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A BRIEF HISTORY OF METAL CASTING FROM THE CREATIVE FIRE TO THE SCIENCE OF HIGH-TEMPERATURE WETTABILITY

This essay offers an interdisciplinary synthesis of the historical and modern foundations of metal casting, linking its mythological origins with contemporary surface physics and hydrodynamics. It proposes a new conceptual framework in which wettability is treated as a dynamic, temperature- and composition-dependent boundary condition for the Navier-Stokes equations rather than a static property. The limitations of the classical Young's equation under high-temperature, reactive, and transient foundry conditions are highlighted, and an extended formulation incorporating vertical interfacial forces and a capillary contact-line term is introduced. Dimensionless numbers governing thin-film flow, mold filling, and defect formation are discussed to emphasize the hydrodynamic relevance of dynamic wettability. The essay argues that future advances in metal casting science should integrate coupled flow-surface energy models with predictive tools such as numerical simulation, additive manufacturing, and AI-based optimization, offering a unified perspective on liquid metal behavior at solid-gas interfaces.

Keywords: Metal casting history; high-temperature wettability; Young's equation; Navier-Stokes coupling; liquid metal engineering

Motto:

*How many roads we've walked as one,
How many paths our feet have run,
Hard hours faced in places wide,
Yet still the will to rise with pride,
Talk again – and reach the goal in time...*

*Dear Paweł, a foundryman by profession and a faithful friend – may this dedication,
and the pages that follow, be a modest yet sincere expression of my friendship, gratitude, and
deep respect, as well as an appreciation of your life's journey...*

Introduction (or rather a sentimental journey back in time)

Since the dawn of time, humankind has lived both in the glow of fire – and in its shadow. Fire was the first element that not only warmed and protected, but also became a teacher: stern, unpredictable, yet remarkably generous. In its light were born courage, curiosity, and the ability to cross boundaries that no one had previously been able even to name. In ancient myths, fire is always both a gift and a curse. Prometheus steals it from

the gods and pays an eternal price for his defiance. Hephaestus, combining strength with precision, uses fire to forge weapons and magnificent, almost magical artifacts [1-5].

The biblical figure of Tubal-Cain – the first metallurgist and blacksmith, the founder of “all tools of bronze and iron”^{a)} –

^{a)} In the biblical tradition, the blacksmith's craft encompassed the entire spectrum of metallurgical work – from smelting and casting to forging and plastic forming. Thus, the biblical “blacksmith” was in fact both a foundryman and a metalworker in the broader, fully integrated sense of the profession [Wertime 1964; Muhly 1988; Encyclopaedia Judaica].

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brings fire into the workshop, transforming it into an instrument of creation. In our Slavic tradition, fire has its own guardian: Svarog (Fig. 1). It was Svarog, the divine blacksmith, who, according to ancient legends, forged the sun itself, taught humans the craft of working metal, and entrusted them with the secret of fire. In this god, elemental power, craftsmanship, the sacred, and the courage to create converge into a single, timeless archetype.



Fig. 1. Artistic depiction of Svarog, the Slavic god of blacksmithing, fire, and cosmic order. His cult is attested in both Western and Eastern Slavic traditions. Source: <https://blog.slowianskibestiarusz.pl/bogowie/swarog/>

All these stories – Greek, Biblical, Slavic – tell us one thing: fire is the beginning of transformation. Yet fire alone is not enough, for it needs matter – something that allows the flame to leave its mark. And this is precisely where metal enters the stage: mysterious, ambiguous, hidden in the earth like a secret waiting to be revealed.

Gold, silver, copper, iron-rich meteorites falling from the sky – all of these were signs to early humans that the world concealed a deeper structure waiting to be uncovered. The first encounter between fire and metal may have been accidental, but accident soon became experience, and experience became knowledge. Humanity realized that it could not only observe the world, but also shape it – that matter could be given a form it had never possessed before. That something could be created which nature itself would never have fashioned. Thus, metal casting – an art, a technique, and a cultural act – was born [6].

Metal casting was never merely a method of making tools. It was a story about humanity's relationship with matter – about the desire to leave a mark more enduring than words, gestures, or memory. A casting is always a testament: a testament to skill, courage, and imagination. It captures the moment when fire,

metal, and human intention meet at a single point – and create something that will endure for centuries.

This is why the history of metal casting is so much more than just the history of technology. It is the history of humanity – of its dreams, fears, ambitions, and discoveries. It is the story of our ability to see the world not only as it is, but also as it could become.

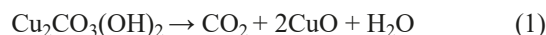
And this is where our story begins.

The birth of metal casting: From chance to technology

Before man became a metallurgist, he was an observer. He watched the world with a keenness that today seems almost a forgotten art. He saw lightning split a tree, lava shaping itself as it cooled, and the earth revealing lumps of metal unlike anything known. These were the first lessons – unwritten, unnamed, yet remembered.

It all began by chance. With the moment when someone threw a stone that was not a stone into a fire. With the moment when malachite – green and inconspicuous – began to change color, soften, and melt. At that time, no one knew the words “oxide reduction,” “melting point,” or “reaction kinetics.”

And yet the process continued, and man watched and learned [6]. He was like the Mesopotamian potter who suddenly noticed droplets of liquid metal on a pot in the kiln – on a decorative pattern previously applied with a paste containing malachite. This thermal decomposition of malachite, according to the reaction (at temperature range $\geq 300\text{--}350^\circ\text{C}$):



This reaction led to the formation of copper oxide ($300\text{--}600^\circ\text{C}$), which – in the presence of carbon or carbon monoxide, especially in the oxygen poor atmosphere of a kiln with limited air access – was reduced to pure copper ($\geq 900^\circ\text{C}$):



Finally, for pure copper to flow, a temperature above 1085°C is required. According to this scheme, the same technological pathway could also have originated from peat pyrolysis – where the extraction of tar or pitch produced an unintended but significant effect: the birth of metal from minerals and ores, from chemical compounds in their most primordial form [7].

It was at this moment that the conscious transformation of matter was born, and man understood that fire not only destroys but also creates. That it can draw from stone something previously hidden. That metal is not merely a gift from the gods, but the result of a process that can be repeated – mastered.

The first metallurgist had no furnaces, crucibles, or molds. He had only fire, intuition, and curiosity. Yet this was enough to begin the path that would lead to the emergence of civilization. Over time, chance gave way to experience, experience to

knowledge, and knowledge to technology. Man began building primitive furnaces in which the temperature was higher and more stable. He learned that some ores “flow” more readily during transformation, while others remain more stubborn. That mixing certain minerals changes the properties of the metal. That a mold – even the simplest one made of clay – allows one to give shape to something that, moments earlier, had been as fluid as water.

Thus, the first conscious casting was born. It was by no means perfect. It was crooked, porous, full of flaws. But it existed – and that was enough for man to understand that he could create things nature itself could never have fashioned. From that moment on, metal casting became not only a technique but also an act of courage. Every casting carried a risk. Would the metal flow or refuse to flow? Would the mold endure or collapse? Would fire prove an ally or an adversary?

And yet man kept trying. And with each attempt, he became increasingly aware that this art was not merely the proper use of tools. It was something more. It was about understanding matter – its whims, its limits, its possibilities. And it was precisely at this moment that the true history of metal casting began. Not in the Bronze Age, not in great civilizations, not in monumental castings, but in the very instant when man first saw that metal could flow – and that it could flow for him.

Ancient metal casting – art, technology, audacity

When man mastered the first techniques of smelting and casting metal, it became a language of culture. A way of expressing power, faith, memory, and ambition. And antiquity was the era in which this language resonated most fully – the

moment when metal casting ceased to be a craft and became a monumental art.

Bronze – an alloy of copper and tin – was to the ancients what steel is to modern times: a material that changed everything. It was hard enough, durable, corrosion resistant, and perfectly suited for casting. In Mesopotamia, it gave rise to the monumental figure of Queen Napirasu; in Egypt, it became a sacred material; in Greece, it reached a level of perfection that remains a benchmark to this day (Fig. 2). Each of these castings was a bold manifestation of the technical capabilities of its era.

In the foundry community, there is one artifact that holds almost symbolic status: a cast stand with a frog. For decades, it was considered the oldest known metal casting in the world. Small, seemingly simple, yet technically brilliant, it testifies to the fact that even very early societies knew how to prepare a mold, control temperature, conduct the casting process with precision, and achieve high-quality, repeatable results (Fig. 3(a)). The casting of a frog is to foundry workers what a fibula is to archaeologists: a small object that reveals more about an era than many a monumental sculpture.

It was the discovery of the Nahal Mishmar treasure that pushed the boundaries even further back in time. Over 400 copper and bronze objects – technical masterpieces dating from the 6th-4th millennia BCE – were found in a cave in the Judean desert (Fig. 3(b)). Among them were scepters, maces, crowns, intricate cult objects, and items whose functions remain elusive. These were not ordinary castings; they were masterpieces of craftsmanship, created by the lost-wax process and of a complexity that astonishes even modern metallurgists. The Nahal Mishmar find demonstrated that advanced casting techniques existed much earlier than previously believed.

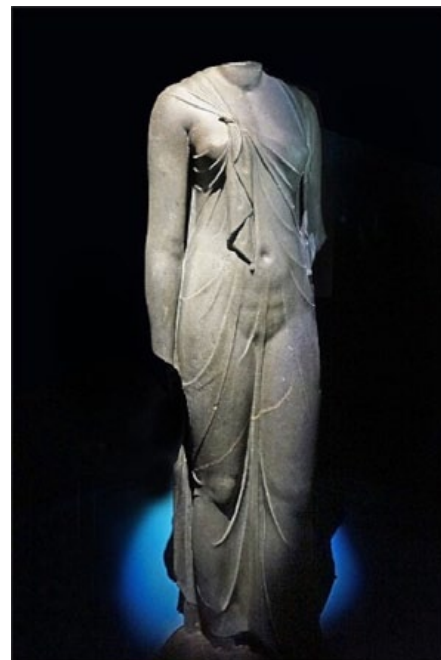


Fig. 2. Considered one of the most beautiful bronze artifacts of prehistory, the statue of Queen Napirasu, wife of Untash-Napirisha, an Elamite king of the 12th century BCE, found in Susa, Iran, c. 1250 BCE (a); and a statue of Arsinoe II, wife of Ptolemy II, depicted as Aphrodite and wearing a diaphanous robe, c. 300 BCE (b).

Sources: <https://www.meisterdrucke.pl/wydruki>, https://commons.wikimedia.org/wiki/File:GrandioriteStatue_ofArsinoe_II_from_Canopus.jpg

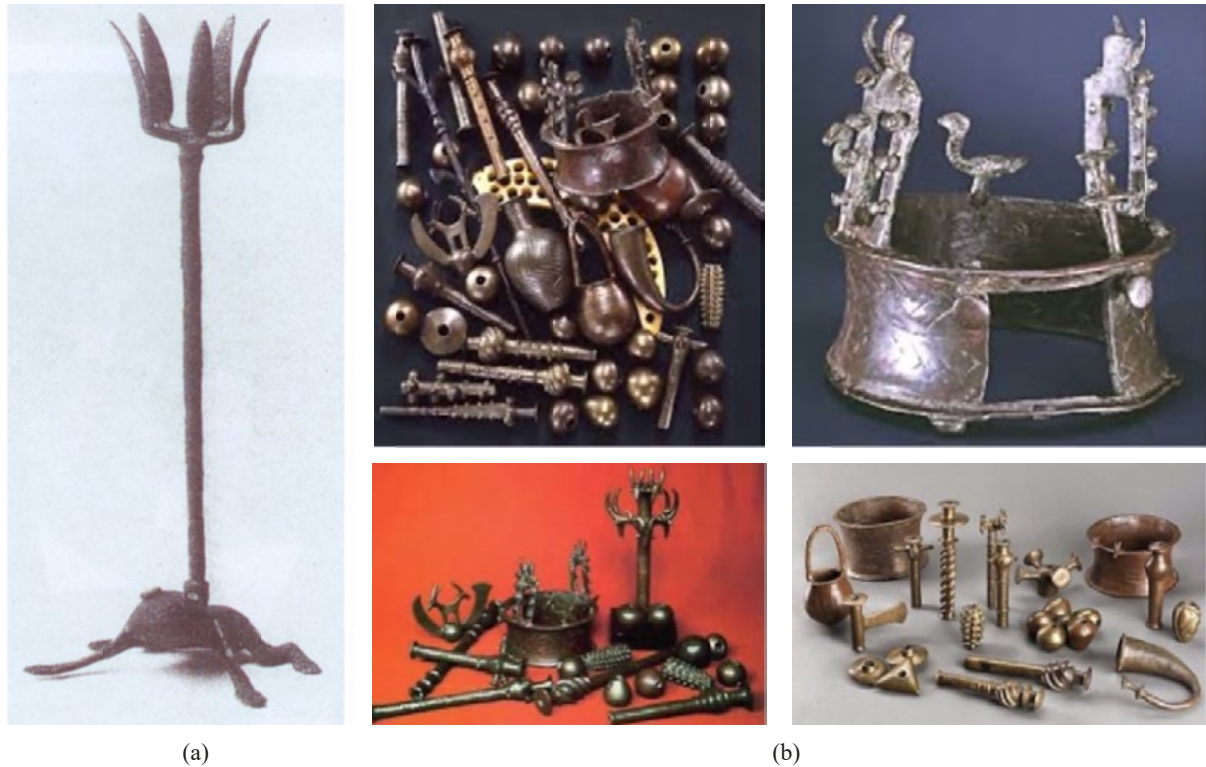


Fig. 3. “Cast Copper Frog,” a cast stand produced by the investment-casting process in Mesopotamia around 3200 BCE (a); and what are likely the oldest preserved castings in the world, from the Nahal Mishmar hoard, dated to approximately 6000-4000 BCE (b).
Sources: private collection; <https://oldeuropeanculture.blogspot.com/2020/05/nahal-mishmar-hoard.html>

The ancients were not afraid of size or scale, and although their ambitions often exceeded their tools, they were able to turn intentions and dreams into reality. The Colossus of Rhodes, the Jupiter of Taranto, the gigantic bronze statues – these



Fig. 4. Artistic depiction of the Colossus of Rhodes, a monumental statue of the Greek sun god Helios standing at the entrance to the harbor. Considered one of the Seven Wonders of the Ancient World, the statue was designed and crafted in bronze and iron by Chares of Lindos between 294 and 282 BCE.
<https://www.facebook.com/AncientGreeksHistory/photos/>

were works that required not only knowledge but also audacity (Fig. 4). It was in antiquity that foundry tools and methods were refined: disposable and reusable molds, the lost-wax technique (today known as the investment process), systems of ingates, and ceramic cores of complex geometry. And in this way, metal casting was slowly becoming a science – even though no one yet used that word.

Every ancient casting is a technological, cultural, and symbolic document: it reveals how people understood the world, the gods, and themselves. Antiquity laid the foundation upon which the entire history of metal casting would later rise.

Chronology of metal casting development – from the Stone Age to the Middle Ages

The history of the development of the art of making castings did not follow a straight line. It was full of leaps, breaks, returns, and breakthroughs; each era brought something new, yet each also carried echoes of earlier discoveries. It is within this chronology that we see how humans learned to master metal – slowly, patiently, sometimes by accident, sometimes with bravado. In the Stone Age, humans did not yet know metal, but they knew fire, clay, and processes that would later prove crucial to the foundry craft.

Bronze – a metal we now classify as an alloy – appeared simultaneously in several regions of the world: in Mesopotamia, Anatolia, Egypt, and China. And everywhere it changed

everything. The Bronze Age was the first moment in history when metal casting became a conscious technology rather than a sequence of incidental events. “But the mastery of bronze was not the end of the story. With the advent of the Iron Age, metal casting entered a new chapter – more demanding, more complex, and ultimately transformative.”

Iron was a material far more demanding than bronze, requiring higher temperatures and more advanced furnaces. But once humans learned to smelt it, the world changed a second time. The bloomery produced sponge iron, which was then forged and could be remelted. This was not yet iron casting, but it was a necessary step forward – and the true breakthrough came in China, where furnaces capable of melting iron, processing it in its liquid state, and casting it into molds were invented. This was the beginning of cast iron, a material that would dominate the following millennia. Chinese metallurgy was several centuries ahead of Europe; it was there that the first cupola furnaces, the first iron castings, and the first techniques for refining cast iron into steel were developed (Fig. 5).

The Chinese cast not only tools and weapons, but also structural elements, water pipes, and even machine parts. Their technology was so advanced that Europe did not catch up until the late Middle Ages (Fig. 6). In Europe, the development of metallurgy was slower but steady. Blast furnaces appeared, cannon casting expanded, and the first attempts at process standardization emerged. Foundry work became a political and military instrument: every cannon was proof of a state’s technical might, and every casting a testament to the growing understanding of metals. It was during this era that the foundations were laid upon which the Industrial Revolution would later rise.

Modern art of metal casting and the birth of industry

As the world entered the modern era, metal casting was no longer a mysterious art of druids or a craft passed down from father to son. It was becoming an orderly, described, analyzed, and ultimately standardized field of knowledge. This

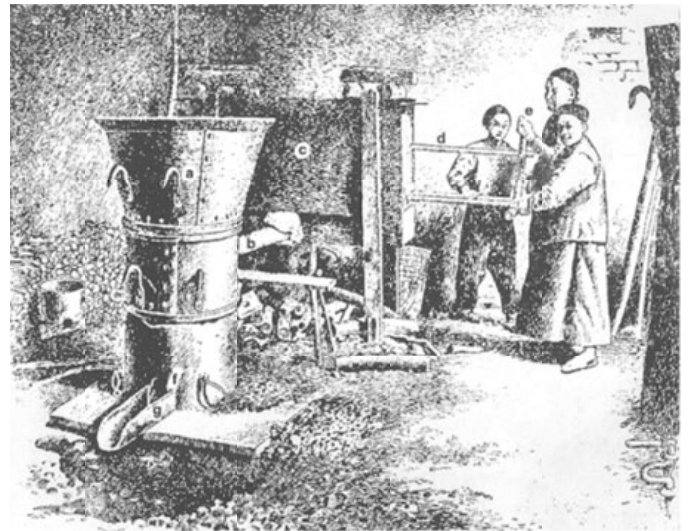


Fig. 5. Sketch of a Chinese cupola furnace from the Warring States period (403-221 BCE). The Chinese cupola was technologically advanced for its time: it featured a tilting upper section (a), nozzles (b) and bellows (c), a dual air-injection system with controllable airflow (d, e), and a lower section (f) equipped with a molten-metal discharge spout (g) [7]

was a time when experience began to be recorded, and practice was expressed in the language of observation, measurement, and experimentation. It was also an era in which metal ceased to be merely a tool or a symbol. It became the foundation of a civilization that advanced faster than ever before.

The invention of mass printing was one of the greatest breakthroughs in human history – and it was made possible thanks to the art of metal casting. The typefaces created by the German goldsmith Johannes Gutenberg were small, precise castings that had to be identical in form, durable, abrasion resistant, and perfectly repeatable. This was the first moment when metal casting became a mass-production technology. It was no longer a matter of making a single casting, but of producing thousands of identical components – a challenge that demanded new precision, new work organization, and a new way of thinking about the entire process.

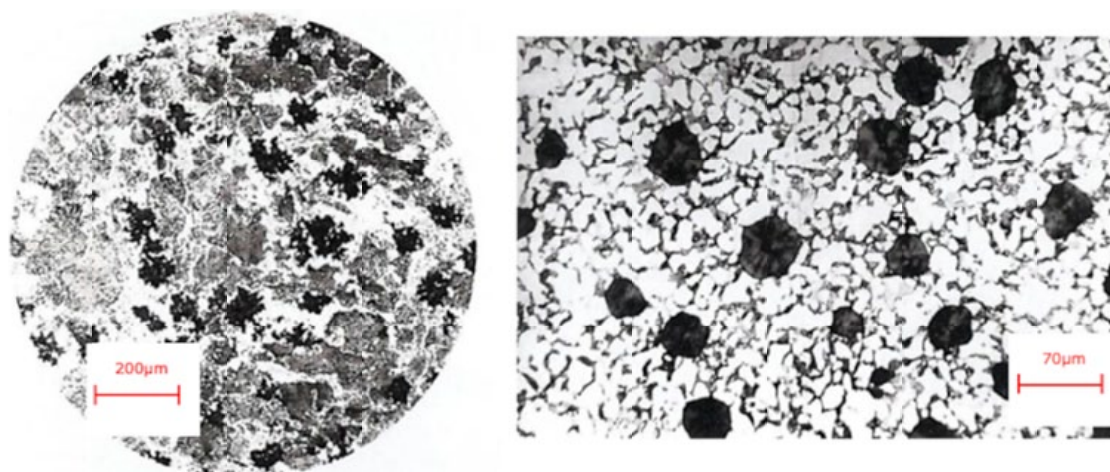


Fig. 6. Structure of black malleable cast iron (left) and spheroidal graphite cast iron (right), alloys already produced in ancient China around 500 BCE. Scales are approximate

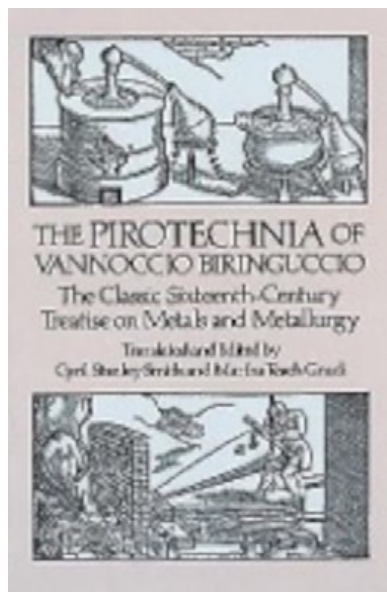


Fig. 7. Photographs of the first foundry monograph, *De la Pirotechnia*, written by Vannoccio Biringuccio in 1540
[https://de.wikipedia.org/wiki/Vannoccio_Biringuccio#/media/Datei:Pirotechnia_del_signor_Vannuccio_Biringuccio,_venezia_1558_\(poi_per_gioseffo_longhi,_bologna_1658\)_43.jpg](https://de.wikipedia.org/wiki/Vannoccio_Biringuccio#/media/Datei:Pirotechnia_del_signor_Vannuccio_Biringuccio,_venezia_1558_(poi_per_gioseffo_longhi,_bologna_1658)_43.jpg)

Vannoccio Biringuccio, the Italian metallurgist and author of the landmark work *De la Pirotechnia*, was the first to provide a systematic description of metallurgy, including foundry work (Fig. 7). His treatise was not a collection of secrets or mysterious operations, but a knowledge-based craft. He introduced technical language, process descriptions, error analysis, and the earliest attempts at standardizing metallurgical practice. His work bridged the gap between the medieval workshop and modern science.

In the 17th and 18th centuries, metal became an everyday material [8]. Iron castings began to appear on farms, in workshops, and in growing cities. Grills, pots, cooktops, structural elements, and the first water pipes emerged. Metal casting started to shape the very spaces in which people lived. The second half of the 18th century – and then the beginning of the 19th – brought a revolution that changed the world forever. Blast furnaces with unprecedented efficiency, Bessemer converters, open-hearth furnaces, the first temperature measurements of liquid metal, and the earliest attempts to control chemical composition all appeared. Cast iron became a construction material, steel (cast steel) a strategic one, and metal casting grew into an industry that shaped cities, bridges, machines, and railways. This was the moment when metal ceased to be merely a human tool. It became the structure of the world humans were building.

With the Industrial Revolution, metal casting underwent a transformation unlike anything seen before: melting and pouring furnaces grew larger, casting molds became more precise, manufacturing processes more repeatable, and knowledge more widely accessible [9]. It was no longer the art of a few masters. It had become an industry employing thousands of people and producing millions of tons of metal products each year. And yet – despite this immense scale – something of the old magic remained, for the moment when liquid metal fills the mold has always commanded awe. It still does to this day...

Modern metal casting – between tradition and technology

The modern art of metal casting is like a great river carrying the memories of all eras. It flows forward, but its current still reflects bloomeries, cupolas, open-hearth furnaces, clay molds, and wax patterns used thousands of years ago. It is one of the few fields of technology where continuity is so clear and, at the same time, change so profound. It is true that today we use metals already known to the ancients, but we use them in the way the ancients could not have even imagined.

In the 20th century, metal casting ceased to be a craft guided primarily by experience and became an engineering science. Mathematical models of metal-casting processes, computer simulations, sub-percent precision in controlling chemical composition, temperature measurements of previously unimaginable accuracy, and the analysis of microstructure and solidification kinetics all emerged.

Metal ceased to be a “fickle matter” and became predictable – a material that could be described by equations, models, and algorithms.

The greatest revolution in recent decades has been the introduction of computer simulations. They have made it possible to predict casting defects, optimize gating systems, analyze liquid-metal flow inside the mold, observe solidification in real time, and even design molds with precision down to tenths of a millimeter. What was once the domain of a master’s intuition has become the domain of calculation. Yet intuition has not disappeared – it has simply gained a new partner.

The introduction of CAD/CAM systems (Computer-Aided Design/ Computer-Aided Manufacturing) in the late 20th century revolutionized the way molds, patterns, and casting geometries were designed. For the first time, foundry engineers could create,

modify, and optimize shapes digitally, long before the first mold was made. CAD/CAM became a bridge between traditional craftsmanship and the emerging world of digital manufacturing. The development of automated molding lines transformed foundries into highly efficient production systems, with the most renowned example being the DISAMATIC lines. Processes that once required teams of skilled workers – sand preparation, mold compaction, core placement, mold closing – became mechanized and later fully automated. This shift enabled mass production of castings with unprecedented repeatability and dimensional accuracy.

Pressure die casting also became one of the defining technologies of modern foundries. It enabled the fully automated production of thin-walled, high-precision components made of aluminum, magnesium, and zinc alloys, with cycle times measured in seconds and dimensional tolerances unattainable in traditional sand casting.

Modern metal casting is no longer based solely on cast iron, steel, and bronze. It now includes high-strength aluminum and magnesium alloys, nickel- and cobalt-based superalloys, titanium

alloys, alloys with controlled microstructures (including High Entropy Alloys), materials engineered for specific properties (such as Functionally Graded Structures), and cast metal matrix composite materials [10]. Each of these materials requires a different approach – a different mold, a different temperature, a different pouring speed. This is no longer “universal” metal casting; it has become *strictly* precision metal casting (Figs. 8, 9).

Intricate precision castings feature highly complex geometries, openwork structures, and topology-optimized forms. They are most often produced from special grades of cast iron (including ADI – Austempered Ductile Iron) and cast steel (industrial components), as well as aluminum, magnesium, and titanium alloys containing rare-earth elements. Composites based on these alloys are used for lightweight, airborne structures (Fig. 9a), while high-melting metal alloys serve in high-temperature applications. Each of these materials demands its own approach – its own mold, temperature regime, and pouring dynamics.

Modern foundries increasingly resemble laboratories rather than workshops. Robots now perform not only pouring, but also

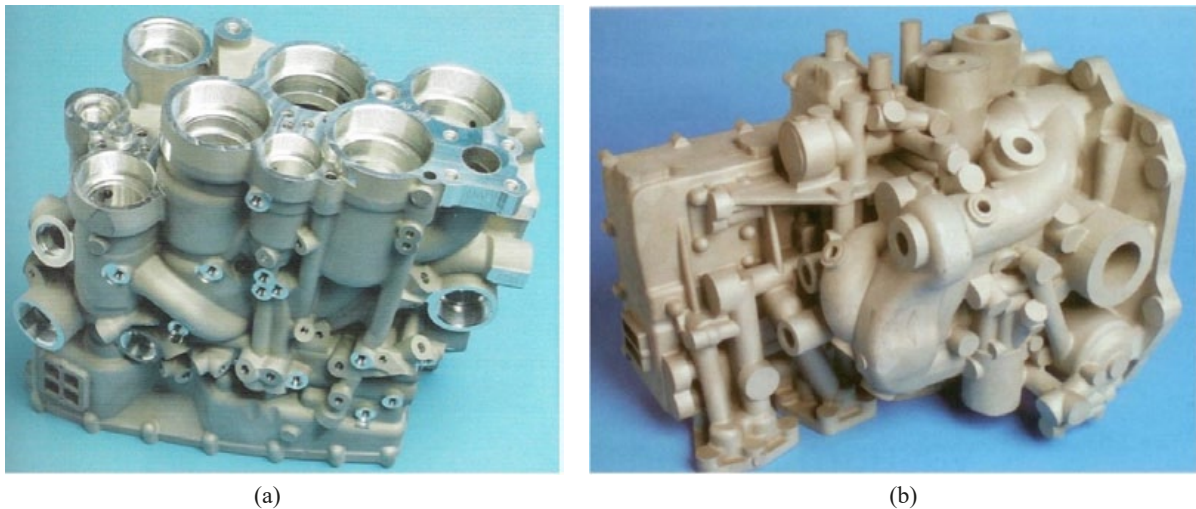


Fig. 8. Castings of a fuel-pump element measuring $35.6 \times 33.0 \times 22.9$ cm (a) and a casting of a spacecraft fuel dispenser measuring $30.5 \times 29.2 \times 40.6$ cm (b), produced by the investment-casting process from a C355 (AlCu5Si2Mg) alloy at Alcoa Howmet, Georgetown, USA

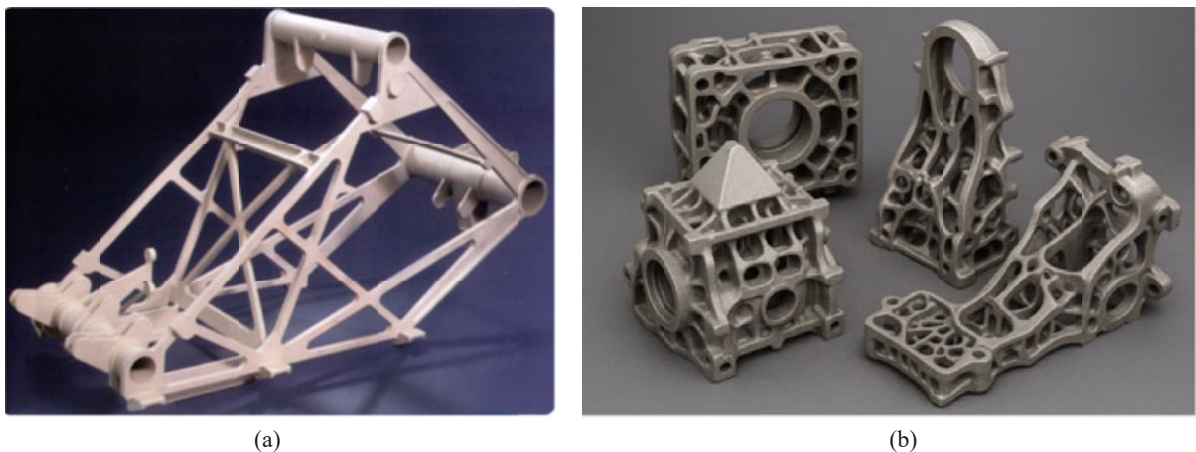


Fig. 9. A modern investment casting of a spacecraft control assembly made from an aluminum alloy at Howmet’s Bethlehem Casting, Pennsylvania, USA (dimensions $91 \times 61 \times 35$ cm) (a). Until recently, such components were manufactured by assembling several dozen separate parts. On the right, an image generated by artificial intelligence, inspired by real-world technologies, requirements, and design principles (b).

Source: private collection

knockout operations, heat treatment, machining, and even quality control. Humans have become process operators rather than physical executors. Yet despite this automation, one element still requires the human eye and human experience – understanding the essence of liquid metal.

The modern world demands a degree of responsibility like never before. Metal casting – as one of the most material- and energy-intensive industries – has become a natural element of the circular economy. Today, metal recycling is standard, molding waste is recovered, and all production emissions are monitored and reduced. Metal circulates – and this is its greatest strength. Despite all the technology, automation, simulation, and mathematics, metal casting remains a deeply human field, because it still captures the same moment that has accompanied humanity for thousands of years: the moment when liquid metal fills the mold. It is the point where fire, matter, technology, and human presence converge. And although we understand this process better than ever before, it still commands respect. There is still something of the ancient magic in it.

Metal casting as a heritage and as a challenge for the future

Metal casting is one of the oldest techniques created by humans. It has accompanied us from the moment we first saw that metal could flow, all the way to modern laboratories where artificial-intelligence algorithms predict every detail of solidification. Yet despite this vast span of time, technology, and culture, one thing remains constant: metal casting is a story about humanity. Every casting – from a simple bronze frog, through the scepters of Nahal Mishmar, to modern aircraft-turbine components – is a testament to the era in which it was created. A casting enshrines technology, culture, beliefs, ambitions, fears, and dreams. A casting does not lie; it cannot be corrected after the fact. It is a record of the moment the decision to create it was made.

In the 21st century, humans have returned to a scale the ancients knew all too well – this time using not only the temperature field of the mold and casting, but also a second state parameter: external pressure. Gigacastings, or megacastings – elements

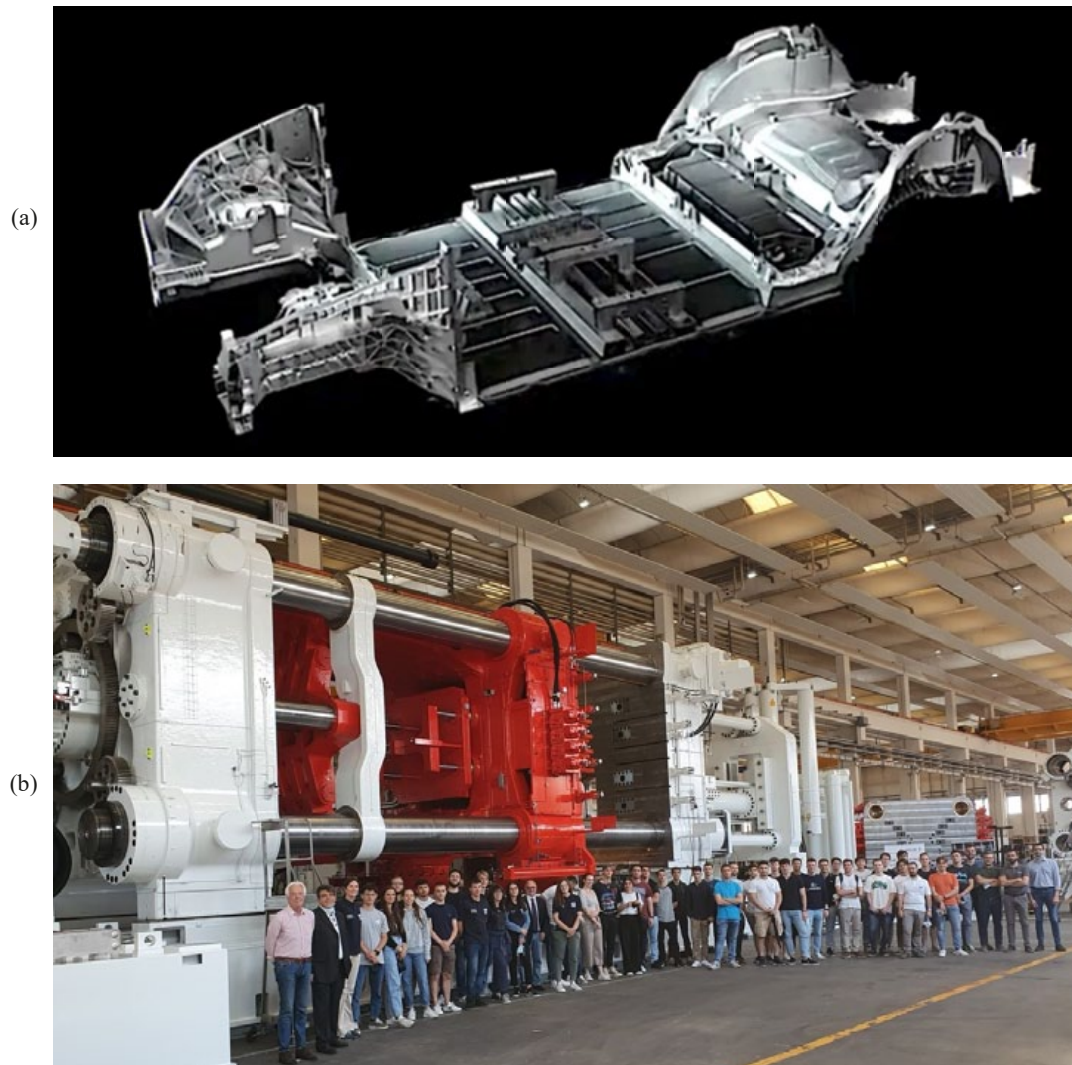


Fig. 10. A megacasting of the chassis for the Tesla Model 3 electric car, manufactured by die casting from an aluminum alloy (a). Production of this component on a Giga Press die-casting machine with a clamping force of 6,000 tons from the Idra Group (Brescia, Italy) (b) made it possible to replace approximately 70 separate parts with a single casting.

<https://www.designnews.com/automotive-engineering/tesla-s-switch-to-giga-press-die-castings-for-model-3-eliminates-370-parts>

weighing tens of tons and exceeding the dimensional limits of traditional molds – have become symbols of modern technological audacity. These are primarily light-metal castings intended for the aerospace, automotive, and energy industries, but also for heavy industry, defense applications, critical infrastructure, and emerging technologies (Fig. 10).

Manufacturing heavy iron megacastings using “traditional” sand-casting technology has also become an everyday foundry practice. One striking example is one of the longest ductile-iron castings ever produced – 23.5 meters in length and weighing 120 tons (Fig. 11). It was made in a sand mold with walls up to 400 mm thick, a mold that required three weeks of preparation and incorporated more than 30 cores. Filling the mold at a temperature of 1350°C required 140 tons of molten iron, and the casting was knocked out only after two weeks, when its temperature had dropped to 300°C.

Gigacastings are the modern equivalent of the Colossus of Rhodes – the clearest proof that humans still strive to push boundaries and turn the impossible into the possible.

Artificial intelligence (AI), as the new language of the foundry, is no longer an auxiliary tool. It has become a new way of thinking about the process itself. Today, AI algorithms predict defects before a mold is created, optimize gating systems, analyze metal flow in real time, design microstructures for specific properties, learn from mistakes faster than humans, and support the design of gigacastings with minimal margins

of error. This is a revolution comparable to the invention of the lost-wax technique. It transforms not only the tools we use, but also the very way we think about metal.

And yet, despite the fact that technology has changed beyond recognition, the core of foundry work remains the same. The tasks once performed by a Mesopotamian founder are performed today by a melting-furnace operator – although now he is accompanied by a simulation screen and a predictive algorithm. The modern metallurgist is not just an engineer; he or she is the heir to a tradition that spans thousands of years, for every casting – from the smallest detail to the largest gigacasting – is part of the same story. It carries the weight of responsibility, but it also gives a sense of participating in something larger and profoundly important – a history that is still being written.

The phenomenon of high-temperature wettability, equations of motion, and the future of metal casting

Modern foundry engineering stands at the intersection of two worlds: the world of matter, which obeys the laws of physics, and the world of humans, who attempt to understand, describe, and tame this matter. Yet there are still areas where the foundryman’s intuition outshines mathematics. One of these is high-temperature wettability – a seemingly simple, yet fundamentally important phenomenon.



Fig. 11. Giant iron casting of a two-piece machine bed manufactured at Siempelkamp Giesserei GmbH (Krefeld, Germany): sand-mold preparation process (a) and the finished casting ready for transport (b).

Source: Siempelkamp Giesserei GmbH (2016), Casting Technology and Plant

High-temperature wettability is the language in which liquid metal speaks to the solid state – it is the first dialogue between the liquid metal and the mold. It is not simply a matter of wetting angle, surface tension, or force balance. It determines whether the metal will penetrate the smallest recesses of the mold, whether it will faithfully reproduce detail, and whether it will avoid the formation of gas bubbles and shrinkage porosity. Wettability determines the smoothness of the casting surface and the character of its microstructure – whether it develops in a columnar/dendritic or equiaxed form. It also governs the effectiveness of composite-material synthesis, the quality of soldered and welded joints, and, crucially, the reliability of additive layer manufacturing, ensuring that 3D-printed metal components achieve their intended properties.

It all starts with Young's equation: the classical expression describing the equilibrium of interfacial tensions (surface energies) in a solid-liquid-gas system at their three-phase contact line (often referred to as the "triple point"):

$$\cos \theta = \frac{\sigma_{sg} - \sigma_{ls}}{\sigma_{lg}} \quad (3)$$

where θ is the contact angle (the angle formed by the tangent to the surface of a liquid droplet resting on a solid surface), σ_{sg} is the interfacial tension at the solid-gas interface, σ_{ls} is the interfacial tension at the solid-liquid interface, and σ_{lg} is the interfacial tension (surface tension) at the liquid-gas interface.

This equation is beautiful in its simplicity, but at the same time fundamentally incomplete, because it describes a state of equilibrium, whereas metal casting is a dynamic process. In its most critical stage, the filling of the mold, the liquid metal is never still; it moves, flows, accelerates and decelerates under the influence of pressure, gravity, surface forces, and Marangoni-driven motion arising from thermocapillary (temperature-induced), solutocapillary (concentration-induced), and electrocapillary (electrically driven) effects.

Young's equation describes the state of an idealized system, including a solid surface that is rigid, perfectly smooth, chemically homogeneous, insoluble, and exhibits no contact-angle hysteresis. Under real-world conditions, modified formulations are used instead, such as the Wenzel equation (applicable to rough but chemically homogeneous surfaces) [11] or the Cassie-Baxter equation (for rough and chemically heterogeneous surfaces, often superhydrophobic, with contact angles exceeding 150°) [12].

The work of adhesion of a liquid on a solid surface, W_{adh} , which quantifies the attractive forces between dissimilar substances, is described by the Dupré equation [13]:

$$W_{adh} = \sigma_{sg} + \sigma_{lg} + \sigma_{ls} \quad (4)$$

Combining equations (3) and (4) yields the Young-Dupré equation:

$$W_{adh} = \sigma_{lg}(1 + \cos \theta) \quad (5)$$

According to this relationship, the work of adhesion can be determined knowing only the surface tension of the liquid and the contact angle. When a liquid spreads completely over a solid surface ($\theta = 0^\circ$), reaches the condition of complete wetting,

and W_{adh} attains its maximum value, equal to the work of cohesion of the liquid. In this limiting case, the liquid molecules attract the solid surface with the same strength as they attract each other.

It is very intriguing that Eq. (3), commonly attributed to Young, was never stated *expressis verbis* by him. In 1805, he described the concept of force balance and the phenomenon of wettability in his seminal text *An Essay on the Cohesion of Fluids* [14]. Young presented the relationship not as a mathematical formula, but verbally, in prose. The mathematical form we now refer to as Young's equation was introduced by other scientists in subsequent years. In 1806, only a year after Young's publication, Pierre-Simon Laplace developed the mathematical theory of capillarity, formalizing the description that Young had provided only qualitatively [15]. In 1830, Carl Friedrich Gauss unified the contributions of Young and Laplace, deriving the differential equation and boundary conditions and applying the principle of virtual work, thereby establishing a full mathematical foundation for capillary phenomena and the contact angle [16]. Thus, Thomas Young was the originator of the concept, but it was Laplace and Gauss (and later others, such as Franz Ernst Neumann) who provided its mathematical formalization [17,18].

Let us allow our thoughts to return to the early nineteenth century [Young, 1805] and, with a touch of immodesty, indulge in our own reflections on the phenomena taking place in a system composed of a liquid droplet resting on a solid substrate and enveloped by a gaseous phase (Fig. 12).

To visualize the forces acting at the interface, let us conventionally represent the interfacial tensions (surface energies) in the system as vectors, fully acknowledging that these quantities are, in fact, scalars. Such vectors illustrate the direction of the forces that tend to minimize the contact area between the phases. At the three-phase contact line (point), where all phases meet, this vector representation helps us understand how these forces interact.

For a wettable system ($\theta < 90^\circ$) (Fig. 12a), projecting these vectors onto the x-axis gives the following relationship:

$$\sigma_{ls} + \sigma_{lg} \cos \theta = \sigma_{sg} \quad (6)$$

which, after minor transformation, yields:

$$\cos \theta = \frac{\sigma_{sg} - \sigma_{ls}}{\sigma_{lg}} \quad (7)$$

For the non-wettable system ($\theta > 90^\circ$) (Fig. 12b), the projection of vectors onto the x-axis allows for the following sequence:

$$\sigma_{ls} = \sigma_{lg} \cos(180 - \theta) + \sigma_{sg} \quad (8)$$

$$\sigma_{ls} = -\sigma_{lg} \cos \theta + \sigma_{sg} \quad (9)$$

$$\sigma_{ls} = \sigma_{sg} - \sigma_{lg} \cos \theta \quad (10)$$

$$\sigma_{lg} \cos \theta = \sigma_{sg} - \sigma_{ls} \quad (11)$$

Thus, once again:

$$\cos \theta = \frac{\sigma_{sg} - \sigma_{ls}}{\sigma_{lg}} \quad (12)$$

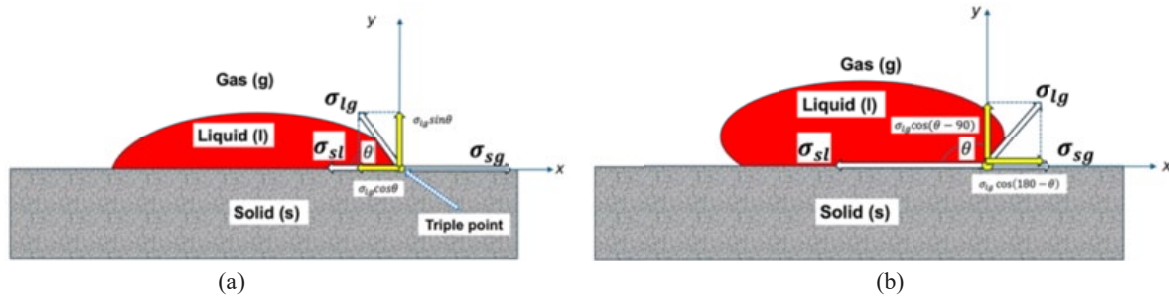


Fig. 12. Graphical interpretation of the energy balance at the equilibrium line of three phases (triple line): solid (s), liquid (l), and gas (g), shown for a sessile drop of a wetting liquid ($\theta < 90^\circ$) (a) and a non-wetting liquid ($\theta > 90^\circ$) on a solid substrate (b)

In both cases – Eqs. (7) and (12) – we obtain the well-known form of Young’s equation.

What happens when the vectors are projected onto the ordinate axis (Fig. 13)?

The diagram above shows that, without introducing the additional vector σ_{nj} , full mechanical equilibrium in the analyzed system cannot be achieved. This issue was first clearly articulated by Tony Tomsia and Brian Cantor in their 2001 publication with Enrique Saiz [19]. As in the present essay, they pointed out the neglected vertical component of the forces (the component perpendicular to the solid surface) in the standard, “horizontal” Young’s equation. The authors argued that the classical Young’s equation accounts only for the balance of forces acting parallel to the surface. In real systems – and especially in reactive systems – vertical forces (such as those arising from meniscus formation, upward or downward bending of the liquid surface, or long-range interactions) play a crucial role in determining the stability of the system and the shape of the droplet.

Therefore, it can be argued that the standard Young’s equation is insufficient, and that analyzing the full force balance – both horizontal and vertical – is essential for a precise description

of the phenomenon, especially in real reactive systems. Hence, it becomes necessary to consider the complete vector force balance, including the vertical component, represented here by the σ_{nj} vector. This is not the only argument. Young’s equation, by virtue of its origin, applies to static conditions and idealized systems. Under high-temperature conditions – in a furnace, crucible, or foundry mold; in reactive atmospheres; and in the presence of oxides, sulfides, or nitrides – the equation requires reinterpretation. Wettability becomes a dynamic phenomenon, dependent on time, chemical reactions, and surface chemistry and microstructure. The contact angle is no longer determined solely by the interplay of surface tensions in the liquid metal-solid substrate-gas system; it also reflects the history of the solid surface – its activation, roughness, and composition. An extended, modified form of Young’s equation should therefore account for the effects of temperature, surface reactivity, and wetting kinetics, including a correction term describing the time- and temperature-dependent evolution of surface tension, as well as a parameter characterizing the degree of surface reactivity.

Such an approach would make it possible not only to describe wettability – and therefore interfacial interactions – un-

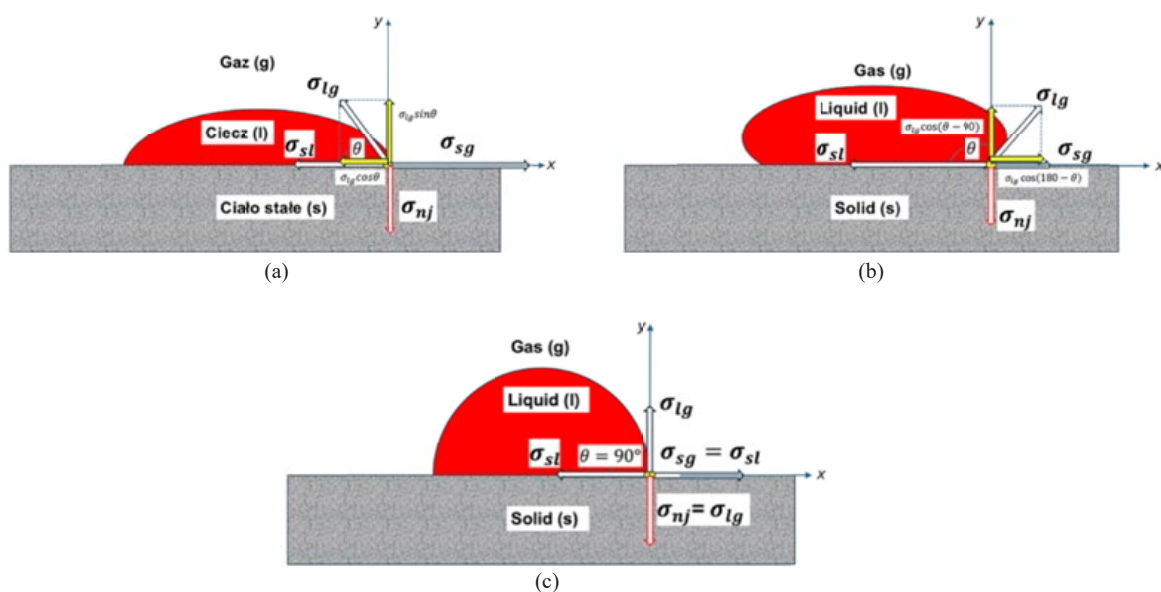


Fig. 13. Projections of vectors in the liquid-solid-gas system illustrating the vertical component of the energy balance: wetting system (a), non-wetting system (b), and boundary system (c). In the boundary system, the magnitude of the vector σ_{lg} equals the magnitude of the vector σ_{lg} in the liquid-gas system

der extreme conditions, but also to predict its evolution. This capability is particularly important in precision casting, in additive layer manufacturing (3D printing) of metals and alloys, and in levitation-based processes.

Wetting, although seemingly a surface phenomenon, affects the very essence of contact – between phases, between materials with different microstructures, and between the world of liquids and the world of solids surrounded by a gaseous atmosphere. In foundry engineering, especially under high-temperature conditions, wettability determines whether the liquid metal is able to “recognize” the mold as friendly or reject it. It is not only a matter of contact angle but also of energy, time, reactivity, and surface history. Moreover, the presence of the gaseous phase in wetting phenomena – particularly at elevated temperatures – is not only important but often decisive [20, 21].

In the classical approach to Young’s equation, the gaseous phase is treated as a passive background – the medium in which the liquid contacts the solid substrate. However, in the real world of foundry engineering, often at temperatures in the range of 1000–1600°C, gas is not the background. It is an active participant in the process, affecting:

- the composition of the solid surface – through oxidation, nitridation, and sulfurization processes;
- the composition of the liquid metal surface – through surface reactions, adsorption, and evaporation/deposition phenomena;
- the surface tension – by altering its values in both time and space; and, finally,
- the contact angle – through dynamic chemical and physical changes, i.e., the intrinsic reactivity of the system driven by the activity of the gaseous phase itself.

Therefore, it is not unreasonable to claim that wetting behavior is a fundamental component of modern liquid-metal engineering. Let us look at a coherent picture in which Young’s equation formulates the boundary condition for surface energy, and the Navier-Stokes equations describe the driving force behind the movement of the wetting front and the filling of the mold with liquid metal.

By defining the balance of surface forces at the three-phase contact line, the contact angle θ determines the ability of the liquid metal to spread along the mold wall. Low values of θ promote capillary flow and the formation of a stable liquid metal film, whereas high values of θ enhance beading (droplet formation), which may lead not only to film discontinuity but even to interruption of the liquid metal flow and the risk of incomplete pouring, i.e., incomplete mold filling. Young’s equation does not “solve the flow”; rather, it establishes a local equilibrium state at the phase interface – in other words, it provides a boundary condition for the dynamic flow problem.

Let us now focus on one of the Navier-Stokes equations, specifically the momentum equation, which defines the dynamic condition governing the motion of the wetting front as the liquid metal advances along the mold surface. In this formulation, the tangential and normal stress components are balanced by the Laplace curvature pressure (the Laplace condition) [15] and

by surface-tension-driven forces (the so-called Marangoni effect [22]):

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g} \quad (13)$$

where:

- ρ – density of the liquid metal, g/cm³;
- $\mathbf{u}$ – velocity vector of the liquid metal, m/s;
- ∇ – gradient operator, 1/m;
- $\nabla \mathbf{u}$ – velocity gradient (rate-of-strain tensor), 1/s;
- $\frac{\partial \mathbf{u}}{\partial t}$ – local (unsteady) acceleration, m/s²;
- $\mathbf{u} \cdot \nabla \mathbf{u}$ – convection, i.e., the change in velocity associated with the motion of the liquid metal, m/s²;
- p – pressure in the liquid metal, Pa;
- μ – dynamic viscosity (drag in the metal fluid), Pa · s;
- $\nabla^2 \mathbf{u}$ – momentum diffusion – velocity Laplacian (dissipation of velocity by viscosity), 1/s · m;
- $\mathbf{g}$ – gravitational acceleration vector, m/s².

For a complete description of the wetting behavior and the flow of liquid metal in a casting mold, dimensionless criteria must also be considered – namely the capillary number (Ca) and the Bond number (Bo) [23], which evaluate the influence of gravity relative to surface tension in large castings (gigacastings poured under gravity without applied pressure). In addition, the Weber (We) and Reynolds (Re) numbers are essential for assessing the likelihood of film breakage, liquid metal splashing, and the transition from laminar flow (with a continuous film) to turbulent flow, particularly in thin mold channels where heat exchange is more intense [24–27].

In a broader perspective, these dimensionless numbers provide a coherent framework for identifying the dominant forces in the system, thereby enabling reliable predictions of liquid metal flow behavior and wettability under various casting conditions and mold geometries. These considerations translate directly into practical foundry phenomena, which can be organized as follows:

1. Spontaneous infiltration of liquid metal into channels within molding sand, described by the imbibition rate in the pores of the mold or core. This process depends on quantities such as σ , θ and μ , as expressed by the Washburn equation [28].
2. Young’s equation defines the “driving force” of wetting, whereas the Navier-Stokes equations describe both viscous drag (the classical internal-friction effect slowing the motion of the flow/wetting front) and geometric drag (additional resistance arising from the shape and dimensions of the channel or mold, i.e., from flow geometry).
3. Formation of surface defects in castings (misruns, cold shuts, non-metallic inclusions) caused by elevated θ values or strong gradients $\sigma(T, c)$, where c denotes alloy-component concentration. These conditions increase the risk of wetting-film disruption. The Navier-Stokes model with capillary boundary conditions enables prediction of local pressure and velocity minima when the advancing liquid metal front becomes inactive.

4. Flow in thin channels and veins, where the capillary number often approaches unity. Under such conditions, subtle variations in σ due to temperature changes or oxidation processes (e.g., formation of a continuous Al_2O_3 layer in aluminum alloys) significantly modulate the velocity of the contact surface (liquid metal jet).
5. Microstructure formation and the adhesion of metal to cores, ceramic coatings, or molds depend on wetting, which controls the thickness and continuity of the liquid-metal film on the substrate. This directly influences heat transfer and local directional solidification. In such cases, the Navier-Stokes equation, coupled with thermal boundary conditions involving conduction and convection, defines a fully coupled solidification problem.
6. Synthesis of metal-matrix composites, particularly dispersion-reinforced systems (e.g., $\text{Al}/\text{Al}_2\text{O}_3$, Al/SiC , Al/C , or Cu/C), requires a detailed understanding of interactions at the interface between the ceramic reinforcement and the metallic matrix. Comprehensive wetting characterization is essential, as it governs both the progression and the overall effectiveness of composite formation. This encompasses the preparation of the reinforcing phase, the selection of processing parameters for the molten matrix and any wetting-enhancing additives, as well as the assessment of how these factors shape the evolution of the composite during fabrication.

Together, these phenomena demonstrate that wettability and liquid metal flow constitute a unified foundation of modern foundry engineering, where fluid mechanics, surface thermodynamics, and interfacial chemistry converge into a single coherent description of processes occurring within the mold.

Summing up, Young's equation provides the energetic boundary condition governing the contact between a liquid metal and a substrate, while the Navier-Stokes equations describe how this contact evolves in time and space under the influence of viscosity, pressure, and capillary forces. In foundry engineering, where the liquid metal film must remain continuous and stable, this coupling is not an "additional" consideration – on the contrary, it is fundamental to planning pouring rates, temperatures, the choice of coatings for molds and cores, and the control of the ambient atmosphere. Wetting becomes a measurable lever of the process, and the Navier-Stokes equation system serves as its dynamic language.

Is it possible to combine both fundamental equations, namely Young's equation and the Navier-Stokes equation? The following expression represents one such attempt:

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g} + \sigma_{lg} (\cos \theta - \cos \theta_a) \delta_C \mathbf{t} \quad (14)$$

where, in addition to the equilibrium contact angle θ , the actual, instantaneous dynamic contact angle θ_a appears, together with new quantities characterizing the motion of the contact line between the liquid and its surroundings: the vector $\mathbf{t}$ (which is parallel to the solid surface when the contact line moves

along it) and δ_C (Dirac distribution [29,30]), acting as a kind of "pin" that localizes the force precisely at the contact lined physically signifies: "the force acts here". Together, these terms allow the capillary contribution to be embedded directly into the momentum balance, ensuring that the motion of the three-phase contact line is represented in a physically consistent and mathematically localized manner.

The Navier-Stokes equation, supplemented with the term $\sigma_{lg}(\cos \theta - \cos \theta_a) \delta_C \mathbf{t}$ "remembers" the Young's equation: the difference between the actual angle θ_a and the equilibrium angle θ appears as a capillary force localized on the contact line („capillary drive“), which couples the wetting geometry with the flow of the liquid metal. If the actual angle θ_a is larger or smaller than Young's angle θ , a force appears that "pulls" the contact line toward the equilibrium state. This is the physical mechanism of dynamic wetting, and it is truly beautiful in showing that the classical Young's equation is not merely a static relation – it also governs motion.

High-temperature wettability is not only a technical parameter but may also be the key to the future of foundry engineering. It determines the quality, durability, and aesthetic value of a casting, while at the same time becoming a metaphor for the human relationship with nature. It lies at the heart of the philosophy of liquid metal engineering – a field that brings together science, craftsmanship, and reflection on the limits of what is possible.

In conclusion, wettability is not a static phenomenon but a dynamic one, governed by local flow conditions. Consequently, the dynamic Young's angle, referred to above as the actual angle θ_a , should be regarded as a function of velocity, pressure, and the local surface tensions within the liquid-metal-solid-gas system. This angle varies in time, depending not only on the temperatures of the metal and the mold, viscosity, and surface tension, but also on the velocity of the liquid metal jet and the intensity of local turbulence.

In recent years, another breakthrough has emerged in foundry engineering with the advent of additive layer manufacturing (3D printing). This technology does not replace traditional methods but opens entirely new possibilities – above all in the domain of mold geometry. Printed patterns, molds, and cores make it possible to create structures that were previously unattainable or economically unjustified. Moreover, additive technology enables control over surface roughness and microstructure, thereby directly influencing wettability, local flow conditions, and, consequently, the solidification process of the casting. In this sense, 3D printing becomes a natural partner for both Young's equation and the Navier-Stokes equations, as it allows the mold to be designed in such a way that boundary conditions are not imposed by the material but consciously shaped by the engineer. This is not merely a new technique – it is a new way of thinking about casting: from geometry, through the dynamics of liquid metal flow, to the microstructure of the finished product.

The future of metal casting science will depend on dynamic models capable of describing wetting in real time, on AI-assisted prediction of local variations in Young's angle, and on fully integrating the Navier-Stokes equations with surface-energy

formulations within the casting-mold system. The foundryman of the future will be not only an operator but, above all, an interpreter of observed and measured phenomena. Metal casting of the future is about surface physics and chemistry, hydrodynamics, thermodynamics, process simulations, artificial intelligence, and – above all – people capable of bringing all these elements together. And this may well be the most ambitious task of modern liquid metal engineering...

The motto used in this study is a paraphrased excerpt from Konstanty Ildefons Gałczyński's poem "Saved from Oblivion."

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