; h_2 – distance between the attachment point of the weight Q_2 from the centre of the trolley's rotation (distance between the axles in the trolley); k – elasticity coefficient of the bridge and the crane beam for the least favourable settings of the mechanisms; φ – rotation angle of the trolley; J_0 – inertia moment of the trolley with respect to the axis passing through the point O_1 and perpendicular to the plane x, y; α – deflection from the vertical of a rope to which the weight is attached, x – displacement of the trolley; V_0 – hoisting velocity.

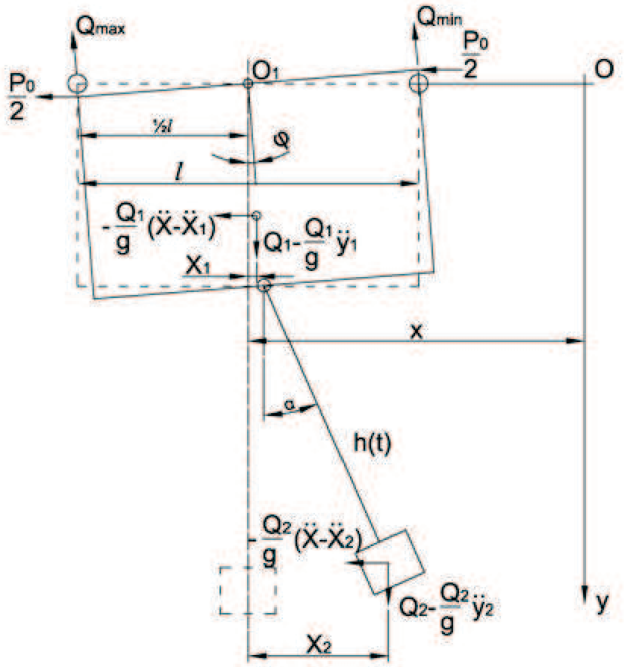

Fig. 3. Model of the crane system- the weight being hoisted and unsteady ride of the trolley

Applying the d''Alambert's rule, the equation of motion is formulated for the model shown in Fig. 3, in the coordinate system x, y.

$$-\frac{Q_2}{g}\left(\ddot{x}-\ddot{x}_2\right)h_{(t)}\cdot\cos\alpha+\left(Q_2-\frac{Q_2}{g}\ddot{y}\right)\cdot h_{(t)}\cdot\sin\alpha=0$$

$$J_0\ddot{\varphi}-\frac{Q_1}{g}\left(\ddot{x}-\ddot{x}_1\right)y_1+\left(Q_1-\frac{Q_1}{g}\ddot{y}_1\right)x_1-\frac{Q_2}{g}\left(\ddot{x}-\ddot{x}_1\right)y_2+$$
$$+\left(Q_2-\frac{Q_2}{g}\ddot{y}_2\right)x_2+\frac{k\cdot l^2}{2}\varphi=0 \tag{9}$$

$$-\frac{Q_1}{g}\left(\ddot{x}-\ddot{x}_1\right)-\frac{Q_2}{g}\left(\ddot{x}-\ddot{x}_2\right)+P_0=0.$$

For small angles α and for small φ, we write:

$x_1 \cong h_1\cdot\varphi;\ x_2 \cong h_2\cdot\varphi+h_{(t)}\cdot\alpha;\ y_1 \cong h_1;\ y_2 = h_{(t)}+h_2;$
$h_{(t)} = h_0 \pm V_0\cdot t;\ \cos\alpha \cong 1;\ \sin\alpha \cong \alpha$

To exemplify things, the solution is given which yields the maximal angle of the trolley's rotation:

$$\varphi_{(l)} = W(1-\cos pt)+S\left[\cos pt-\left(1\pm\frac{3}{4}\frac{V_0}{h_0}t\right)\cos\gamma t\right] \tag{10}$$

where:

$$W=\frac{P_0\left[h_1+\frac{Q_2}{Q_1+Q_2}(h_2-h_1)\right]}{Q_1\cdot h_1+Q\cdot h_2+\frac{kl^2}{2}},\ p^2=\frac{Q_1\cdot h_1+Q_2\cdot h_2+\frac{kl^2}{2}}{J+\frac{Q_1}{g}\cdot h_1^2},$$

$$S=\frac{\frac{Q_2\cdot P_0(h_2-h_1)}{Q_1+Q_2}}{\left(1-\frac{\gamma^2}{p^2}\right)[Q_1\cdot h_1+Q_2\cdot h_2]+\frac{kl^2}{2}},\ \gamma^2=\frac{g\cdot\varepsilon}{h_0}.$$

In order to find the maximal pressure of the trolley wheels onto the rails, it is required that the maximal angle of the trolley's rotation $\varphi_{\max}$ should be determined. Maximal pressure on the pair of riding wheels is derived from the formula:

$$Q_{\max} = Q_s+\varphi_{\max}\cdot\frac{kl}{2} \tag{11}$$

where:

Q_s – static pressure exerted on the wheels by the weight of the trolley and of the hoisted material.

The expression (10) assumes its maximal possible value when $cospt = cos\varphi = -1$ for the time $t = \frac{\pi}{p}$. This case becomes possible since the frequencies p are not multiples of frequency γ but are associated with the nature of the crane's operations. Recalling these constraints, after the necessary transformations Eq (10) becomes:

$$\varphi_{\max}=\frac{P_0}{Q_1+Q_2}\cdot\frac{Q_1\cdot h_1+Q_2\cdot h_2}{\frac{kl^2}{2}+Q_1h_1+Q_2h_1}\left(2\pm\frac{3}{4}\frac{1-\frac{h_1}{h_2}}{1+\frac{Q_1h_1}{Q_2h_1}}\cdot\frac{\frac{V_0}{\sqrt{h_0}}}{\sqrt{1+\frac{Q_2}{Q_1}}}\right) \tag{12}$$

When the weight is not being hoisted or dropped during the acceleration or braking phase, $V_0 = 0$, then:

$$\varphi_{0\max}=\frac{2P_0}{Q_1+Q_2}\cdot\frac{Q_1h_1+Q_2h_2}{\frac{kl^2}{2}+Q_1h_1+Q_2h_2} \tag{13}$$

The influence of the hoisting velocity on the maximal loading of the trolley's riding wheels is expressed by the ratio φ_{max} to $\varphi_{0\max}$.

$$\delta=\frac{|\varphi_{\max}|}{|\varphi_{0\max}|}=1\pm\frac{3}{8}\frac{1-\frac{h_1}{h_2}}{1+\frac{Q_1\cdot h_1}{Q_2\cdot h_2}}\cdot\frac{1}{\sqrt{1+\frac{Q_2}{Q_1}}}\cdot\frac{V_0}{\sqrt{h_0}} \tag{14}$$

4. Summing-up

The dynamic analysis covering the two phases in the crane's duty cycle allows for finding the maximal loading of the crane in the two stages of its operation and for establishing how this loading should vary in time. That will become the starting point for the design of a crane beam using the fatigue endurance methods.

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